Wading Through Batchelor's Book

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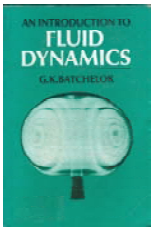

An Introduction to Fluid Dynamics
G K Batchelor
Cambridge University Press, 1967
Price: Rs.395/-

It is a real pleasure for me to review this immensely popular text on fluid dynamics. I will begin with a comment on the general character of the book, and on a few of the features that set it apart from most other books on the subject. Subsequently, I attempt a detailed chapter-wise discussion of the book contents. An effort has been made in this latter part to highlight the significance of the topics covered, to discuss the intricacies of the physical arguments presented by the author (the very arguments that make the text a must-read for a proper understanding of the subject), and further, to point out connections with more recent research when possible. It is hoped that the points brought out will guide a beginner in finding material in the book that wouldn't have been apparent except from a detailed reading. As would be the case in any review, the material that follows is, on one hand, biased by my experiences in the field, and on the other hand, limited by my inadequate knowledge of the subject.

The book is a comprehensive treatise on the dynamics of incompressible laminar flow. The first three chapters set out the basic principles and equations, this being done without the aforementioned constraint of incompressibility. At the end of Chapter 3, the author presents a careful discussion of the conditions under which one may assume a flow to be incompressible; all too often, one tends to equate incompressibility with low speeds (small Mach numbers), although this is hardly the only circumstance under which effects of compressibility become unimportant. The following two chapters discuss the various phenomena related to viscous fluid flow. The situations examined range from uni-directional flows wherein the Reynolds number (Re), at least as a measure of the relative magnitudes of the inertial and viscous forces, is irrelevant, to flows where viscosity is dominant and the Reynolds number is restricted to small values. The concluding section in Chapter 4 discusses in detail the nature of the flow past a bluff body with increasing Reynolds number. Thus, by the end of Chapter 4, the Reynolds number has become large enough that one is naturally led to the discussion of the dynamics of vorticity at the beginning of Chapter 5. The latter parts of this chapter include a detailed exposition of laminar boundary layers. Chapter 6 deals with irrotational flow theory and its applications. The reader is, however, continually reminded of the role of viscosity, and the implied relevance of irrotational flow theory only in the absence of separation. Batchelor's text was one of the few, at the time of its publication, that included a detailed description of the effects of viscosity (Chapters 4 and 5) prior to a discussion of potential flow

despite the mathematical simplicity of the latter (and the attractive analogy with the equations underlying the phenomena of electricity and magnetism). The justification is provided at the start of Chapter 6 where the author rightly emphasizes the need to understand the nature of viscous forces well enough to (hopefully) be able to determine the location of the singular streamsurfaces in the inviscid limit, surfaces where effects of viscosity remain important even as $\mathrm{Re} \to \infty$. It is these surfaces that then determine the nature and extent of the domain where the flow continues to remain approximately irrotational. Finally, Chapter 7 presents an excellent and elaborate discussion of the various aspects and subtleties of inviscid rotational flow.

The one feature of the book that stands out, and certainly one that frustrates the first-time reader no end – a seemingly undue emphasis on detailed physical (verbal) exposition; particularly, when a shorter mathematical derivation would appear to serve the purpose. The preponderance of text tends to draw the reader out of his or her comfort zone wherein the argument might be followed more easily as a series of mathematical steps. Indeed, on a casual flip through the pages of the book, one is hard-pressed to find a page where equations occupy more space than text or figures. Thus, the style adopted forces one to think and reason physically before delving into a mathematical formulation. A sense of frustration may be linked to some of the other classics on hydrodynamics too; for instance, the ones by Lamb [1] or Milne-Thomson [2]. But, the annoyance in these books is more due to the language and the rather archaic yet heavy mathematical notation[1]. (The overwhelming emphasis on physical rather than mathematical understanding throughout is, in my opinion, an admirable and a unique trait of Batchelor's text.) In a subject that entails as many mathematical intricacies as fluid mechanics does, physical insight is often an invaluable guide in wading through the equations, keeping in mind the final result. Moreover, physical rather than mathematical insight always provides one an access to a greater range of problems. Serving as an invaluable aid to a physical understanding of the phenomena discussed in the book are the series of images contained between pages 364 and 365 (in this regard, *An album of fluid motion* by Van Dyke [3]) is a valuable source of illustrations of fluid mechanical phenomena). References to these pictures, as well other experimental observations, during the course of any explanation help the reader to develop a sense of the reality. As the author admits, credit for this particular style must go to Prandtl. It is worth noting that, with its constant emphasis on the connection between theory and experimental observations, Batchelor's text also

[1] It must be said that the books by Prandtl (see, for instance, Prandtl and Tietjens [5]), although written in the same era, remain a glorious exception in this regard, and the preface in Batchelor's book indicates that the author was indeed inspired by Prandtl's expository style based on a balance of theory and observation.

tends to differ from another classic, the one by Landau and Lifschitz [4], which, although more comprehensive in its overall coverage of topics, has been written primarily with a theoretician in mind.

Aside from a mere emphasis on verbal explanation, a peculiarity is the manner in which the prose is formatted. For instance, it is often not obvious as to which sentence, or group of sentences, in a big paragraph (of which there are many) describing a fluid mechanical phenomenon is the most important. Thus, it often turns out that an equation that follows a paragraph of explanation ends up befuddling the reader merely because he or she wasn't careful enough in reading every sentence in the preceding explanation. Almost every sentence in the book is there for a reason, and the manner of writing makes the reader work hard; nevertheless, a rewarding excercise at the end of it. In fact, Batchelor's research papers are written in a similar style, and reading them is an equally rewarding experience.

The general level of exposition in the book is such that it is certainly no 'introduction' to the subject, and the contents are best appreciated by a reader reasonably well acquainted with the basics of fluid dynamics.

In the preface, the author expresses his regret in being unable to cover all of fluid dynamics in a single book. Rather interestingly, contingent on the success of the 'introductory' volume, the author also reveals an intent to make the coverage more complete by attempting to include, in a second volume, topics such as gas dynamics, surface waves, buoyancy effects, turbulence, heat and mass transfer and magneto-fluid dynamics; that a single person could cover such a wide range of topics with the singular authority displayed in the first (and only) volume does come to mind! Rather unfortunately for the fluid dynamics community, the second volume never materialized.

A chapter-wise discussion of the book follows.

Introduction to Reader's Companion

Batchelor's book is quite demanding for a first-time reader and may be so even for one familiar with fluid mechanics. The following pages include discussions that may aid a reader on several topics covered in the book. Many of the references at the end point to recent advances in the field. We have added some figures along with brief write-ups to explain some of the phenomena described in the Reader's Companion. Some of the frequently occurring unfamiliar terms are also explained.

Resonance has also featured many fluid mechanicians in earlier issues: Prandtl (December 2000), Kármán (August 2005), Lighthill (January 2009), Taylor (October 2009), Bernoulli brothers (August 2000), and Wright brothers (December 2003). Prof S P Govindaraju has also written a series of articles on Aerobasics starting from September 2008. Readers may be interested in the articles on boundary layer theory, Kármán vortex street and others, which appeared in these issues.

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