OF THE MANY BOOKS by Johannes Kepler, the *Rudolphine Tables* of 1627 is at first sight the least inspiring. A tall, slender volume, the *Tabulae Rudolphinae* is half filled with numerical tables, half with Latin explanations — scarcely exciting reading. Yet the tables are extraordinarily important, for they document in a unique way Kepler’s great contributions to astronomy.

When he became an astronomer in the closing years of the 16th century, Kepler found a science in which planetary predictions typically erred by several degrees on the sky; the legacy of the *Rudolphine Tables* was a prediction scheme nearly 50 times more accurate. For Kepler these tables were the proof of the pudding, the substantiation of his laws of planetary motion. He called them “my chief astronomical work.”

Johannes Kepler was born just 400 years ago, on December 27, 1571. Two events from his childhood left a lasting impression: in 1577 his mother showed him the great comet, and later his father let him watch a lunar eclipse. In 1589 Kepler entered the University of Tübingen, where the senate was soon to note that he had “such a superior and magnificent mind that something special may be expected of him.”

Yet Kepler himself wrote that nothing indicated to him a special bent for astronomy. Hence he was surprised and distressed when, midway through his last year as a theology student, he was summoned to Graz, far away in southern Austria, to become an astronomy teacher and the provincial mathematician.

In Graz, Kepler’s active and ever-speculative mind soon hit upon what he believed to be the secret key to the construction of the universe. He knew that there were five regular polyhedra, that is, solid figures each with faces all the same kind of regular polygon. (The cube is such a solid.) By inscribing and circumscribing these figures with spheres (all

Johannes Kepler (1571–1630), from a recently discovered painting by an unknown 17th-century artist. The original is 4.3 by 2.5 inches in size, and was acquired in 1941 by the Upper Austrian Land Museum in Linz. This portrait was one highlight of the Kepler exhibition at Linz this past summer.

nested in the proper order), he found that the positions of the spheres closely approximated the spacings of the planets. Since there are five and only five regular polyhedra, Kepler thought that he had explained the number of planets in the solar system. In 1596 he published his scheme in his Mysterium Cosmographicum, “The Cosmographic Secret.”

Before dismissing the idea as the work of a crank, we must remember the revolutionary context in which it was proposed. The Mysterium Cosmographicum was the first unabashedly Copernican treatise since De Revolutionibus itself, for without a sun-centered universe the entire rationale of the book would have collapsed.

Kepler also realized that, although in Copernicus’ system the sun was near the center, it played no physical role. But he argued that the sun’s centrality was essential, for the sun itself must supply the driving force to keep the planets in motion. This idea, which appears in the latter part of the book, establishes Kepler as the first scientist to demand physical explanations for celestial phenomena. Although the principal idea of the Mysterium Cosmographicum was erroneous, never in history has a book so wrong been so germinal in directing the future course of science.

Kepler sent copies of his remarkable book to various scholars, including the most famous astronomer of the day, Tycho Brahe. Although unwilling to accept all these strange arguments, In Weil der Stadt (near Stuttgart in southwestern Germany), where Kepler was born, a seated statue honors the great astronomer. This view of the market-place is from a color transparency taken by the author in 1966.
the Danish astronomer immediately recognized the author’s genius, and invited Kepler to visit him. However, the long journey was out of the question for the impecunious young man. Thus, wrote Kepler, “I ascribe it to Divine Providence that Tycho came to Bohemia.”

Tycho, fearing the loss of royal support by the King of Denmark, had in the meantime resolved to join the court of Rudolph II in Prague. Emperor Rudolph was a moody, eccentric man whose twin loves were the occult and his collection of curiosities. He was more than willing to support a distinguished astronomer whose accurate planetary positions could make horoscopes more accurate. Tycho arrived at Prague in 1599, and Kepler, forced out of Graz by religious controversy, joined him there the following January. To Kepler was assigned the analysis of the observations of Mars.

Kepler’s Laws in Modern Form

1. Law of the Ellipse (1609)
The orbit of each planet is an ellipse with the sun in one of its foci.

2. Law of Areas (1609)
The radius vector of each planet sweeps out equal areas in equal intervals of time.

3. Harmonic Law (1619)
The squares of the periods of revolution of the planets are proportional to the cubes of their mean distances from the sun.

(The dates are those of publication, not of discovery.)

This engraving from Kepler’s Mysterium Cosmographicum illustrates his early idea that the distances of the planets from the sun were related to the five regular polyhedra. The outer sphere, for Saturn, circumscribes a cube which circumscribes the sphere of Jupiter. Continuing inward are a tetrahedron, Mars, dodecahedron, Earth, icosahedron, Venus, and octahedron, Mercury. At lower left, this diagram is dated 1597.
The encounter between the young German theoretician and the famous Danish observer turned the course of astronomy. Otherwise, Tycho’s observations would have remained largely unexploited, and without them Kepler would never have found the true key to planetary motions. Precisely how much Kepler learned from the imperious Tycho himself can never be established. In any event, within two years Tycho had died and his full set of observations, although claimed by his heirs, fell into Kepler’s hands.

The young man’s speculative drive was tempered but not suppressed as he wrestled with the unique observational legacy. Kepler’s eagerness to publish seemed unbounded. We can see from the still-extant manuscript notes that he was already outlining the chapter headings for his *Commentary on Mars* long before he knew that its orbit was anything but a combination of circles. In fact, he had written 58 chapters of his *Astronomia Nova* in virtually their final form before he even discovered Mars moved in an ellipse. Fortunately, several other studies, including an analysis of the supernova of 1604, delayed the completion of this great book until he had fully established the elliptical orbit and law of areas for Mars. The *Astronomia Nova* was finally published in 1609. Its break with the traditional requirement of circular motions for the planets made it truly “the new astronomy.” Seldom has a book been better titled.

At Tycho’s death, Kepler received not only his observations but also his title of Imperial Mathematician. The chief duty of this post was to complete the great set of planetary tables originally envisioned by Tycho, to be based on his incomparable set of precisely determined positions and intended to provide much improved predictions. Kepler’s analysis of Mars could be considered the first step in this great undertaking.

The discovery of the first two planetary laws had created an entirely new foundation for calculating the *Rudolphine Tables*. But since Kepler had derived these laws solely from the observations of Mars, he now had to demonstrate their validity for the other planets. Mercury created special difficulties, and the complex motion of the moon caused a great deal of trouble. Among the papers dating from Kepler’s stay in Prague are hundreds of sheets of calculations that are apparently preparatory work for the *Tables*. Yet more than two decades were to pass before the work was published, and the emperor who had commissioned it was long dead.

The astronomer’s efforts to work on the tables were continually diluted by the time lost in trying to collect his salary, and by a variety of astronomical events. The most important
The second and third stages of the titled page of the Rudolphine Tables, the last exhibiting Kepler’s well-defined esthetic taste. He had neglected to show Tycho’s heirs the title page before it was printed, and they objected to wording implying that he had to correct his mentor’s data. The second printing (left) was hastily prepared at Prague, but Kepler found the typography so bad he had the third one (right) printed at Ulm, presumably at his expense. Among other things, Kepler indicates near the end of the long title that he personally owned the type for the numbers used in the tables.

of these was Galileo’s application of the telescope to the heavens, causing Kepler to take time out to write his Dissertatio cum Nuncio Sidereo – “Conversation with the Sidereal Messenger” of Galileo. Kepler’s enthusiastic reception of the Italian astronomer’s discoveries and his confirmation of the new Jovian satellites served as strong witness for the new findings. Galileo wrote to him: “I thank you because you were the first one, and practically the only one, to have complete faith in my assertions.”
In 1611 the political situation in Prague took an abrupt turn, ending Kepler’s exhilarating atmosphere of intellectual freedom. The gathering storm of the Counter-Reformation reached the capital, and brought about the abdication of Rudolph II. As warfare and bloodshed surged around him, Kepler sought refuge in Linz, where he was appointed provincial mathematician and a teacher in the district school.

The school was even smaller than the one at Graz, but his principal work was to be the *Rudolphine Tables*. The Linz authorities charged him first of all to “complete the astronomical tables in honor of the Emperor and the worshipful Austrian House, for the profit of ... the entire land as well as also for his own fame and praise.”

After Rudolph’s death in 1612, his successor Matthias confirmed Kepler as a court mathematician and agreed to his new residence away from Prague. But Kepler realized that as long as the *Rudolphine Tables* were unfinished, he would be tied to Linz. Thus the work on the tables became part of his fate.

Several other great works took form in Linz. Foremost among them were the *Epitome of Copernican Astronomy*, which outlined the theoretical basis for Kepler’s planetary tables, and the *Harmonice Mundi*, a great cosmological sequel to the *Mysterium Cosmographicum*. In 1618, Kepler discovered what has come to be called his third or harmonic law of planetary motions; it was published next year in the *Harmonice Mundi*. He continued to grapple with the difficulties of the moon’s orbit, achieving success in 1620. This theory was included in the final installments of the *Epitome*, and in the preface Kepler urged his readers to use that work until the *Rudolphine Tables* came out.
Nevertheless, from all sides impatient reminders urged Kepler onward with the tables. “Don’t sentence me completely to the treadmill of mathematical calculations – leave me time for philosophical speculations, my sole delight,” responded Kepler. But he added, “I am as eager for the publication as Germany is for peace.”

Meanwhile, another innovation completely altered Kepler’s original plan for the form of the tables – “a happy calamity,” as he called it. In 1617 Kepler first saw John Napier’s epoch-making work on logarithms and was deeply impressed by it, recognizing how this new invention would simplify the time-consuming computations of astronomy. However, not content to adopt the new aid as he found it, during the winter of 1621-22 Kepler composed his own book on the subject.

He exploited the new logarithms to solve two problems introduced for the first time by the novel form of the *Rudolphine Tables*. The first arises in the solution of what is now called Kepler’s equation. For a planet moving in an ellipse, under Kepler’s law of areas, there is no elementary way to find explicitly the position angle corresponding to a given time. However, the converse is easily calculated. Therefore he solved his equation for a set of uniformly spaced angles, which determine a set of nonuniformly spaced times. Kepler tabulated the logarithms of these intervals as a convenient means for interpolating to the desired times.

The second important use of logarithms arises from the thoroughly heliocentric nature of the book. In previous planetary tables, the motions of the sun and planets were combined into a single procedure. In the *Rudolphine Tables* we must find separately the heliocentric positions of the earth and planet in question. To find the geocentric position of the planet, these two positions must be combined – essentially a problem of vector addition. Kepler facilitated this maneuver by tabulating the logarithms of the radius vectors of earth and planet, and by providing a convenient double-entry table for combining them.

*A modern test compares predictions of the geocentric longitudes of Mars during the years 1625 to 1631, made with the Rudolphine Tables of Kepler (solid line) and the Brandenburg Ephemerides of David Origanus (dashes). The latter man, who lived from 1558 to 1628, followed the Prutenic Tables of Copernicus. The great superiority of Kepler’s predictions is obvious.*
Kepler’s tables, unlike the modern Nautical Almanac, do not show the daily positions of the sun, moon, and planets. Instead, they contain general tables from which it is possible to work out a planet’s position for any time in the past or future. Besides the planetary data, Kepler included tables of logarithms, a catalogue of 1,000 stars, and a list of the geographical longitudes and latitudes of a large number of cities. Approximately half the volume is made up of instructions for the use of the tables and numerical examples of their use.

At long last, in 1624, Kepler completed his Rudolphine Tables in their new logarithmic form. The printing of the tables was very difficult because of wartime conditions. Linz offered no really suitable press, and circumstances in the city became increasingly unpleasant. Kepler’s lodgings were located on the city wall, and eventually he was obliged to open his house for the soldiers guarding the ramparts. To a correspondent he described the noise and smells of battle, but pointed out that he found solace in continuing his calculations. Finally, however, the printing was transferred to Ulm.

Financing the book’s publication caused the astronomer much worry and trouble. A long trip to the imperial court in Vienna won some concessions for him, including an agreement to tax the city of Nürnberg for the arrears in his yearly allowance. Yet a visit to Nürnberg in 1625 was in vain, and he eventually decided to finance the work from his own pocket. Even this was not a simple operation, for Tycho’s heirs claimed both a share in the profits and also censorship rights.
His preface to the tables explains that the long delay in publication was partly from these circumstances, but also from “the novelty of my discoveries and the unexpected transfer of the whole of astronomy from fictitious circles to natural causes.” These, he said, required deep searching, and were especially difficult to explain or calculate, since no one had ever attempted anything of the kind before.

Because the *Rudolphine Tables* was in many ways the long-awaited climax to Kepler’s entire scholarly output, he proposed that this book, alone among his works, should have an appropriate frontispiece. His Tübingen friend Wilhelm Schickard prepared a wash sketch of the proposed engraving. It showed the temple of Urania, modeled after the foyer of Tycho’s observatory, Stjerneborg, on the Danish island of Hven. On the ceiling of the temple was shown the geocentric Tychonic system, and within the building stood a series of notable astronomers, including Tycho himself.

Kepler submitted the sketch to Tycho’s heirs, who at once objected. Their illustrious ancestor should be depicted more formally, they said, with his long ermine robe and the elephant medal around his neck, which he always wore while observing. (The medal was the highest award of the Danish monarchy.)

The frontispiece in its final form (reproduced on the front cover of this magazine) was a great elaboration over the trial sketch. Ten of the temple’s 12 zodiacal columns are visible; their rich variety depicts the increasing elegance of astronomy. Those at the back, merely rough-hewn logs, represent the most ancient traces of the science. Nearby stands

*Part of the frontispiece of the Rudolphine Tables is seen here at a larger scale than on the front cover of this magazine. Tycho, clad in his court costume, stands right of center, pointing upward. The panels on the foundation of the Temple of Urania illustrate the history of the book. At far left, Tycho’s heir passes the observations to Kepler, seated at his worktable. In the center is a map of the island of Hven where Tycho observed. At the right are shown the printers and typesetter in the shop of Jonas Saur at Ulm. This photograph is used by permission of the Harvard College Library.*
The original design drawn by Schickard for the frontispiece of the Rudolphine Tables was much simpler than the printed version and gave less emphasis to Tycho Brahe.

a Chaldean observer who, for want of an instrument, uses his fingers to measure the angular separations of the stars. The Greek astronomers Hipparchus and Ptolemy appear beside brick columns adorned with instruments of the times. Copernicus, seated beside an Ionian column, engages in spirited conversation with Tycho, who stands in his magnificence by an elaborate and splendid Corinthian pillar. The Danish astronomer points to the Tychonic system inscribed on the ceiling, and his Latin comment to Copernicus may be loosely translated, “How about that?”

Surrounding the dome of the temple appear six goddesses, each recalling an important idea of Kepler. To the far right stands Magnetica, with her lodestone and compass, reminding us of the magnetic forces that Kepler believed to control the planets. Next is Stathmica, goddess of the law of the lever and balance; the sun at the fulcrum reminds us that this is a form of Kepler’s law of areas. The third divinity is Geometria, with her mathematical compass, square, and tablet bearing the Keplerian ellipse. The next figure is Logarithmica, who holds in her hands rods in the ratio of one to two, while emblazoned on her halo is the natural logarithm of 1/2. The fifth goddess holds a telescope, and the sixth a globe with its shadow, reminding us of Kepler’s two books on optics: *Dioptrice* (1611) and *Astronomiae Pars Optica* (1604).

Above the temple hovers the imperial eagle, generously dropping golden coins from his beak for the support of astronomy. A few of the coins even manage to fall to the foundations of the temple where, in the left panel, we see Kepler himself, calculating by candlelight. And here is a marvelous personal touch, for prominently on the worktable sits a replica of the dome of the temple of Urania. In his subtle and uncensored fashion,
Kepler reminds us that although Tycho may have built the most splendid column, the temple of Urania would never have been finished without Kepler himself laboring far into the night!

Kepler personally oversaw the printing and worked almost daily with the typesetters. One thousand copies were printed – an enormous edition for a 17th-century science book. (Sixty years later, Newton’s *Principia* was published in an edition of about 300 copies.) As a result, the *Rudolphine Tables* is by far the most common of all Kepler’s books, though examples today generally command a price of nearly $1,000.

Kepler’s tables enabled him to predict the transit of Mercury over the disk of the sun on November 7, 1631, a phenomenon never previously observed. He did not live to see the prediction fulfilled, for he died on November 15, 1630, at Regensburg, where he had gone on yet another effort to collect his back salary. However, this transit of Mercury was actually witnessed by the astronomer Pierre Gassendi at Paris.

In a letter to Kepler’s old friend Schickard, Gassendi wrote: “But Apollo, acquainted with [Mercury’s] knavish tricks from his infancy, would not allow him to pass altogether unnoticed. To be brief, I have been more fortunate than those hunters after Mercury who have sought the cunning god in the sun. I found him out and saw him where no one else had hitherto seen him.”

In 1665, J. B. Riccioli related that the tables of Ptolemy, Copernicus, and Longomontanus each erred by about five degrees in predicting this transit, whereas Kepler missed by less than 10 minutes of arc! The overwhelming evidence of this crucial experiment convincingly established the power of the *Rudolphine Tables*. In this almost forgotten way, the geocentric world view was broken and the heliocentric system, together with Kepler’s laws of planetary motion, triumphed.