K Ramachandran
School of Physics
Madurai Kamaraj University,
Madurai 625 021, India
Email:
thirumalchandran@gmail.com

On the Measurement of Phase Difference using CROs by Lissajous Method

In electronics practical work (laboratory experiment), most of the time the phase difference between any two port network is measured by forming Lissajous figure using an oscilloscope (CRO). Such measurements do not agree with the expected values at least for some frequency regions. This is a perturbing issue for the students as well as teachers in the laboratory. By considering very simple circuits, this problem is approached and solved here. This is not reported in any of the textbooks.

1. Introduction

It is well known that the phase difference between any two port network is measured by forming Lissajous figure using an oscilloscope (CRO). Most of the time, such measurements do not compare well with the expected values, at least for some frequency regions. This is analyzed in detail here, as this is not only a pedagogical problem but also a perturbing issue for the students as well as teachers who are doing the experiments on phase shifts. Very simple circuits are considered here for experiment and analysis.

2. Experiment and Theory

To start with, a simple experiment is considered here, in the form of an RC circuit, as given in *Figure* 1.

The phase difference between input $v_{\rm i}$ and output $v_{\rm o}$ is found by feeding $v_{\rm i}$ and $v_{\rm o}$ to X and Y plates of an oscilloscope. (The input $v_{\rm i}$ is from a signal generator, 0.1Hz–150kHz, sinusoidal, 600 ohms output impedance and the measuring device, the oscilloscope, is a 'Company 1' make having a bandwidth of 150kHz and input

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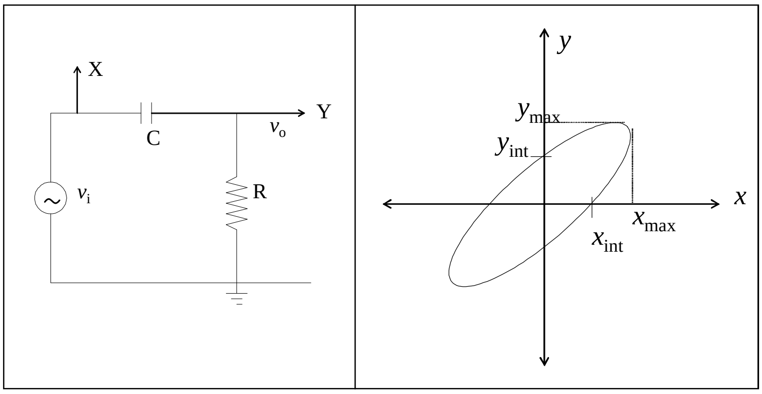
Phase difference,Llissajous figure, oscilloscope, phase offset.

Figure 1 (left). R C circuit. Figure 2 (right). Lissajous figure (ellipse).



impedance 100 kilo ohm). All measurements are only in the range of 100Hz–15000Hz. An ellipse is formed on the screen of the CRO as shown in *Figure* 2.

The phase difference ϕ is measured from this Lissajous figure (*Figure* 2) by finding out the intercepts and maxima for x and y for any frequency (denoted by the subscripts 'int' and 'max').

$$\phi = \sin^{-1}(x_{\text{int}}/x_{\text{max}}) = \sin^{-1}(y_{\text{int}}/y_{\text{max}}) \, . \tag{1}$$

(Proper care is taken in the measurements: Before forming the ellipse, the spot in the CRO is adjusted to the midpoint of the screen for no inputs; only then the ratios in (1) will be equal).

These measurements can be compared with the expected values from simple circuit analysis, which gives

$$\phi = \tan^{-1}[1/(\omega RC)] \, , \tag{2}$$

where ω is the angular frequency, R and C are the resistance and capacitance used.

All these measurements and calculations are given in *Table*1 . It is clear from this table that there is a considerable deviation in the phase difference between the measured and expected values marked as ϕ_{av} and ϕ_{ex} (Column 9 and 10 of *Table* 1).

Before going into the discussions, let us first look at another experiment, to see whether such deviation is present in other circuits. For this, another simple circuit namely 'series resonance circuit' as in *Figure* 3 is considered here.

Freq	X_{int}	X_{max}	Y_{int}	Y_{max}	ϕ_x	ϕ_y	ϕ_{av}	ϕ_{ex}	ϕ_D	ϕ_C
5	1.6	1.63	1.08	1.1	79.8	79.1	79.5	84.6	4.1	83.9
6	1.8	1.85	1.1	1.13	76.4	76.8	76.6	82.4	4.3	80.7
7	1.9	2.0	1.12	1.18	71.8	71.7	71.8	78.5	5.1	76.1
8	1.7	1.85	1.1	1.21	66.8	66.4	66.6	69.9	2.8	69.3
9	1.6	2.3	1.05	1.5	44.1	44.4	44.3	42.1	0.5	44.8
9.5	0.1	4.8	0.1	4.8	1.2	1.2	1.2	4	2	3.2
10	0.4	1.0	0.35	0.9	23.6	22.9	23.3	35.7	7.3	30.9
11	0.9	1.1	0.6	0.75	57.6	59	58.3	65.5	6.4	64.7
12	1.4	1.5	0.8	0.85	69.1	70.3	69.7	74.3	4.7	74.4

Table 1. Phase measurements using CRO along with expected values for RC circuit. (Freq in kHz; X_{int}, X_{max}, Y_{int}, Y_{max} in cm and ϕ in degrees. $\phi_{av} = (\phi_x + \phi_y)/2$; ϕ_{ex}: theoretically expected; ϕ_{av} : offset error in CRO.)

Here again the input is fed to the X plate and the output to the Y plate of the CRO. The ellipse is formed and the phase difference is measured as in the earlier case using (1).

This can be compared with the expected values

$$\phi_{ex} = tan^{-1}\{(L\omega - 1/C\omega)/(R + R_d)\} ,$$

where R_d is the DC resistance of the inductance coil.

Such measurements and calculations above and below the resonance frequency are observed and tabulated in *Table* 2. Here also there is considerable disagreement between the measured and the expected values of ϕ. (Column 9 and 10 of *Table* 2).

From the above two experiments and *Tables* 1 and 2, it is clear that the disagreement is not only for a specific circuit but for any electronic circuits. Also we find a systematic deviation of the phase difference from the theoretically expected values in both the cases.

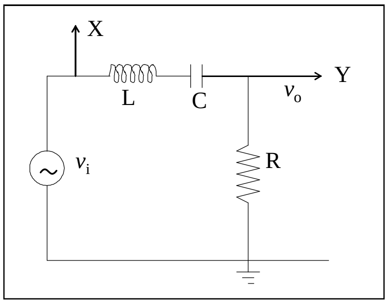


Figure 3. An LCR circuit.

Freq	X_{int}	X_{max}	Y_{int}	Y_{max}	ϕ_x	ϕ_y	ϕ_{av}	ϕ_{ex}	ϕ_D	ϕ_C
5	1.18	1.2	0.49	0.5	80.1	80.1	80.1	84.6	4.1	84.2
6	1.08	1.15	0.44	0.45	79.3	79.3	79.3	82.3	4.3	83.6
7	1.05	1.1	0.38	0.40	73.2	73.2	73.2	78.4	5.1	78.3
8	0.93	1.0	0.32	0.35	68.5	68.5	68.5	69.9	2.8	71.3
9	0.63	0.95	0.20	0.30	41.7	41.7	41.7	42.1	1.5	43.2
9.5	0.03	0.9	0.01	0.30	2.2	2.2	2.2	4.0	2.0	4.2
10	0.61	0.9	0.2	0.3	42.4	42.4	42.4	35.7	7.3	35.1
11	0.95	1.0	0.33	0.35	72.1	72.1	72.1	65.5	6.4	65.7
12	1.18	1.2	0.39	0.4	79.2	79.2	79.2	74.3	4.7	74.5

Table 2. Phase measurements using CRO along with expected values for LCR circuit. Inductance = 20.3 (1)mH, Capacitance = 0.0137(2) micro farad, Resistance = 104(1) ohms, DC resistance of the coil, R_L = 54 ohms, Resonant frequency = 9.6 KHz. (Freq in kHz; X_{int} , X_{max}, Y_{int}, Y_{max} in cm and ϕ in degrees. $\phi_{av} = (\phi_x+\phi_y)/2$; ϕ_{ex}: ϕ theoretically expected; ϕ_D: offset error in CRO.)

3. Reasons for Disagreement

In this section, we shall discuss the various reasons for this disagreement (theory with experiment).

3.1 *Accuracy of the Components*

We are using 1/4 or 1/2 watt carbon resistors with about 10 % tolerance and paper (or polyester) capacitors of about 2 % tolerance from the marked values of the components. We analysed in detail, whether this would explain the deviation of about 5 to 7 degrees in the measurement. For our calculations we did not consider the marked values of the components; instead we used a digital LCR meter to measure the values of the components at different frequencies (1 kHz, 2 kHz, 5 kHz, 10 kHz, 20 kHz) and the errors in the measurements are properly taken into account in the calculation of the phase difference. (the accuracy of the measurements are given in brackets against the components in *Tables*).

When these errors are also considered to work out the compound error in the measurement of ϕ from equation (1), there is a change in the phase upto a maximum of 0.5°. But the actual deviation of the measurement from theory is upto a maximum of about 6°.

This indicates that the values of the components and their accuracies could not explain the deviation in the measured and calculated values of ϕ.

3.2 *Power Fluctuation*

When there is a fluctuation in the operating voltage of the signal generator or CRO, then there would be a change in the ϕ as this would affect the amplitude of the signal. But this will be equally reflected in the measurements of x-intercept ($x_{\rm int}$) and x-maximum ($x_{\rm max}$) measurements and since we take only the ratio of the above two, power fluctuations could not be the possible reason for the measured and expected values of ϕ at different frequencies.

3.3 *Parallax Error*

Since ϕ measurements are made on the oscilloscope by taking the $x_{\rm int}$ and $x_{\rm max}$, there is always a possibility for parallax error. We minimized this not only by proper measurements but also by taking measurements for both upward and downward variation of the frequencies about the resonant frequency in LCR circuit (or characteristic frequency in RC circuit)

This could not again account for the deviation in the phase measurements, as this is up to a maximum of $0.2°$.

3.4 *Signal Generator*

We are using sinusoidal signal generator for all the phase measurements. If this does not give a fairly stable sinusoidal signal of specific frequency f and amplitude $v_{\rm i}$, then the calculation of ϕ will not agree with the measurements. We have checked this signal generator for the following:

- Whether the output impedance varies with the change in frequency.
- Whether there is any loading (mismatch of impedance) in any part of the circuit.
- Whether the frequency of the output from signal generator is stable.
- Whether the connecting wires contribute additional capacitance in the circuit, so that the expected value for the phase difference is altered.

Even when all these things are properly taken into account, they could not explain the discrepancy in the agreement of the phase difference ϕ in the above two circuits, as this is to a maximum of $1^{\rm o}$.

3.5 *Error in the Measuring Device* (*CRO*)

Finally, the measuring device, i.e., the CRO is analyzed in detail for its genuine performance. Before measuring the intercepts and maxima on the CRO, the origin is properly set at the center of the screen. Then at each stage it has been verified for the symmetry of the ellipse, i.e., $(x_{\rm int}/x_{\rm max})$ should be equal to $(y_{\rm int}/y_{\rm max})$. The deviation between these two ratios is negligible.

But, a careful look shows that this measurement using the CRO gives rise to such deviations. This is discussed in the next section, in detail.

4. Remedy

The details of the problem and the remedy are given here, with our measurements and calculations.

Step 1: Generally a CRO will have three input terminals namely x input, y input and ground. Wires are taken from the x and y terminals and shorted simultaneously to the ground. The spot on the CRO screen is adjusted to the center for no input from the audio frequency (a.f.) generator to the oscilloscope.

Step 2: Now these two wires x and y are removed from the ground and connected directly to the a.f. generator's output terminals as shown in *Figure* 4. Oscilloscope ground should be properly connected to the ground of the a.f. generator.

It is known that connecting to the x and y terminals of the CRO means we are connecting to the x amplifier and y amplifier of the CRO.

Now, strictly speaking, we should not get an ellipse for the Lissajous figure, as there is no phase change between the CRO and the signal generator (*Figure* 4). But we find an ellipse, even for no inputs. This is alarming as this ellipse changes with frequency in the a.f. generator. This is the offset error which is the cause of the disagreement of measurements and theory, for the phase difference.

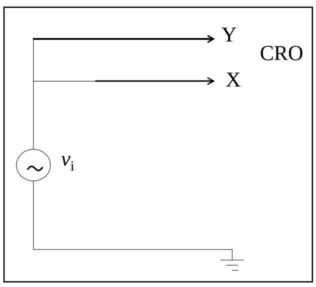


Figure 4. Measurement of device error (offset) ϕ_D.

Table 3. Offset error in the measurement of phase difference with CRO. (Freq in kHz; X_{int}, X_{max}, Y_{int}, Y_{max} in cm and ϕ in degrees; $\phi_D = \phi_{av} = (\phi_x + \phi_y)/2$; ϕ_D: offset error in CRO.)

Freq	X_{int}	X_{max}	Y_{int}	Y_{max}	ϕ_x	ϕ_y	ϕ_D
1	.2	3.6	.1	1.8	3.2	3.2	3.2
3	.3	3.8	.15	1.9	4.5	4.5	4.5
5	.25	3.5	.1	1.4	4.1	4.1	4.1
6	.25	3.3	.2	2.6	4.3	4.3	4.3
7	.3	3.4	.15	1.7	5.1	5.1	5.1
8	.2	4.1	.25	3.1	2.8	2.8	2.8
9	.1	3.8	.1	3.8	1.5	1.5	1.5
9.5	.1	2.9	.1	2.9	2.0	2.0	2.0
10	.4	3.1	.2	1.55	7.3	7.3	7.3
11	.35	3.1	.25	2.2	6.4	6.4	6.4
12	.3	3.7	.2	2.5	4.7	4.7	4.7

This is because there is already a phase difference between the x amplifier and y amplifier of the CRO and the shape of the ellipse varies with the variation in the frequency in the signal generator. This means that the phase difference (offset) between the x and y plate of the oscilloscope is not even a constant but a varying quantity.

Now if additional phase change circuits such as RC circuits or LCR circuits are introduced between the a.f. generator and the CRO, obviously the measurement of the phase difference will be drastically affected. In some frequency regions this 'zero error phase difference or the offset phase difference' will get added (with the RC or LCR phase difference) or get subtracted, depending on the sign of the ϕ. This is why such measurements on the CRO for the offset phase difference are done separately and are given in *Table* 3.

When this offset phase difference due to the CRO is properly considered, the agreement between the measurement and theory, for phase difference, improves to a very good extent. This can be seen in *Tables* 1 and 2, from columns 9 and 11.

5. Discussion

A serious, though simple, problem that we generally come across in the electronics labs is viewed here in detail. In general, the phase difference measured using the CRO by the method of Lissajous figures does not agree well with the expected values (from theory) and the deviation is appreciable in some frequencies. When we analysed this, we found that most of the time this is due to the measuring device, namely the CRO. This is even more alarming when the X and Y amplifiers of the CRO, have inherently a phase difference. The best way to avoid this is to choose a good oscilloscope by checking its performance.

But this may not be possible always. In this article, we have shown how to make better measurements even with a very bad oscilloscope. For example, in *Tables* 1 and 2, we could see the agreement between the measured and expected, only after making the suitable 'offset' phase correction, in the measured value of phase (ϕ_{av}) (see columns 9 and 11 of the *Tables*).

$\phi_{\text{D},}$ the device offset error (i.e., CRO), is specifically given in column 10 (which is from *Table*.3)

6. Offset in Other Oscilloscopes

It is necessary to find whether such offset phase difference exists only in this type of oscilloscopes or even in other costlier oscilloscopes. The following oscilloscopes are taken for sampling, of which the first two are costlier compared to the third.

1. 10 MHz, Dual trace, 1 M- input impedance (make 2).
2. 5 MHz, Dual trace, 1 M- input impedance (make 3).
3. 150KHz, Single beam, 100k- input impedance (make 4).

Measurements for the offset phase difference were made in these CRO's, as discussed in the previous section. It is found that the offset in the oscilloscopes (1) and (2) are less than 0.5° for frequencies between 10 Hz–150 kHz whereas in oscilloscope (3) this is appreciable as observed in the previous section, i.e, it is up to a maximum of 4°. This is alarming when phase measurements are made with this type (3) of oscilloscopes. (It doesn't mean that all the costlier oscilloscopes will not have this offset phase difference).

It is also to be mentioned that students cannot be provided with very costly and sophisticated oscilloscopes for measurements in the laboratories. Also they should be given training in making measurements with even defective oscilloscopes. So, with the available oscilloscopes in the laboratories, if we want to measure the phase difference, it is essential to make measurements on the offset phase difference.

This will also help the students in identifying the bugs in the functioning of the electronic circuits.

Conclusion

The discrepancy between the observed and expected values of phase shifts, by Lissajous method, is now solved. How the measuring device, namely the oscilloscope, plays an important role in this measurement is demonstrated with simple examples.

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Suggested Reading

Any standard text book on basic or advanced Electronics at BSc or MSc level.

N B Ramachandra and
M S Ranjini
Unit on Evolution and Genetics
DOS in Zoology
Manasagangotri
University of Mysore, India.
Email: nallurbr@gmail.com

Experiments to Demonstrate Change in Allelic Frequency by Genetic Drift

Populations may show a change in the frequency of alleles due to a number of factors such as migration from or to other populations, mutation, selection and random changes caused by small size of population. Genetic Drift is a random, non-adaptive change in gene frequencies in small populations. Sewall Wright, one of the giants in synthesizing the modern theory of evolution, was the first to introduce the concept of genetic drift, which is also known as 'Sewall–Wright effect'. The changes due to genetic drift are not driven by environmental or adaptive pressures, and may be beneficial, neutral, or detrimental to reproductive success. The statistical effect of sampling error during the reproduction of alleles is much greater in small populations than in large ones. For instance, if a small random sample of individuals is separated from a larger population, the gene frequencies in that sample may differ significantly from those in the population as a whole; this is because of sampling error.

Keywords
Population genetics, genetic drift, allele frequency.

A founder effect is the special case of genetic drift which occurs when a small group in a population splits off from the original