Eugene Paul Wigner – A Tribute*

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*Reproduced from *Current Science,* Vol.69, No.4, 25 August 1995.

Keywords

Symmetry in quantum mechanics, measurement theory in quantum mechanics, nuclear physics, relativistic quantum mechanics, elementary particles.

One of our last surviving links with the period of the creation and development of quantum mechanics was broken with the passing of Eugene Wigner on 1 January 1995 at Princeton in USA. Wigner was remarkably talented and wide-ranging in his interests, and his work touched innumerable aspects of modern physics. In every area that he turned to, he discovered new and profound insights and interesting viewpoints, often understood and carried further by others much later. He was as much at home in fundamental problems of physics and their mathematical analysis as in engineering and technological matters. In this tribute, I shall first describe briefly his life and career, then turn to a sketch of his work, and conclude with an attempt to capture his personality and philosophy of science and life.

A Brief Life Sketch [1]

Eugene Paul (Jeno Pal in Hungarian) Wigner was born on 17 November 1902 in Budapest, Hungary, to Elisabeth Einhorn and Anthony Wigner. He thus belonged to the same generation as Werner Heisenberg, Enrico Fermi and Paul Dirac. Leo Szilard and John von Neumann were Wigner's classmates at the Lutheran High School in Budapest – 'at that time, perhaps the best high school of Hungary and probably also one of the best of the world' [2]. Wigner retained great regard for his mathematics teacher L. Ratz, who also recognized and encouraged von Neumann's unusual talents.

After a year spent at the Technical Institute in Budapest, in 1921 Wigner joined the Technische Hochschule in Berlin to train as a chemical engineer. He completed his doctorate in 1925 and then worked for a year and a half as a leather chemist. By this time he had become very much a part of the Berlin physics scene; his break came with an appointment as assistant to Richard Becker for 1926–27. This was followed in 1927–28 by a position as assistant to David Hilbert in Göttingen, and then as Privatdozent at Göttingen during 1928–30. At this point he moved to the United States, where he spent the rest of his life.

Wigner's career in the US began as a lecturer in mathematical physics during 1930 at Princeton University, quickly elevated to a Professorship from 1930 to 1936. The year 1937–38 was spent as a professor at the University of Wisconsin at Madison. Upon return to Princeton, he became Thomas D. Jones Professor of Mathematical Physics in 1938, a position he held until 1971. The academic year 1957–58 was spent at Leiden in the Netherlands.

In 1937 Wigner became a naturalized citizen of the United States. He took his citizenship very seriously, and played a very active role in public affairs and matters of government policy. As his contribution to the war effort, he spent 1942–45 at the Metallurgical Laboratory of the University of Chicago, the last two years as the head of the theory group there. Earlier he had joined Szilard and Fermi in persuading Einstein to write the famous August 1939 letter to President Franklin Roosevelt that led to the setting up of the Manhattan Project. He was present at the University of Chicago's Stagg Field Squash Courts on 2 December 1942 to witness the world's first controlled nuclear fission reaction set up under Fermi's leadership. During 1946–47 he served as Director of what later became the Oak Ridge National Laboratory in Tennessee. In 1952 he was full-time adviser to the Du Pont Company to design the Savannah River heavy-water plutonium production reactors. Soon after, in 1954 he was appointed to the General Advisory Committee of the United States Atomic Energy Commission, and served also on many panels of the Science Advisory Committee to the President of the United States.

Of the many awards that came to Wigner, we must mention the Medal for Merit, the Franklin Medal for 1950, the Enrico Fermi Award of the USAEC for 1958, the Atoms for Peace Award for 1960, the Max Planck Medal for 1961, and the 1963 Nobel Prize in physics (shared with Maria Mayer and Hans Jensen) for his wide range of contributions to quantum mechanics.

Wigner's first marriage, to Amelia Franck in 1936, was followed by a second one in 1941 to Mary Annette Wheeler, a professor of physics. His sister Margit Balasz nee Wigner married Paul Dirac in 1937. It appears that Dirac was so shy that he once introduced his wife to an old friend as Wigner's sister. In response (!) Wigner referred to Dirac as 'my famous brother-in-law' [3]. There is a charming account by Margit of her first meeting with Dirac in Wigner's company. At a meeting in Budapest, the von Neumanns had invited Margit to visit and stay with them in Princeton. And then [4]:

'Eugene insisted, "If you come to Princeton, you must stay with me. What would people say if you did not stay with your brother?" I was not terribly thrilled with the idea. The von Neumanns had a lovely home,..., while my brother liked to appear, and act, like a pauper. We sailed in the fall; Eugene had a two-bedroom apartment, proudly boasting that he furnished it to the cost of under $25. It looked like it ... It was soon after my arrival; we were having lunch at one of these restaurants, when a tall, slender young man entered the dining room, looked at Eugene and greeted him. He looked lost, and sad. I asked who he was, still standing undecided and none too happy-looking. I was told, he was an English physicist, whom Eugene knew in Göttingen, where they used to have their meals together. "He does not like to eat alone". "So why don't you ask him to join us?" That was how I met Paul Dirac. That was the fall of 1934. The Institute for Advanced Studies had no building of its own as yet. Its members, like Einstein, von Neumann and Dirac as a visiting member, had adjoining rooms in a large university building, called Fine Hall. I remember so well: to the left was Einstein's room, in the middle Eugene's and to the right of him, Dirac's.'

Wigner, Szilard and von Neumann formed the famous Hungarian trio who contributed so decisively to intellectual life in the United States in the 1930s and later. There is a story that during a meeting of scientists connected with the war effort there was so much confusion due to many languages being used that someone got up and exclaimed: 'Gentlemen, let us use one language we can all understand – Hungarian!'

When Wigner died he left behind his third wife Eileen, a son and two daughters.

Contributions to Science and Engineering

Wigner's work in physics is characterized by hard mathematical analysis based on simple yet profound physical assumptions. While there is a down-to-earth practical quality to some of his work, in others he dealt with the most fundamental issues with great refinement – he was both an artist and an engineer, and quantum mechanics was his medium. To quote John A. Wheeler [5]: 'In the work of Eugene

Wigner one sees the basic harmony between the conceptual framework of physics and the structure of the mathematics associated with that physics.' On the other hand, his grasp of technology is best conveyed by this passage from Lawrence Dresner and Alvin M. Weinberg [6]: '.... the facility with which he could pass back and forth between engineering and physics – from a discussion of the probable distribution of energy levels in U^{235} to a critical examination of the blueprints of the concrete foundations for the Hanford reactors, or from a group theoretical argument in transport theory to the design of aluminium fuel elements!'

Wigner's first important work in physics, completed during his apprenticeship with Becker, was a powerful treatment of quantum many-fermion systems. Around the time of the move to Göttingen, and following a suggestion by von Neumann, he undertook the major task of introducing group theoretical methods into quantum mechanics. By 1928 he had published six landmark papers on the subject; he shares with Hermann Weyl the credit for making this an essential and characteristic component of quantum physics which pervades all its applications. During the 1930s he worked in solid-state physics and at the frontiers of the developing subject of nuclear physics, making a major effort to understand the forces between nucleons, and developing the compound nucleus model to explain resonance phenomena in neutron-induced nuclear reactions. His development later of the R-matrix theory of nuclear reactions was a response to a comment by Fermi that the compound nucleus model needed a firm theoretical foundation. Probably his most remarkable work in mathematical physics – the study of the unitary representations of the inhomogeneous Lorentz group – grew out of a suggestion made to him by Dirac in 1928. This was completed in Madison in 1937, and subsequently became the basic framework for all relativistic quantum theories. He came back to problems of nuclear structure in his supermultiplet theory of 1937, and later in his statistical treatment of nuclear spectroscopy based on random matrices.

In the midst of all this, in the 1940s he worked on the theory of neutron chain reactors and the design of plutonium breeder reactors.

Wigner's concern with the structure of quantum mechanics has led to a series of incisive insights over many years. In the early 1960s he turned to problems of interpretation and epistemology raised by the standard interpretation of quantum mechanics. At this point it is convenient to present briefly and selectively sketches of Wigner's work under several broad areas. This is admittedly an inadequate, incomplete and possibly superficial way to survey his work, yet it may succeed in conveying some idea of the range and magnitude of his achievements. Before embarking on this, we may recall the following important books published by Wigner: (1) *Group Theory and its Application to the Quantum Mechanics of Atomic Spectra* (Academic Press, New York, 1959; the original German version published by Friedrich Vieweg, Braunschweig, 1931); (2) *Nuclear Structure,* with Leonard Eisenbud (Princeton University Press, 1958); (3) *The Physical Theory of Neutron Chain Reactors,* with Alvin M. Weinberg (University of Chicago Press, 1958); and (4) *Symmetries and Reflections – Scientific Essays* (Indiana University Press, 1967). We may also mention that the October 1962 issue of the *Reviews of Modern Physics,* published on the occasion of his 60th birthday, contains many articles surveying Wigner's work in several areas.

Structure and Content of Quantum Mechanics

Any serious user of quantum mechanics is sure to find herself employing repeatedly, either explicitly or implicitly, one or another of the many basic concepts and methods invented by Wigner. One of the earliest is the concept of parity [7]. In classical physics, space inversion is merely a geometrical operation or transformation, a rule to map each point in space to its image by inversion through a chosen origin. The time is left unaffected. A particle trajectory, for example, would be mapped on to another possible trajectory.

Classical space inversion:

$$P : x \to -x, t \to t,$$
$$x(t) \to -x(t).$$

Wigner showed that in quantum mechanics, parity is more than a transformation, it is a physical observable whose value can be experimentally measured. The possible results of measurement are ± 1, and the corresponding quantum states are said to possess even or odd parity, respectively.

Quantum space inversion:

$$P\psi(x,t) = \psi(-x,t),$$
$$\psi(-x,t) = \pm\psi(x,t) \Rightarrow P = \pm 1,$$
$$\text{even/odd states.}$$

It was this role of parity in quantum mechanics that was shown by Wigner to be the explanation for Laporte's selection rule in atomic spectroscopy [8]: the matrix elements of the electric dipole moment operator, and hence the corresponding transitions, vanish unless the two concerned states have opposite parities.

The deep connection between invariance principles and conservation laws, both in classical physics and in the quantum domain with specifically new and subtle features, remained a lifelong concern for Wigner, something he came back to time and again. In the particular case of rotational symmetry, the general programme of incorporating group theoretical methods into quantum mechanics led to Wigner's impressive body of results concerning angular momentum in quantum mechanics [9]. The detailed representation theory of the rotation group SO(3) and its covering group SU(2), which is basic to quantum mechanics, was developed by him in a form suited to practical application. The angular momentum addition theorem, the concept of tensor operators, the Wigner–Eckart theorem for their matrix elements, explicit expressions for the Clebsch–Gordan coupling coefficients (also called the Wigner $3j$ symbols), leading on to the intricate Racah–Wigner calculus for coupling of tensor operators and computing the resulting matrix elements, the generalizations to other symmetry groups – all these oh-so familiar tools of the trade in atomic, nuclear and particle physics originate from his work.

In his book on group theory, Wigner formulated and proved a fundamental theorem concerning the representation of symmetry operations in quantum mechanics [10]. This is a very deep and subtle result, and a brief explanation would not be out of place. The relation between physical states and wave functions (or Hilbert space state vectors) in quantum mechanics is one-to-many. This is because a change in the overall phase of a wave function is physically unobservable:

$$\text{vectors } \psi, e^{i\alpha}\psi, e^{i\beta}\psi, \ldots \rightarrow \text{same physical state},$$
$$\text{vectors} \overset{\text{Many-to-one}}{\leftrightarrow} \text{physical states.}$$

Therefore, what physical states correspond to in a one-to-one manner are not vectors but rays: a ray is an equivalence class of vectors, two vectors being declared equivalent if they differ only by a phase. The ray to which a vector ψ belongs can be unambiguously described by the corresponding projection operator or density matrix ρ_ψ:

$$\text{vector } \psi \rightarrow \text{ray} \rho_\psi = \psi\psi^+,$$
$$\text{rays} \overset{\text{one-to-one}}{\leftrightarrow} \text{physical states.}$$

Rays do not form a vector space, so their geometry is somewhat harder to visualize than that of vectors ψ. Wigner's theorem then shows that any mapping Γ of rays (i.e., physical states) onto themselves preserving quantum-mechanical probabilities – and any symmetry operation must be of such a nature – can be 'lifted' to either a linear unitary or an antilinear unitary (antiunitary) transformation T on vectors.

Unitary-Antiunitary Theorem:

Symmetry operation Γ,

$$
\begin{aligned}
&\rho_\psi = \psi\psi^\dagger \rightarrow \Gamma(\rho_\psi) = \rho_{\psi'=\psi'\psi'^\dagger}\\
&\rho_\varphi = \varphi\varphi^\dagger \rightarrow \Gamma(\rho_\varphi) = \rho_{\varphi'} = \varphi^{'}\varphi^{'\dagger}\\
&\quad \psi^{'}, \varphi^{'}, \ldots \text{determined up to phases,}\\
&\quad |(\varphi^{'}, \psi^{'})| = |(\varphi, \psi)| \Rightarrow\\
&\textit{either}\\
&\quad \psi^{'} = T\psi, \varphi' = T\varphi, \ldots,\\
&\quad T \text{ linear unitary,}\\
&\quad (\varphi^{'}, \psi^{'}) = (\varphi, \psi) \rightarrow \text{unitary alternative;}\\
&\textit{or}\\
&\quad \psi^{'} = T\psi, \varphi^{'} = T\varphi, \ldots,\\
&\quad T \text{ antilinear unitary,}\\
&\quad (\varphi^{'}, \psi^{'}) = (\varphi, \psi)^*\\
&\quad = (\psi, \varphi) \rightarrow \text{antiunitary alternative.}
\end{aligned}
$$

(Here the inner product of the Hilbert space vectors φ, ψ is denoted by (φ, ψ)). This remarkable theorem has been extended and proved under different conditions by others over the decades.

Most symmetries in quantum mechanics turn out to be of the unitary type; time reversal is one example where the antiunitary alternative is realized. The analysis of this transformation in quantum mechanics was given by Wigner [11] in 1932. In Schrödinger's quantum mechanics, time reversal acts on wave functions thus:

$$T\psi(x, t) = \psi(x, -t)^*.$$

Unlike parity, however, this operation does not have the status of a physical observable in quantum mechanics, and its eigenvalues are not invariantly defined and are not experimentally measurable.

Continuing with the theme of symmetry in quantum mechanics, Wigner and von Neumann proved a very interesting result in 1929, which is of great importance especially in molecular physics [12]: if the electronic states in a molecule are classified according to their symmetry, i.e., the representation of the full group of symmetry of the relevant molecule, and if we have two distinct eigenvalues and eigenstates sharing the same symmetry (two electron terms), these eigenvalues will not cross (become accidentally equal) as one varies the internuclear distances in the molecule. On the other hand, electron terms of distinct symmetry can cross. This is a general theorem of quantum mechanics, applicable to a generic hamiltonian possessing some symmetries and dependent on a continuous parameter: as the parameter is varied, distinct eigenvalues 'of the same symmetry' will not accidentally cross but will repel each other.

Many years later, Wick, Wightman and Wigner [13] brought to light another aspect of symmetry in quantum mechanics, namely the existence of superselection rules. This amounts to a restriction on the applicability of the superposition principle in quantum mechanics. In general, the Hilbert space of states of a quantum system breaks up into sectors, and the formation of complex linear combinations to produce new states from old is limited to one sector at a time, not cutting across sectors. This is the reason why the phase of a spinor field – a field with half-odd integer spin – is nonobservable. So, for instance, a nontrivial linear combination of states with integer and half-odd integer angular momenta cannot be prepared. As another example, one finds that linear superpositions of states of distinct electric charge are unphysical. It is suspected that these results had long been known to Wigner, and he was persuaded by his coauthors to join them and say so in print.

In the preface to his book on group theory, Wigner relates a conversation with von Laue on the use of group theory as the natural tool with which to tackle problems in quantum mechanics [14]. He says: 'I like to recall his question as to which results ... I considered most important. My answer was that the explanation of Laporte's rule (the concept of parity) and the quantum theory of the vector addition model appeared to me most significant. Since that time, I have come to agree with his answer that the recognition that almost all rules of spectroscopy follow from the symmetry of the problem is the most remarkable result'.

The exponential decay law for unstable states has been well known since the days of Rutherford's experiments on radioactivity. The first properly quantum-mechanical discussion and derivation of this law is due to Weisskopf and Wigner [15]. They were able to provide the basic theory for the natural linewidths and lifetimes of atomic states decaying via transitions to other states with emission of radiation. Their use of second-order perturbation theory along with judicious and delicate assumptions also disclosed that the exponential decay law is only an approximate, not an exact, consequence of quantum mechanics; so departures from it for both very short and very long times are to be expected.

The linear superposition principle of quantum mechanics, already referred to above, finds its most natural expression at the level of state vectors in Hilbert space. The ray space or density matrix description of physical states, which is closer to a classical description, obscures this principle somewhat – it is present but not manifest. In 1932, while studying thermodynamic equilibrium in quantum mechanics, Wigner introduced another description of states for quantum systems possessing classical canonical analogues [16]. Thus, each quantum state is describable by a certain real distribution or function on the classical phase space. In one dimension with classical phase space variables x and p, the construction is as follows:

$$\psi(x) \rightarrow W(x,p) = \frac{1}{h}\int_{-\infty}^{\infty} \mathrm{d}x^{'}\psi\left(x-\frac{1}{2}x^{'}\right)\psi\left(x+\frac{1}{2}x^{'}\right)^{*}\exp(ix^{'}p/\hbar)\,.$$

These distributions – named after Wigner – are at the level of density matrices, not state vectors. They are suggestively like classical probability distributions on phase space, such as one uses in classical statistical mechanics. However, since in general $W(x,p)$ can become negative for some arguments, we do not have a classical statistical picture with well-defined probabilities. This is as it should be, since quantum features must be preserved. This description of states in quantum mechanics turned out to be the counterpart, or companion, to a rule or convention given by Weyl for associating a quantum-mechanical operator with every classical dynamical variable; and these ideas were further extended, particularly by Moyal [17].

Wigner contributed a great deal to the formal description of scattering and reaction processes in quantum mechanics, especially in the context of nuclear physics. One of his results concerns the physical meaning of phase shifts. In general, scattering cross-sections are determined by the squared magnitudes of S-matrix elements, and in these the phases get washed out. On the other hand, the spatiotemporal development of a scattering process described within the limits set by quantum mechanics, involves these phases. The beautiful connection found by Wigner is the expression for time delay caused by interaction and its relation to the energy dependence of the scattering phase shift [18]:

$$\Delta T(E) = 2\frac{\mathrm{d}}{\mathrm{d}E}\delta(E) \ .$$

Here $\delta(E)$ is the phase shift at energy E; thus, an attractive (repulsive) interaction leads to $\delta(E)$ increasing (decreasing) with energy, hence to a delay (advance) in the appearance of the final-state products of a collision after undergoing interaction.

We conclude this account with a couple of 'curios'. Classically, one expects the possible states of a system of interacting particles – especially, a two-body system – to separate into two types: unbounded or scattering states, having positive energy, and bound states, having negative energy. In quantum mechanics we expect the energy eigenvalues to behave analogously: a continuum of unbound, positive-energy scattering states sitting on top of a set of discrete negative-energy, bound states. Only the latter have normalizable wave functions. In a remarkable paper in 1929, Wigner and von Neumann produced an example of a two-body potential which possesses a bound state embedded in the continuum [19]! This is an unexpected and essentially quantum-mechanical result. The potential is 'artificial' in that it has to be carefully engineered to produce the desired result, and the state involved is unstable even under small perturbations.

The passage from classical to quantum mechanics results, at the level of dynamical variables, in the loss of *commutativity* in multiplication. Thus, for two physical quantities represented by operators A and B, in general $AB \neq BA$. However, this departure from the classical is limited in the sense that *associativity* is maintained: for three (or more) quantities multiplied in a given sequence the product is unambiguous: $(AB)C = A(BC) = ABC$. One can ask how quantum mechanics might be modified if one takes the nonclassical path one step further and, along with commutativity, one gives up associativity as well. This was examined by Jordan, von Neumann and Wigner [20] in 1934. It did not, however, lead to any alternatives with sufficiently interesting and flexible properties to give a further extension of quantum mechanics.

Going over this rich list of contributions, one is tempted to say that Wigner took his revenge for not having been involved in the discovery of quantum mechanics, and compensated for it accordingly!

Nuclear Forces, Structure and Reactions

Following the discovery of the neutron by Chadwick in 1932, there was a great deal of work exploring the nature of the strong nuclear forces between neutrons and protons. It was realized that these would be strikingly different from the familiar Coulomb forces between protons, of very short range, and with complicated distance dependences. Further dependences on spin and space exchange were also anticipated. Wigner was one of the earliest contributors to this field, and his name is associated with one of the four basic types of terms in the potential energy expression [21]:

$$\begin{aligned}\text{potential energy between proton and neutron} = \\ \text{purely distance-dependent Wigner term} + \\ \text{spin exchange Bartlett term} + \\ \text{space exchange Majorana term} + \\ \text{spin and space exchange Heisenberg term.}\end{aligned}$$

Thus, the Wigner force is the simplest of all; the others either distinguish between singlet and triplet spin states, or between even and odd orbital angular momenta, or both. Such phenomenological potentials are useful in analysing low-energy nuclear bound states, scattering processes, etc.

The low-energy (in the keV to few MeV range) scattering cross-sections of neutrons off various nuclei were experimentally studied by Fermi and his collaborators, and many other groups, around 1936. They found striking resonance structures in these cross-sections, with sharp maxima separated by very small values in between. Soon after, a theoretical explanation was offered independently by Niels Bohr on the one hand, and by Gregory Breit and Wigner [22] on the other. This is the so-called compound nucleus model. It pictures the scattering and reaction processes as taking place in two steps. At first the incoming low-energy projectile (which could be some light nucleus rather than a neutron) and the target combine to produce a compound nucleus in one of several possible metastable states. In this process the projectile energy is quickly shared with all the nucleons in the compound structure, and then the mode of formation of this structure is 'forgotten'. In the second step, the decay of the compound nuclear state into various energetically allowed channels is governed by probability laws.

It is the probability of occurrence of the first step that shows an extremely sensitive energy dependence and gives rise to the observed resonances. In their work Breit and Wigner derived the famous bell-shaped single-level resonance formula known after their names:

Probability of formation of compound nucleus:

$$\alpha\, \Gamma_\lambda / \{(E - E_\lambda)^2 + \frac{1}{4}\Gamma_\lambda^2\},$$

E = total initial energy,
$E_\lambda, \Gamma_\lambda$ = average energy, width, of compound nuclear state λ.

The partial cross-sections for subsequent decays into each of the several available final channels retain this characteristic energy dependence.

Sometime after this, around 1944, Fermi remarked to Wigner (as was mentioned earlier) that a good theoretical basis for the compound nucleus model was lacking. Thereupon Wigner set about formulating one. This was the starting point of the R-matrix theory of nuclear reactions, developed by him largely in collaboration with Eisenbud [23]. The basic idea is to separate the total multidimensional configuration space of all the nucleons in the compound nucleus (i.e., the projectile nucleons plus the target nucleons) into two parts: an interior region where they are *all* within the range of nuclear forces acting between every pair, and an exterior region where this is not so. In the latter region, one then defines or picks out essentially nonoverlapping subregions, one for each possible (two-body) final channel into which the compound nucleus can decay. Instead of posing a multichannel hamiltonian eigenfunction and eigenvalue problem, a series of matching conditions connecting the interior and exterior channel wave functions and their radial derivatives, across the borders between the interior and each exterior region, are set up. The R-matrix elements are quantities that enter these relations, they are a multichannel generalization of the logarithmic derivative of a wave function in a one-dimensional radial problem. The parameters entering the R-matrix are the energy values and the various partial decay widths of all possible compound nucleus levels. Thus, the R-matrix became simultaneously a convenient method for parametrization of scattering and reaction amplitudes using phenomenologically accessible compound nuclear state energies and widths, and with further developments a way to embody general physical principles, such as unitarity and causality, governing reaction processes. *Inter alia* this led to a multi-level generalization of the Breit–Wigner resonance expression given above, and to a criterion for the validity of the single-level formula.

Returning to the problem of nuclear forces and structure, in 1937 Wigner came up with the SU(4) supermultiplet theory to systematize the low-lying energy levels of light nuclei [24]. The idea was that the interactions among protons and neutrons, regarded as nucleons possessing the isospin degree of freedom introduced by Heisenberg [25] as early as 1932, might to a good approximation be both spin- and isospin-independent. More generally, it might be invariant under all four-dimensional unitary transformations mixing up the four independent spin-isospin states of a nucleon. (This assumption actually leads to specific spin and isospin dependences in the interaction.) It would then be possible to arrange the energy levels of 'neighbouring' nuclei with a common mass number into various unitary irreducible representations (UIRs) of SU(4), consider systematically the breaking of this symmetry, etc. Each UIR of SU(4) is made up of several spin-isospin multiplets in a definite way. While the idea was physically well motivated as a useful first approximation, it was pursued only to a limited extent. However, many years later, in 1964, Wigner's theory provided the inspiration for a similar SU(6) invariant theory of baryons and mesons in the framework of the quark model [26].

At the other end of the scale from low-lying well-separated energy levels of light nuclei, we have the relatively highly excited and closely spaced levels of heavy nuclei with many degrees of freedom. Here Wigner proposed a completely different physical approach, one which has stimulated work by many others and led to connections with several other problems [27]. The physical ideas may be motivated as follows. As the excitation energy (of a complicated nucleus) increases, one expects the energy levels to get closer and closer, and one also loses hope of being able to obtain them individually from a first-principles Hamiltonian. Instead, what would be more accessible and physically interesting are various statistical properties of the levels; the probability distributions for successive levels to occur at various energies, for the spacing between neighbouring levels to have different values, and so on. To obtain these statistical features, and at the same time to reflect the fact that one is dealing with a very complex system with many degrees of freedom, Wigner proposed that the basic Hamiltonian (after truncation to a large but finite dimension) be itself regarded as a random matrix, belonging to an ensemble with specified properties. Once one specifies the nature of this ensemble, regarded as a primary input, the statistical properties of the eigenvalues of the Hamiltonian, the spacing distribution, etc., can all be derived, in principle, as secondary consequences. It turns out that in using this approach one must deal with one 'simple sequence' of nuclear levels at a time; this is a set of levels possessing the same exactly conserved quantum numbers

– 'belonging to the same symmetry' – such as the total angular momentum and parity. Properties of different simple sequences are independent. Thus, Wigner's hypothesis was that the local statistical behaviour of the levels in a simple sequence is given by the properties of the eigenvalue spectrum of a random matrix drawn from a suitable ensemble. The type of ensemble to be used depends on the integer or half-odd integer nature of the total angular momentum, behaviour under time reversal, and presence or absence of rotational symmetry. Later work has shown that there are three natural types of ensembles, in correspondence with the three great families of classical compact simple Lie groups: the Gaussian real orthogonal, the Gaussian complex unitary, and the Gaussian symplectic ensembles. These ensembles consist respectively of real symmetric, complex hermitian and real quaternionic matrices (of suitable dimensions, even in the last case). The probability distribution defining the ensemble is invariant under a real orthogonal, complex unitary or unitary symplectic group of transformations applied to its elements; moreover, the matrix elements of the Hamiltonian are assumed to be independent random variables. It is the combination of these two properties that makes these ensembles Gaussian.

A great deal of sophisticated mathematical analysis has gone into these objects, and this activity continues [28]. One very interesting feature that was recognized very early was that the spacing distribution vanishes as a power of the spacing as the spacing tends to zero. The rate of this vanishing, the power involved, is characteristic for each of the three families of ensembles. The physical meaning of this result – borne out by experiments and reminding us of the no-crossing theorem of Wigner and von Neumann for electron terms of the same symmetry – is that within a simple sequence neighbouring levels do not like to come very close to one another. Had we imagined that the energy levels themselves were independently statistically distributed, there would have been no cause for such level repulsion. This only emphasizes Wigner's idea that the properties of the ensemble of Hamiltonians must be chosen first, and other properties then obtained as consequences.

Quantum Field Theory, Relativistic Classical and Quantum Mechanics

The rules for canonical quantization – creating a quantum theory from a classical one – were originally invented in the context of nonrelativistic particle quantum mechanics. The first successful application of these rules to a classical field theory came with Dirac's quantization of the electromagnetic field. This led to his classic 1927 paper in which he treated the processes of emission and absorption of radiation by matter, using quantum principles and the photon concept [29]. The quantized field led to a synthesis of complementary classical particle and field languages, and could describe states with variable numbers of identical particles. The canonical quantization method led to commutation relations of the form

$$a_r a_s^\dagger - a_s^\dagger a_r = \delta_{rs},$$

$$a_r a_s - a_s a_r = a_r^\dagger a_s^\dagger - a_s^\dagger a_r^\dagger = 0.$$

Here $a_r(a_r^\dagger)$ are the annihilation (creation) operators for photons in various states indexed by r. These states are an independent, orthogonal and complete set of modes of the electromagnetic field. The operators $a_r, a_r^\dagger$ are quantum analogues of the classical complex coefficients in an expansion of the classical field in these modes. In this case the appearance of commutation relations led naturally to Bose–Einstein statistics for photons. Very soon after Dirac's paper, Jordan and Wigner showed that to describe fermions (such as electrons) obeying Pauli's exclusion principle and Fermi–Dirac statistics, the particle annihilation and creation operators must obey anticommutation relations [30]:

$$a_r a_s^\dagger + a_s^\dagger a_r = \delta_{rs},$$

$$a_r a_s + a_s a_r = a_r^\dagger a_s^\dagger + a_s^\dagger a_r^\dagger = 0.$$

For a finite number of modes, they proved that up to equivalence there is only one irreducible representation of these relations, and it is finite-dimensional. This uniqueness is similar to a corresponding result in the case of commutation relations. The major difference is that from a mathematical point of view systems of operators obeying the anticommutation relations are quite 'harmless', while in the case of commutation relations they are unbounded and the space is infinite dimensional – even for a finite number of modes. Of course, in the Jordan–Wigner case there is no sensible classical limit.

It is interesting to note that Dirac's initial reaction to this work of Jordan and Wigner was decidedly negative [31]. Wigner later attributed this to Dirac's being very committed to the Hamiltonian point of view in dynamics – 'a captive of the Hamiltonian formalism'. However, it became clear very soon that this was the correct way to set up quantum field theory for fermions, and it became part of the foundations of the subject.

The first attempts at uniting quantum mechanics and special relativity were due to Klein and Gordon. This resulted in the wave equation named after them, but it faced problems of interpretation at the one-particle level. The next, spectacularly successful, attempt was Dirac's work in 1928 that led to his wave equation for the electron and its series of amazing consequences [32]. Probably soon after, in 1928 itself, Dirac suggested to Wigner a comprehensive study of all possible unitary irreducible representations of the inhomogeneous Lorentz group (IHLG), i.e., of the homogeneous Lorentz group (HLG) supplemented by space–time translations. By about 1932, Majorana had constructed many of these UIRs, and later these were simplified by Dirac and Proca [33]. The solution of this problem posed by Dirac to Wigner became a herculean effort, being completed only in 1937. The result was an all-time classic paper in mathematical physics [34]. In it, Wigner acknowledges the help and guidance he received not only from Dirac but also on mathematical aspects from von Neumann. At some stage Dirac advised Wigner to be careful, and the latter replied [35]: 'You point out that care is needed in the analysis of the representations of the Lorentz group; I promise you that I will be careful'.

Wigner's paper contains a detailed analysis of the structure of the HLG and the IHLG, and of general unitary representations (URs) of the IHLG in the context of quantum mechanics; it then turns to a study of the UIRs. The result was that these could be classified into four broad types, depending upon the nature of the possible values of energy–momentum p^μ occurring within the UIR, and the allowed 'states of polarization' for each energy–momentum. The helicity λ is defined as the component of angular momentum in the direction of momentum. For each kind of p^μ (provided it is not identically vanishing) the allowed values of λ are determined by some UIR of a corresponding subgroup of the HLG, the so-called 'little group' for that p^μ; it consists of all elements of the HLG which leave p^μ invariant. The pattern of UIRs of the IHLG is displayed in *Table* 1* (here space inversion or parity has been included in the HLG, except that for neutrinos this operation is undefined) [36].

While many of these UIRs were known earlier to Majorana and Dirac, the so-called infinite-spin or continuous-spin representations in cases (b) and (c) were genuinely new. In his work, Wigner did not carry the investigation of these, or of case (d), to completion. He mentioned their existence, and only remarked: '... the last case may be the most interesting from the mathematical point of view. I hope to return to it in another paper. I did not succeed so far in giving a complete discussion of the 3rd class.' Wigner's 'last case' and '3rd class'

* At the time this article was written it was generally believed, as indicated in *Table* 1, that neutrinos are massless. Over the past decade this situation has changed, some of them definitely have (very small) nonzero masses.

Nature of p^μ	Little group within HLG(SL(2,C))	Number of polarization states, spectrum of λ	Remarks
(a) Time-like (positive or negative)	SO(3)(SU(2))	$2s+1$ for $s=0,1/2,1...$ $\lambda = s, s-1, \ldots, -s$	Massive particles with zero or finite spin, $s=0$ for πmeson $s=1/2$ for electron
(b) Light-like (positive or negative)	E(2), two-dimensional Euclidean group	One: $\lambda = 0$ Two: $\lambda = \pm s, s = 1/2, 1, \ldots$ Infinite: $\lambda = 0, \pm 1, \pm 2.\ldots$ or $\lambda = \pm 1/2, \pm 3/2, \ldots,$	No known particles $s=1$ for photons $s=1/2, \lambda=-1/2$ for neutrinos No known particles
(c) Space-like	SO(2,1) (SL(2,R))	One: $\lambda = 0$ Infinite: $\lambda = s, s+1, \ldots$ or $-s, -s-1, \ldots,$ $s = 1/2, 1, \ldots$ or $\lambda = 0, \pm 1, \pm 2, \ldots$ or $\lambda = \pm 1/2, \pm 3/2 \ldots$	Imaginary mass, unphysical
(d) Vanishing	HLG(SL(2,C))	–	–

Table 1.

correspond respectively to (c) and (d) in our table. We also see that not every mathematically acceptable UIR of the IHLG is acceptable on physical grounds.

Relativistic quantum systems described by any UIR of the IHLG are called 'elementary systems'. Truly elementary particles, able to exist in isolation, are described using them. Examples are photons, neutrinos, electrons and muons. The phrase 'elementary systems' conveys the meaning that all their properties are revealed by studying their behaviour under all elements of the IHLG – there is no internal structure involved. In the above listing, only cases (a) and (b) for finite helicity are realized in nature.

The UIRs of case (d) are actually UIRs of the HLG SO (3,1) (or of the closely related group SL (2, C)). It remained for Harish-Chandra and for Gel'fand and Naimark to determine them independently [37]. The inputs needed to construct the UIRs of case (c) for infinite spin were provided by Bargmann through his construction of the UIRs of SO (2,1) and SL (2,R)[38].

In his contribution to RMP, Dirac made the following comments on Wigner's work [39]. 'The problem of working out all unitary representations of the IHLG has been dealt with by Wigner, taking the mathematical point of view that two representations are equivalent if they are connected by a unitary transformation.

He decomposes the representations into their irreducible constituents and finds that the irreducible constituents provide theories of elementary particles with various spins. This work does not lead to any interaction between particles. To bring in interaction, one must depart from the point of view of looking at two representations as equivalent if they are connected by a unitary transformation, a point of view which involves looking upon all unitary transformations as trivial. To a physicist, some unitary transformations are trivial, whereas others (for example, the S matrix) are far from trivial, so a physicist cannot look upon two representations connected by a unitary transformation as necessarily equivalent.' The point is that for any really interesting relativistic quantum system, such as a relativistic quantum field theory, it is not only important to know which UIRs of the IHLG are present, but also how they are put together. However it must be pointed out that as early as 1949 Wigner himself had drawn attention to this situation [40]: 'The elementary systems correspond mathematically to irreducible representations of the Lorentz group and as such can be enumerated... However, in the description by irreducible states, the form of almost all physically important operators remains unknown and, in fact, depends on the system, the types of interactions, etc. This leads to a rather strange dilemma: in the customary description the form of the physically important operators is known but the time dependence of the states is unpredictable or difficult to calculate. In the description just mentioned, the situation is opposite: the time dependence of the states follows from the invariance properties, but the form of the physically important operators is hard to establish.'

Wigner returned on many occasions to a description of the results of his classic work. He also constructed with Bargmann a unified set of wave equations whose solutions would lead to UIRs of types (a) and (b) in our table [41]. His work with Newton on the problem of position is particularly interesting, so I describe it in a little detail [42].

The starting point of nonrelativistic particle quantum mechanics is the set of positions and momenta as primary dynamical variables, out of which all other variables are built up. (Later work has shown that these positions and momenta can be derived as secondary objects starting from suitable quantum-mechanical representations of the Galilei group). Now, from Wigner's point of view in the relativistic context, the primary things are the UIRs of the IHLG. After having set them up, one must examine within which UIRs one can construct position operators with physically desirable properties. Such an analysis was first undertaken by Newton and Wigner. They were able to show that in every finite mass and finite spin UIR (case (a)) a unique set of position operators possessing several physically reasonable properties does indeed exist. However, contrary to naive expectation, they do not form the space components of a relativistic four-vector. This illustrates the fact that in quantum theory the unitary transformation law is more basic than the geometric one or manifest covariance. In the massless case with finite nonzero helicity even this much cannot be done. Thus, neither photons nor neutrinos can be localized in space.

In other related work we mention the study by Inonu and Wigner of the process of 'group contraction' by which the IHLG goes over in the nonrelativistic limit to the Galilei group [43]; Salecker and Wigner's analysis of deep conceptual problems in bringing together quantum mechanics and general relativity, caused by quantum limitations on position measurements [44]; and van Dam and Wigner's construction of classical relativistic direct-interaction theories resting upon integro-differential equations for particle trajectories [45]. One of Wigner's conclusions was that while special relativity and quantum mechanics could at least conceptually be combined, with general relativity and quantum mechanics there was no common ground at all.

Interpretation of Quantum Mechanics

In the early 1960s Wigner turned to a serious examination of the problems of interpretation of quantum mechanics, and a clear expression of the orthodox position which essentially coincided with his own [46]. As evidence for the latter, here is his own statement: 'The orthodox view is very specific in its epistemological implications ...A large group of physicists finds it difficult to accept these conclusions and, even though this does not apply to the present writer, he admits that the far-reaching nature of the epistemological conclusions makes one uneasy.' He also often said that he was adding hardly anything new to London and Bauer's classic 1939 exposition [47]. He accepted the treatment of measurement theory that had been articulated by his friend von Neumann [48] as early as 1932, and wanted to restate it for a new generation and extract its ultimate consequences for epistemology.

Wigner emphasized that the state vector of a quantum system changes in two mutually exclusive ways – continuous, deterministic Schrödinger evolution when not subject to observation, and discontinuous, probabilistic, collapse when measurements are made. He went to much length to show that the linear Schrödinger equation – even including the apparatus and the system's coupling to it – can never produce the macroscopically desired collapse phenomenon, and stressed repeatedly that pure states and mixtures have very different physical properties. He also presented a pragmatic answer to the question 'What is the state vector?'. It was that it codifies in a compact way all past information about a system, on the basis of which we can state the probabilistic connections that quantum mechanics gives among a series of measurements carried out subsequently and sequentially in time; all the consequences of quantum mechanics are just such statements. So, as the orthodox view claims, 'the laws of quantum mechanics can be expressed only in terms of probability connections', and cannot be formulated in terms of objective reality.

Pursuing this analysis further, Wigner came to the conclusion that human consciousness is an essential external ingredient needed to make complete sense of quantum mechanics. The collapse of the state vector occurs when and only when an observation is registered in some individual consciousness: 'It is at this point that the consciousness enters the theory unavoidably and unalterably. If one speaks in terms of the wave function, its changes are coupled with the entering of impressions into our consciousness'. And again: '... it was not possible to formulate the laws of quantum mechanics in a fully consistent way without reference to the consciousness'. In support of this declaration, Wigner appeals to Heisenberg and says: 'W. Heisenberg expressed this most poignantly (*Daedalus*, 1958, **87**, 99): "The laws of nature which we formulate mathematically in quantum theory deal no longer with the particles themselves but with our knowledge of the elementary particles ... The conception of objective reality... evaporated into the... mathematics that represents no longer the behaviour of elementary particles but rather our knowledge of this behaviour" '.

As one can imagine, this line of thinking led Wigner inexorably to a kind of solipsism, and to the delineation of two kinds of reality – the content of one's own consciousness, the only absolutely real, and everything else external to oneself, including every other person's consciousness. To support the former he turned to Schrödinger; '... the most eloquent statement of the prime nature of the consciousness with which this writer is familiar and which is of recent date is on page 2 of Schrödinger's *Mind and Matter:* "Would it (the world) otherwise (without consciousness) have remained a play before empty benches, not existing for anybody, thus quite properly not existing?"' But there was a sign of asymmetry – the only absolutely real, one's own consciousness, does depend on food, air and water for its own survival and functioning, as we are painfully aware; so he made a case for devising experiments which might show up the effects of consciousness on matter. In talking of the first kind of reality, Wigner also realized and stated its obvious limitations – its awakening with birth and infant growth, its extinction at death. So he argued for a deep study of the former phase, to understand the nature of consciousness.

Wigner felt that the development of quantum mechanics had widened the outlook of most physicists, and also in a sense made them inward-looking: 'Until not many years ago, the "existence" of a mind or soul would have been passionately denied by most physical scientists ... Even today, there are adherents to this view though fewer among the physicists than – ironically enough – among biochemists'. He also saw that quantum mechanics reinforces the circumstance that any observation and interpretation of measurement rests on previously constructed and understood theory. Thus, we are linked in a chain to the very beginnings of our acquisition of knowledge of our surroundings and its regularities – indeed to phylogenesis and ontogenesis.

Today it may seem that these conclusions of Wigner were premature. Certainly, efforts are aplenty to find more 'acceptable' interpretations of quantum mechanics, without appeal to ourselves as essential prerequisites. Was Wigner then 'a victim of his generation'? Should we smile at these conclusions which he found inescapable? Or was he only being ruthlessly honest and expressing clearly what others hesitated to put into words?

Solid-State Physics, Reactor Theory and Technology

I will touch upon these areas only briefly. Wigner's interest in problems of solid-state physics and materials science stemmed from a very early date. There must have been links to his original training as a chemical engineer; later on his detailed knowledge of properties of materials played a key role in his work on reactors. Among his gifted students in solid-state science in the 1930s we may mention John Bardeen, Gregory Wannier and Frederick Seitz. It was Wigner who suggested to Wannier [49] 'that there ought to be a way to reconcile the local and the band concept for electrons, and that such a reconciliation would probably be useful in understanding the spectra of insulators'. Wigner also worked on radiation damage in solids – the detailed microscopic picture of lattice defects occurring when materials are irradiated with neutrons, the resulting changes in heat and electrical conductivity and ductility, and also the ways in which the material seems to recover from the damage as time goes on [50].

Wigner was the source of much of the theory and the major technological developments connected with nuclear reactors. His contributions began in 1940. As briefly mentioned earlier, he was a leader at the University of Chicago Metallurgical Laboratory during 1943–45. He contributed to the development of research reactors, power reactors and plutonium production reactors. On the theoretical front he made major contributions to the spectrum of the Boltzmann equation, neutron thermalization, thermal utilization and resonance absorption. All very practical contributions 'which one would hardly, *a priori,* have associated with the same man who introduced group theory into quantum mechanics' [51].

Views on Science, Philosophy and Life

Wigner was a gifted and articulate expositor of science and its principles to general audiences. However, he frequently indulged in a kind of mock humility – as his Princeton colleagues explained his language [52], 'A piece of work is "amusing" if it is correct and beautiful; it is "interesting" if it is wrong and messy.' And in describing the epistemology of quantum mechanics to an audience of non-physicists, he said of himself the writer [53]: 'He realizes the profundity of his ignorance of the thinking of some of the greatest philosophers and is under no illusion that the views to be presented will be very novel. His hope is that they will appear sensible.' He could convey sharp ideas pithily: 'Someone once said that philosophy is the misuse of a terminology which was invented just for this purpose'.

These apart, his grasp of and concern for the grand principles of science were very deep. The role of invariance principles and their associated conservation laws captivated him – he dwelt upon them at length on many occasions [54], and said: 'A large part of my scientific work has been devoted to the study of symmetry principles in physics....' He titled his Nobel lecture 'Events, laws of nature, and invariance principles'. He often described as a miracle the fact that human understanding could uncover laws of nature, and separate them from the accidents of initial conditions. The laws provide structure and coherence to events, and, in turn, the symmetry principles provide these qualities to laws; thus one has the ascending progression: events to laws to symmetry principles.

Turning to the role of mathematics in natural science, he expressed wonder at the way in which mathematical concepts and connections show up in unexpected ways and places, and also at the fact that tentative theories turn out upon further development to be far more accurate than could reasonably have been expected at the outset. This led him to conclude that, since we do not quite know why we succeed so well so often, we must be cautious and not immediately regard a successful explanation as the truth!

Pondering on the likely future of science, Wigner wondered whether it might not wind down under its own weight, and lose its attractiveness to the young. The increasing extent of science makes it go beyond the reach of any one individual. But the response to this cannot just be an increase in team efforts, because this can never capture true creative thinking in the individual subconscious. There is a need here to find deeper ways of sharing information and insight, of harmonizing the collective conscious with the subconscious in each individual.

Continuing on the theme of the growth of science and the emergence of large collaborative efforts, he argued for protecting the individual and giving value and esteem to little science: 'One does not have the satisfaction which creative work, as we know it today, provides, if one's activities are too closely directed by others'. About the emergence of deep insights, 'It is hard to imagine how they can be developed other than in comparative solitude'. And as for the pleasures of pursuing science: 'It has been said that the only occupations which bring true joy and satisfaction are those of poets, artists, and scientists, and, of these, the scientists are apparently the happiest.'

Through the description of his work I have tried to convey the fact that Wigner acknowledged very graciously his debt to some of his most gifted contemporaries. He was also generous in his assessment of them. Of von Neumann he wrote: '... whenever I talked with the sharpest intellect whom I have known – with von Neumann – I always had the impression that only he was fully awake, that I was halfway in a dream.' And about Richard Feynman: 'He is a second Dirac, only this time more human'.

Two persons that Wigner had been very close to – Enrico Fermi and von Neumann – both died in their fifties. Wigner described and contrasted their attitudes to the inevitable. With Fermi, 'On a heroic scale was his acceptance of death ...He was so completely composed that it appeared superhuman'. But with von Neumann it was very different: 'It was heartbreaking to watch the frustration of his mind, when all hope was gone, in its struggle with the fate which appeared to him unavoidable but unacceptable'. These experiences must have affected Wigner deeply; at a convocation address to an audience of young students soon after, he said: 'Our culture is committing a sin by covering our eyes against the realization that none of us will be here always'. And to a general audience some time later:

'The recognition that physical objects and spiritual values have a very similar kind of reality has contributed in some measure to my mental peace'. These various expressions seem related.

Wigner was a physicist who achieved rare range and depth in his life and work. He was a product of the old world who flowered during the golden age of theoretical physics, and carried the fragrance of his subject to the new world. The time seems past when such another can appear.

Suggested Reading

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