Maxwell's Demon and the Second Law of Thermodynamics

P Radhakrishnamurty

Dr Radhakrishnamurthy obtained his PhD from the Indian Institute of Science, Bangalore, after which he worked as a scientist at the Central Electrochemical Research Institute, Karaikudi. He is now retired and enjoys reading and writing on topics in science. His present research interests include classical thermodynamics and elastic collision theory.

A century and a half ago, Maxwell introduced a thought experiment to illustrate the difficulties which arise in trying to apply the second law of thermodynamics at the level of molecules. He imagined an entity (since personified as 'Mawell's demon') which could sort molecules into hot and cold and thereby create differences of temperature spontaneously, apparently violating the second law. This topic has fascinated physicists and has generated much discussion and many new ideas, which this article goes over. Particularly interesting is the insight given by deeper analysis of the experiment. This brings out the relation between entropy and information, and the role of irreversibility in computing as we understand it – topics still under discussion today.

Introduction

The science of thermodynamics is nearly 200 years old. It took birth in the works of Carnot around the year 1824 and grew into a subject of great significance. Just four simple laws – the zeroth law, the first law, the second law and the third law – govern thermodynamics. The first and the second laws are the bones and the flesh of thermodynamics, and by comparison the zeroth and the third laws are mere hat and slippers, in the language of Sheehan. Thermodynamics derives its strength and power because of two very important factors. One, its independence from the concept of time – time has no role to play in settling issues of thermodynamics – and the second, its independence from the nature of the system under study – whether the system is solid, liquid or gas, whether it consists of molecules or not, etc. Yet another important aspect is that, it gives results of unparalleled reliability in simple terms of 'yes' or 'no'. It decrees whether a given process under prescribed conditions, is or is not possible, in principle[1].

[1] It does not say anything about the process being possible in practice. The distinction is exemplified by the simple example: Theromodynamics says the combination of hydrogen and oxygen to give water under conditions of room termperature and pressure is a spontaneous process, that is, a process possible in principle; however, as we know very well we don't find a drop of water after keeping pure hydrogen and oxygen in contact in a vessel with smooth (polished) walls, at room temperature and pressure even for extended periods of time.

With the onset of atomic theories of matter, Newtonian dynamics was applied to the study of the behavior and properties of matter. Kinetic theory of gases was developed and Maxwell's theory of velocity distribution of molecules in a gas was established. Maxwell and Boltzmann applied statistics and probability theory to study the behavior of gases in such works. Those works gave birth to a new branch of research 'Statistical Thermodynamics', making the erstwhile thermodynamics – 'the classical thermodynamics'. The concept of entropy assumed greater significance than it had in classical thermodynamics. It even got a new definition: $S = k \ln (W)$, where W is the number of microstates available to a system corresponding to a given macrostate, and k is the Boltzmann constant. It also brought with it a lot of problems such as Gibb's paradox. Statistical thermodynamics predicts the final state a system reaches from a given initial state under given conditions with a specific maximum value (which is necessarily less but as close as one wishes to 1) for the probability; classical thermodynamics gives the final state with 100 percent probability – certainty. Today, students have less scope to be exposed to the classical thermodynamics – it being less complex –and therefore considered as of lower significance!

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Keywords
Maxwell's demon, second law of thermodynamics, irreversibility.

Entropy from information theory entered the scene as well, during 1930s. Information processing in computers, logical manipulation of information, the concept of logical irreversibility, etc., have entered the scene during 1960s. The boundaries of thermodynamics thus expanded into many modern areas of research. Thermodynamics became more of statistical thermodynamics and less of classical thermodynamics.

The Second Law

Of the four laws, the second law arouses the greatest interest. Since its original appearance in Carnot's memoir, the principle was always formulated as a completely general statement, as free of exceptions as Newton's laws of motion or his law of universal gravitation. Its standing among natural laws is such that, aside perhaps from the standard conservation laws, no other physical

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axiom engenders more support from the scientific community. To wit, it grew beyond the reach of serious scientific discussion.

In the words of Arthur Eddington: "*The second law of thermodynamics holds, I think, the supreme position among the laws of Nature. If someone points to you that your pet theory of the universe is in disagreement with Maxwell's equations - then so much the worse for Maxwell's equations. If it is found to be contradicted by observation, well, these experiments do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation*". It got similar endorsements in the works of many a great scientist – Einstein, Max Planck, Maxwell to name a few.

Challenges to Second Law

The second law assumed, in the words of Sheehan, 'a mystic faith'. It is believed to be inviolable. However, the second law is understood deeply by few and taken on faith by most. Surprisingly, despite the deep-rooted faith in its absolute status, it is suspected to have shallow roots. Its validity has been challenged almost from its inception. The challenges resumed greater impetus with the developments in other fields – information technology, computers and computations, low temperature quantum systems, biotechnology, nanotechnology, etc. Ironically, one of the foremost among the challenges was from Maxwell!

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What Maxwell challenged really was the universal inviolability of the second law. Maxwell associated kinetics with the treatment of mutual action between bodies rather than with the treatment of motion in its purely geometrical aspect. He distinguished the dynamical method from the statistical method. The former was concerned with the behavior of entities, which could be observed individually, whereas the latter was used for investigation of entities collectively. He made it clear that while the equations of dynamics as applied to matter gave results with 100 percent certainty, their application implied a perfect knowledge of all the data (positions and momenta). In cases where the individual particles were unobservable, all the data could not be known and the equations of dynamics could not be applied and so the statistical method was appropriate. Thus in the study of gases, "The smallest portion of matter which we can subject to experiment consists of millions of molecules, not one of which ever becomes individually sensible to us and statistical method has to be used. In this method our attention is fixed not on the bodies themselves, but on the *number* belonging at any instance to one particular group". Maxwell considered that the certainty provided by the two methods was different – the dynamical method gave 'absolute certainty' but despite the unvarying regularities and stability revealed by the statistical method, it is possible that in a particular case a very different event might occur than that expected from the regularity of the averages.

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Birth of Maxwell's Demon

A significant illustration, which Maxwell gave of the distinction between individual molecules and an immense number of molecules, was a thought experiment. The conception, which appeared in print in Maxwell's book, *The Theory of Heat*, (1871), was first formulated in a letter to his schoolmate and friend, Tait in 1867. We quote it in Maxwell's words:

"... if we conceive of a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are as essentially finite as our own, would be able to do what is impossible to us. For we have seen that molecules in a vessel full of air at uniform temperature are moving with velocities by no means uniform, though the mean velocity of any great number of them, arbitrarily selected, is almost exactly uniform. Now let us suppose that such a vessel is divided into two portions, A and B, by a division in which there is a small hole, and that a being, who can see the individual molecules, opens and closes this hole, so as to allow only the swifter molecules to pass from A to B, and only the slower molecules to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and

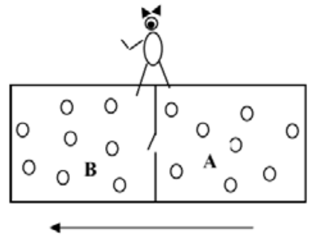

Figure 1. Maxwell's Demon.

lower that of A, in contradiction to the second law of thermodynamics."

The purpose of this illustration was to show that the second law applied only to a system of large numbers of molecules and could be violated by individual molecules. The second law of thermodynamics thus had only a statistical certainty.

Maxwell's challenge took the strange form of, what W Thomson (later knighted Lord Kelvin) subsequently (1874) nicknamed, 'Maxwell's intelligent demon' – a quaint character indeed! Thomson apparently did not envisage the creature as malicious. The being's subversive effects on the order of natural things gave it the name demon. Thus was born the 'Maxwell's Demon'.

Maxwell's conception is depicted in *Figure* 1. The demon played a dual role – on one side it appeared to ring the death knell to the second law and on the other end it helped in firmly establishing the statistical nature of thermodynamics. Because of this dual nature the demon continued to attract the attention of serious thinkers and has been debated for the past 150 years or more.

In Maxwell's view, the demon did not undermine Clausius' axiom: a process whose only final result is transfer of heat from a body at a given temperature to another body at a higher temperature is impossible; or in terms of entropy: entropy of an isolated system (say, universe) tends to a maximum, as a basis for the second law.

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Maxwell asserted that the second law of thermodynamics was not the type of law that could be reduced ultimately to dynamics, but it expressed instead the statistical regularity of systems composed of unimaginably large numbers of molecules. Maxwell was insisting on the statistical character of the second law at a time when Clausius and Boltzmann were trying to show that it was a strictly mechanical theorem.

Maxwell indicated that he did not wish to engage in physical speculations as to the mode of action of such a being, and for him the notion merely expressed the distinction between laws applicable at the micro and macro level. As he emphasized in his letter to Tait, his aim had been to show that the second law had only a statistical certainty and his illustration served that purpose. Later he elaborated on this in his book, *Theory of Heat*, in a section, 'Limitations of the second law of thermodynamics'. It became one of the most heavily quoted passages in physics. It reads:

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One of the best established facts in thermodynamics is that it is impossible in a system enclosed in an envelope which permits neither change of volume nor passage of heat, and in which both temperature and pressure are everywhere the same, to produce any inequality of temperature or pressure without expenditure of work. This is the second law of thermodynamics and it is undoubtedly true as long as we can deal with bodies only in mass, and have no power of perceiving or handling the separate molecules of which they are made up. But if we can conceive of a being whose faculties are so sharpened that he can follow every molecule.. would be able to do what is at present impossible to us. Later, in a letter to Rayleigh, Maxwell remarks –"I do not see why even intelligence might not be dispensed with and the thing made self acting".

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One should not infer from the above that Maxwell himself was seriously intent on breaking the second law; in fact, he was strongly convinced of its statistical truth as is evident from the statement in a letter he wrote to J W Strutt in Dec. 1870, "the second law of thermodynamics has the same degree of truth as the statement that if you throw a tumblerful of water into the sea, you can not get the same tumblerful of water out again".

Growth of the Demon – I Phase

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the past more than a century and a quarter, the puzzle of whether a Maxwell's demon can or cannot violate the second law has been studied and debated. Its story can conveniently be divided into three stages for ease of understanding.

(1) From its birth in 1867–1929; (2) 1929–1961; (3) 1961 onwards. H S Leff and A F Rex give a detailed account of the story and valuable bibliography on Maxwell's demon.

Loschmidt, earlier to Maxwell, proposed a thought experiment to show that, though Clausius' statement that 'it is impossible for heat to pass from a colder to a hotter body without an equivalent compensation' was admittedly supported by ordinary experience, it was not true for all conceivable cases. The experiment consisted of so fixing the conditions that fast moving molecules striking a partition that separated the gas from another portion not containing any gas, be allowed to go through the partition and the slow moving ones stopped will bring about a temperature difference between the two portions. It is therefore not theoretically impossible, without the expenditure of work or other compensation, to bring a gas from a lower to a higher temperature, or even to increase its density.

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Meanwhile, Boltzmann who pioneered the idea of using probability theory came up with his famous *H* -theorem to account for the increase of entropy associated with irreversible processes. Using probability theory was a new idea in 19th century physics, thermodynamics included, since those subjects were regarded as describing processes that had no fundamental uncertainty. Connecting probability theory with the highly controversial idea of the existence of atoms made it even more susceptible to criticism from many leading scientists of the time. Interestingly, even Max Planck was initially an opponent of atomic theory! He changed his views only in 1900 when he himself succeeded in explaining the black body radiation spectrum using probability theory. Boltzmann's *H*-theorem led to a great deal of controversy, especially from his friend, Johann Josef Loschmidt, and Zermeleo.

Loschmidt argued that the laws of dynamics being time-reversal invariant, an instantaneous reversal of momenta of all particles in an isolated system demands the system retrace its trajectory. So, if Boltzmann's *H*-function decreases for a given microstate of a system, the value of *H* for the time-reversed system must increase, invalidating Boltzmann's conclusions. This criticism came to be known as Loschmidt's reversibility paradox.

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Ernest Zermeleo, Max Planck's student and a prominent mathematician, came up with a different objection, after more than 20 years of Boltzmann's original paper. His argument was based on a result from Poincare, which required the behavior of any isolated system to be quasi-periodic. That is, the system must return repeatedly to points that are arbitrarily close to its original configuration. This contradicts Boltzmann's result, which asserts that a system in equilibrium will never leave it.

These criticisms revealed to Boltzmann that his attempt to find a dynamical function of molecular motion which would reflect the behavior of entropy could only lead to a dead end, for whatever the mathematical function might be, the mere reversal of velocities would also reverse the supposedly unidirectional behavior of that function. Then Boltzmann concluded that no purely dynamical proof of the second law would ever be possible and that the irreversible increase of entropy must reflect not a mechanical law, but states of differing probabilities. Furthermore, systems move towards equilibrium (not because of dynamical behavior of molecular motion, which is necessarily reversible) but, simply because the number of molecular states which correspond to equilibrium is vastly greater than the number of more ordered states of low entropy. Boltzmann thus provided the key for quantifying the statistical interpretation of the second law in terms of relative numbers of molecular states that correspond to equilibrium and non-equilibrium.

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Thus the chief end of Maxwell's creature, 'to show that the second law of thermodynamics had only a statistical certainty' became established as a cardinal principle of classical physics.

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Thus ended the first phase of the story of 62 years of quiet existence of Maxwell's demon.

Growth of the Demon – II Phase

In 1929 Leo Szilard launched the demon into the information age. The objective of his investigation was to find the conditions which apparently allow the construction of a perpetual motion machine of the second kind (a device that converts energy input in the form of heat to energy in the form of work with 100% efficiency), if one permits an intelligent being to intervene in a thermodynamic system. When such a being makes measurements, they make the system behave in a manner distinctly different from the way a mechanical system behaves when left to itself. It is not clear from his paper whether the thermodynamic cost is from measurement, remembering or forgetting. However, he stated that intervention is merely a type of coupling, or a measurement and so one need not be concerned with the intervention of living beings. It is enough if the coupling is accompenied by memory. Thus Szilard regarded *memory* as an important feature of the demon's operation.

In order that Maxwell's demon may not violate the second law, the mechanism of operation of the demon needed to dissipate energy. The general doctrine of dissipation (not destruction) of energy was first clearly stated by W Thomson in his papers on dynamical theory of heat. It has also been treated by Clausius under the name entropy.

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To account for the dissipation of energy in the workings of Maxwell's demon, Szilard proposed a thought experiment. It consisted of a heat engine – a cylinder divided by a partition/piston with a one-molecule working substance. The piston is driven by the single molecule. The cyclic process consists of 4 steps (see *Figure* 2).

Step 1. Place a partition in a cylinder (containing one molecule) dividing it into two equal chambers.

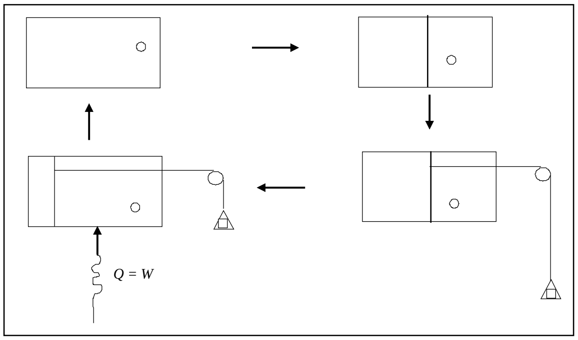

Figure 2. Szilard's engine.

Step 2. Maxwell's demon decides on which side of the partition the one-molecule fluid is on (that is, makes a measurement) and records the result.

Step 3. Partition is replaced by a piston, and the recorded result is used to couple the piston to a load upon which work W is done. The gas pressure pushes the piston to one end of the cylinder returning the gas to its initial volume. In the process the one-molecule gas has energy $Q = W$ delivered to it via heat transfer from a heat reservoir.

Step 4. The piston is decoupled from the load and removed from the cylinder, completing one cycle of operation.

If successful this cycle would allow extraction and conversion of heat from a surrounding heat reservoir into work in a cyclic process, apparently violating the second law.

Szilard correctly identified three important and previously unappreciated aspects of the demon – measurement, information and memory. Unexpectedly, the demon helped lay down the foundation of modern information theory. Szilard found that, in order, to comply with the second law, the measurement process must entail a compensatory entropy production of $S = k \ln (2)$. This *bit* of entropy and information creates a new line between the two seemingly disparate concepts, later established more solidly by Shannon and others. Over the last ninety years a lot has been written on Szilard's engine and the informational aspects of Maxwell's demon.

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Brillouin was stimulated to extend Szilard's ideas into an extensive mathematical theory connecting measurement and information. He concluded that the second law was linked to the quantum nature of light. The photons used for the measurement dissipate energy and lead to a compensatory increase of entropy saving the second law. The use of light signals for *seeing* the molecules to gather information about their location and velocity is critical to Brillouin's exorcism of Maxwell's demon.

The works of Szilard and Brillouin gained widespread acceptance: A demon must gather information in order to operate, and information acquisition is sufficiently dissipative to save the second law. Following the widespread acceptance of this reasoning it appeared that Maxwell's demon was dead.

This ended the second phase of the story of Maxwell's demon.

Growth of the Demon – III Phase

For a long time after Szilard, most researchers assumed the use of light signals to gather information on molecules. This assumption was a key element in the exorcism of Maxwell's demon in Brillouin's work. But light is not the only way to carry information and a complete solution of Maxwell's demon's puzzle ought not to depend on use of light. The next major progress toward banishing of Maxwell's demon was a side effect of research by R Landauer of IBM, on thermodynamics of data processing.

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In 1961 Rolf Landauer introduced the principle of 'logical irreversibility' in connection with information-discarding process in computers. His principle has come to be regarded as the thermodynamic principle of information processing. It states that, any logically irreversible manipulation of information, such as the erasure of a *bit* or the merging of two computation paths must be accompanied by a corresponding increase in non-information-bearing degrees of freedom of the information processing apparatus or its environment. Conversely, it is generally accepted that any logically reversible transformation of information can in principle be accomplished by an appropriate physical mechanism operating in a thermodynamically reversible fashion. The real reason Maxwell's demon could not violate the second law dawned with the onset of Landauer's principle. It is the unexpected result of a very different line of research: research on the energy requirement of computers.

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Bennett extended the work of Landauer. Drawing upon Landauer's results on dissipation in computing, he discovered the importance of memory erasure. He argued that the real reason Maxwell's demon could not violate the second law was that in order to observe a molecule, it must first forget the results of previous observation. Forgetting results or discarding information is thermodynamically costly, i.e., uses energy thereby dissipating energy and increasing entropy. According to him, if a logically irreversible operation, such as erasure is applied to random data, the operation may still be thermodynamically reversible, because it represents transfer of entropy from data to the environment, rather like reversible-transfer of entropy to the environment when an ideal gas is compressed isothermally. But if the logically irreversible operation is applied to known data, then the operation is thermodynamically irreversible because, the increase in entropy in the environment is not compensated by any decrease in entropy of the data. This wasteful situation, in which an operation that could have reduced the data's entropy is applied to known data whose entropy is already zero, is analogous to the irreversible expansion of an ideal gas into vacuum without doing any work, then compressed back isothermally to its original volume.

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Bennett proposed a clever mechanical detector that did not involve any light signal to detect the presence of molecules. In principle, the detection can be done, according to him, with an arbitrarily little work and energy dissipation. However, Leff and Rex observe that no specific device can operate with arbitrarily little work and energy dissipation for the velocity measurement. Hence they note that the possibility of measurement without entropy generation is not universally accepted.

By eliminating the requirement of light signals to gather information Bennett's argument nullified Brillouin's exorcism of Maxwell's demon.

Penrose also suggests that erasure of information is an irreversible process. It contributes to the increase of entropy compensating the decrease of entropy brought about by Maxwell's demon.

Sentient and Non-Sentient Demons

Maxwell's demon and Loschmidt's demons are sentient demons – ones which perceive (make measurements) and think (store, manipulate and erase information) to guide their molecular separations. Non-sentient demons, on the other hand would not necessarily perceive or think, but merely respond to and manipulate molecules as by natural processes.

Whiting proposed what could be considered as non-sentient demon, in the sense, that its sorting process, is replaced by a natural, automatic process (as in evaporation of a liquid – a molecule on the surface leaves the liquid and goes into vapor).

It is evident from the above that, not only the existence of Maxwell's demon is debated, but also the mechanism of entropy generation in the workings of the demon is debated with no consensus.

The story of Maxwell's demon is a continuing story and the end is not on the horizon.

Suggested Reading

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Address for Correspondence
P Radhakrishnamurty
42, Shivakrupa 1st Floor
3A Main
Manjunathanagar II Phase
Bangalore 560 010, India.
Email:
padyala1941@yahoo.com