Aerobasics – An Introduction to Aeronautics

10. Airplane Performance

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Previous Parts:
Resonance, Vol.13, p.836, p.971, p.1009, p.1107; Vol.14, p.19, p.191. p.272, p.328, p.650.

Keywords
Airplance performance, flight speeds, endurance, range, take-off, landing, pull-ups, turns.

Performance of an airplane defines its usefulness and is the primary reason for designing and manufacturing aircrafts. In this article we present simple methods for approximately evaluating the performance of an airplane in normal and accelerated flight conditions. Methods are presented for calculating maximum and minimum flight speed, maximum flight altitude, endurance range, etc. Some flight conditions involving acceleration like take-off, landing, pull-ups and turns are also considered.

1. Introduction

The user of an airplane is interested in answers to questions of the type: How much payload can it carry? How fast can it fly? How much fuel does it consume? How far can it go nonstop? The answers are relevant for an assessment of an airplane from an economic point of view. The purpose of performance analysis of an airplane is to answer questions of this type. The performance of an airplane is determined by a combination of the aerodynamic characteristic of the airplane configuration and the propulsion characteristic of its propulsive device. Here, we use simple but realistic analytical models of these characteristics to illustrate how one may calculate the various items of performance. The actual calculations of airplane performance follow the general principles given here, but are more involved and are normally performed by a computer. This is so because the actual characteristics of the airplane and propulsive device are somewhat more complex and are often available only as tables of values of various parameters.

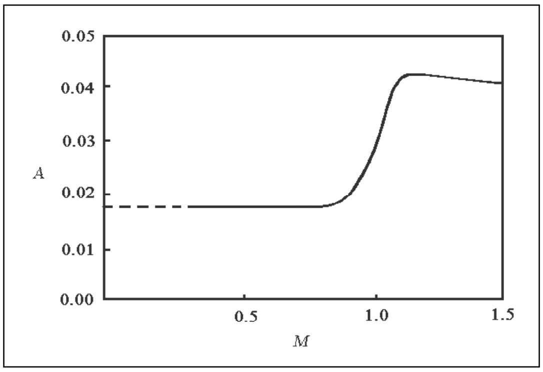

Figure 1. Variation of zero-lift drag coefficient with Mach number: The parameter A which represents the drag coefficient of an airplane configuration at zero lift includes skin friction and flow separation effects. It is nearly independent of Mach number at subsonic speeds. At supersonic speeds, there is a substantial increase due to wave drag. The illustration is typical of a combat airplane configuration.

The aerodynamic characteristic of an airplane can be closely approximated by the drag polar which is the relation between the drag and lift coefficients:

$$C_{\mathrm{D}} = A + B\, C_{\mathrm{L}}^2 \ . \tag{1}$$

In (1), A and B are parameters which depend on the airplane configuration. The coefficient A is the zero-lift drag coefficient and is largely due to skin friction and flow separation effects on the configuration. The coefficient B represents the lift-dependent drag coefficient and includes a large contribution due to induced drag. An airplane is said to be in the clean configuration in cruise. During take-off and landing phases of flight, the airplane flies with the landing gear lowered and the flaps deflected by suitable amounts. The values of A and B for a given airplane will be different in these configurations as compared to cruise and must be considered. They also depend to a small extent on the position of centre of gravity of the airplane which influences the elevator setting for steady flight and hence the drag of the tail plane. However, we shall neglect this effect in the analysis that follows. The parameters also show some dependence on the flight Mach number. In particular, the parameter A shows a significant increase at supersonic speeds due to a contribution from wave drag. *Figures* 1 and 2 are

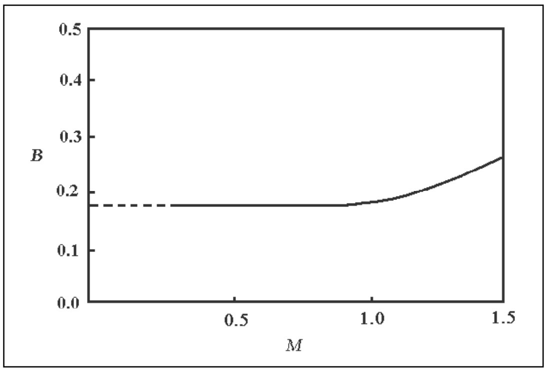

Figure 2. Variation of lift-dependent drag coefficient with Mach number. The parameter B which represents lift-dependent drag is largely due to induced drag. It shows a slight increase with Mach number at supersonic speeds. The illustration is typical of a combat airplane configuration.

typical of the Mach number effect on A and B for a combat aircraft configuration. It is seen that the parameter B is weakly dependent on M.

The propulsion characteristic of any propulsive device is defined by its net thrust T as a function of altitude h and flight speed V

$$T = T(h, V) \ . \tag{2}$$

For a jet engine, the relation shown in (2) is obtainable from the engine manufacturer. This thrust may have to be corrected for installation effects as the thrust of an engine estimated using the test bed measurements is somewhat higher than as installed on an airplane due to installation effects. This thrust is obtained at 'full throttle' and represents the maximum the power plant can deliver. Any lower thrust up to idle thrust can be obtained at part throttle which the pilot can select. Further, the thrust is under ISA conditions and may need correction for atmospheric temperature variation from ISA. Thus, on a hot day the net thrust of the engine could be significantly reduced and this needs to be considered particularly during take-off.

As an illustration, the propulsion characteristic of an engine suitable for a combat airplane is shown in *Figure* 3. The engine is normally operated with the afterburner inoperative. This condition is indicated as 'dry' operation. When extra thrust is needed, the afterburner is lit and the engine is said to operate with 'reheat on'. With reheat, the engine thrust is higher by up to about 50% for the static (zero flight speed) condition. It is even higher at high flight speeds as indicated in the figure. However, this extra thrust comes at a high price in terms

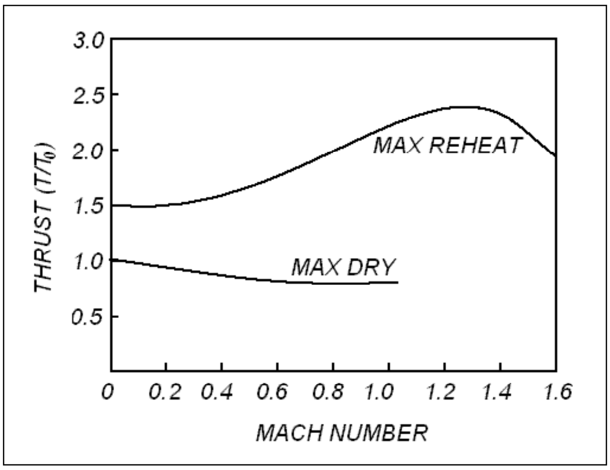

Figure 3. Variation of engine thrust with Mach number. Thrust of a jet engine without reheat (dry thrust) decreases slowly with flight speed in the subsonic range. Flight at supersonic speeds is generally not possible without reheat for want of thrust. With reheat, the static thrust is about 50% higher and rises substantially before decreasing at high supersonic speeds. The illustration is typical of a combat airplane engine.

of fuel consumption. The fuel consumption per unit thrust (the specific fuel consumption) under this condition is about twice that without reheat.

For airplanes equipped with an engine-driven propeller, the relationship of (2) is derivable by combining the individual characteristics of the engine and the propeller for any given flight condition. These details will not be indicated here. Thus (2) can be used as a basis for performance estimates of all types of airplanes.

2. Performance in Un-Accelerated Flight

Flight along a straight line with insignificant acceleration constitutes an important flight condition. All airplanes spend a large part of their flight time in this condition. This includes the climb to altitude, cruise, and descent phases of flight. During this flight condition, the airplane is in equilibrium subject to aerodynamic, propulsive and gravity forces as shown in *Figure* 4. Here, the airplane flight path makes an angle γ with the horizontal. By resolving the forces along and at right angles to the flight direction, we have for equilibrium:

$$L = W\cos\gamma \ , \qquad (3)$$

$$T = D + W\sin\gamma \ . \qquad (4)$$

Equations (3) and (4) can be approximated for small γ by taking cos $\gamma = 1$ as

$$L = W \qquad (5)$$

and

$$\sin\gamma = \left(\frac{T-D}{W}\right) \ . \qquad (6)$$

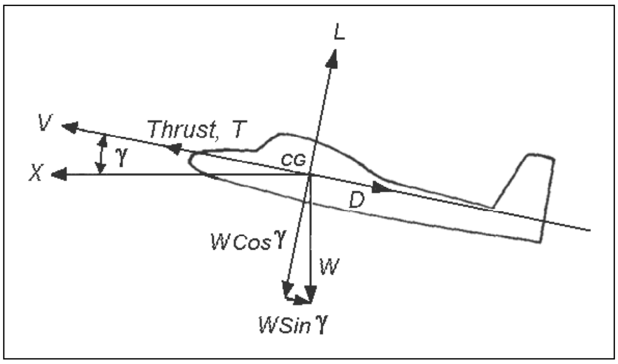

Figure 4. Equilibrium of an airplane in steady flight. The forces on an airplane in steady flight at a climb angle γ include the aerodynamic forces, drag and lift. They respectively act along and at right angles to the flight direction. The other forces include the airplane weight acting vertically down and engine thrust along the flight direction.

Using (6), we can derive the rate of climb of the airplane, R/C, which is the vertical component of the flight velocity as

$$R/C = V\sin\gamma \;=\; V\left(\frac{T-D}{W}\right)\ . \tag{7}$$

Equations (5) and (7) indicate that during steady flight at a small angle of climb (a condition satisfied by transport aircraft generally, but not always by combat aircraft) lift balances weight. The rate of climb R/C equals the excess power $V\,(T-D)$ divided by the airplane weight. Level flight implies that γ is zero and hence thrust balances drag in this condition.

Equations (1), (2), (5) and (7) are sufficient for calculating the rate of climb of an airplane at any given flight speed V. We proceed by expressing D in terms of flight speed by eliminating C_{L} as

$$L \;=\; \frac{1}{2}\rho\, V^2 C_{\mathrm{L}} S \;=\; W\ ,$$

or

$$C_{\mathrm{L}} \;=\; \frac{2W}{S\rho V^2}\ .$$

Using this in (1),

$$D \;=\; A\,\frac{1}{2}\rho V^2 S \;+\; \frac{2BW^2}{\rho V^2 S}$$

or

$$D \;=\; CV^2 \;+\; \frac{D}{V^2}\ . \tag{8}$$

Equation (8) shows the variation with flight speed of the drag of an airplane in un-accelerated flight. Here C, D are some constants which depend on the air density ρ and hence on flight altitude. This drag is the sum of two parts. The first part increases as the square of the flight speed and is obviously very important at the upper end of the flight speed range The second part varies inversely as the square of flight speed and is more important at the low end of the flight speed range (i.e., during take-off and climb). *Figure* 5 illustrates (7) graphically at one value of air density (and thus at one altitude). It also illustrates the variation of engine thrust at full throttle with flight speed for this airplane at the same altitude.

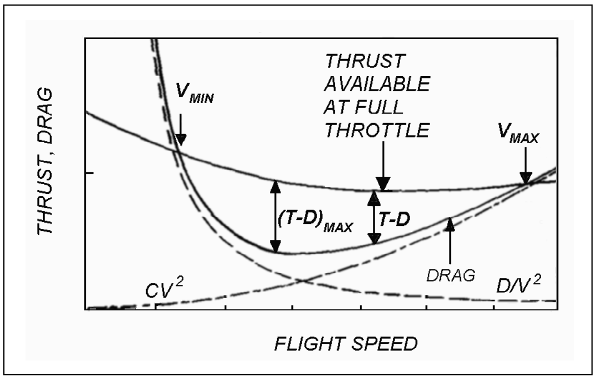

Figure 5. Variation of thrust and drag with flight speed. At any given altitude, the drag of an airplane at a low flight speed is large and is dominated by the lift-dependent drag. At a high flight speed, the drag is again large due to the zero-lift drag which increases as the square of flight speed. Thus there is an intermediate speed at which drag is a minimum. The airplane has a good rate of climb near this speed.

In this figure, it is seen that the engine thrust is larger than drag over a range of flight speeds from $V_{\min}$ to $V_{\max}$ within which range only level flight is possible. Outside these limits, thrust is less than drag and steady level flight is not possible.

Within the range of $V_{\min}$ to $V_{\max}$, the airplane will have a positive rate of climb at full throttle as thrust is larger than drag. At any flight speed in this range, this rate of climb can be calculated using (7). *Figure* 6 illustrates the results of such calculation performed at various flight speeds. At any altitude, there are two flight speeds, $V_{\min}$ and $V_{\max}$, at which the airplane rate of climb is zero. There is an intermediate speed which results in the maximum rate of climb and it would be desirable to fly at this speed when climbing after take-off. There is an altitude $h_{\max}$ at which $V_{\max}$ and $V_{\min}$ come together and the rate of climb at this speed is zero. This altitude represents the highest altitude at which level flight is possible for the given airplane and engine configuration. This altitude is termed the 'absolute ceiling' for the airplane. The maximum flight altitude of an airplane,the 'service ceiling', is slightly lower than this altitude. Any obstacles in the flight path of an airplane (like mountain peaks) should be at a lower level than the service ceiling of the airplane.

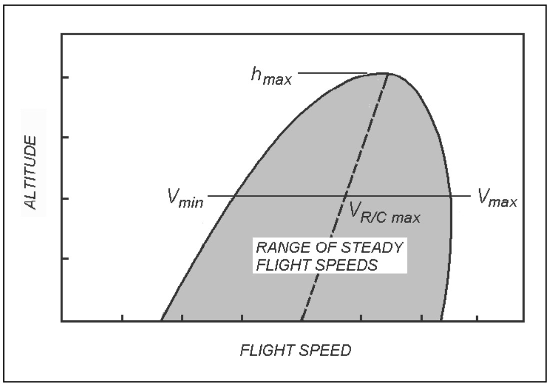

Figure 6. Variation of flight speeds with altitude. At any altitude, there is a minimum and a maximum level flight speed. The speed for the best rate of climb is somewhere in between the maximum and minimum speeds. As the flight altitude increases, all these speeds approach each other. At the absolute ceiling, there is only one flight speed possible. The rate of climb is zero at the absolute ceiling. Airplanes operate only below a service ceiling at which the rate of climb is about 100 ft/min.

It may be remarked here that flight at $V_{\min}$ is risky. Any reduction of flight speed results in a negative value for excess power and thus results in a sink rate. Any effort by the pilot to arrest this (by operating the elevator) results in a further loss of speed and consequent increase in sink rate. This may quickly lead to stall. In general, airplane flight at any speed below the speed for drag minimum, V_{md} has this characteristic and airplanes always come for landing above this flight speed. Configuration changes which increase A (like lowering landing gears, deployments of flaps) help in reducing this speed. Some transport aircraft employ airbrakes in the landing configuration to increase A and thus lower V_{md}.

3. Endurance and Range

How far and for how long an airplane can fly are questions of obvious relevance to aircraft operations. The first question is more relevant for a transport airplane while the second is more relevant for a surveillance airplane. As the flight of an airplane has to terminate before all the fuel is burnt, the answers to the above questions involve the airplane's fuel capacity and fuel consumption. Thus the specific fuel consumption of the aircraft engine, C, defined as the weight of fuel burnt per unit thrust per hour plays a major role in these calculations. Here we shall only consider the turbofan type of engines for which C is independent of flight speed and altitude within broad limits. A typical value for C is about 0.6 kg per kg of thrust per hour. The rate of fuel consumed is the product of thrust, T and the specific fuel consumption C. As the rate of fuel consumed by an aircraft is also the rate at which the aircraft weight W decreases, we have

$$\frac{\mathrm{d}W_{\mathrm{fuel}}}{\mathrm{d}t} = TC = -\frac{\mathrm{d}W}{\mathrm{d}t} \ . \tag{9}$$

We may use (8) to get a relation between elemental time duration and the corresponding airplane weight reduction due to fuel burned as

$$\mathrm{d}t = -\frac{\mathrm{d}W}{TC} \ . \tag{10}$$

Endurance E defined as the time of flight during which the airplane weight changes from the initial weight with fuel, W_{i}, to the final weight W_{f} is given by

$$E = \int_{W_{\mathrm{i}}}^{W_{\mathrm{f}}} \frac{-\mathrm{d}W}{TC} \ . \tag{11}$$

In (9), the values of T and C in general depend on the airplane weight, flight speed and altitude. Thus, it cannot in general be easily integrated to get the total flight time E. However, it is easily integrated in the special case of flight at constant C_{L} so that L/D is a constant. In this case, the thrust required is directly proportional to the instantaneous airplane weight. This implies that as the fuel is consumed, the airplane makes an adjustment (reduction) of flight speed or altitude (increase) or both so as keep the lift equal to weight at constant C_{L}. This can be done in more ways than one. We shall only consider the case of flight at constant speed but varying altitude. This means that the airplane is in a slowly climbing flight at a constant speed. Under this assumption, we can obtain the time of the flight (endurance) as

$$E = \int_{W_{\mathrm{i}}}^{W_{\mathrm{f}}} -\frac{1}{C}\frac{L}{D}\frac{\mathrm{d}W}{W}$$

or

$$E = \frac{1}{C}\frac{L}{D}\ln\left(\frac{W_{\mathrm{i}}}{W_{\mathrm{f}}}\right) . \tag{12}$$

In (12), W_{i} and W_{f} refer to the initial and final weights of the airplane. $W_{\mathrm{i}} - W_{\mathrm{f}}$ thus represents the weight of fuel consumed. Endurance E clearly increases with increase of fuel carried. This equation also shows that the endurance is a maximum at the flight speed corresponding to maximum L/D.

One may calculate in a similar manner the horizontal distance traveled by the airplane, the range R starting from (10). An elemental distance traveled by the airplane, $\mathrm{d}R$, is easily seen to be

$$\mathrm{d}R = V\mathrm{d}t = -\frac{V\mathrm{d}W}{TC}.$$

Thus for the same flight profile as indicated above (at constant C_{L}) we have

$$R = \frac{V}{C}\frac{L}{D}\ln\left(\frac{W_{\mathrm{i}}}{W_{\mathrm{f}}}\right) . \tag{13}$$

Equation (13) clearly shows that the range of an airplane is a maximum when the factor $(V/C)(L/D)$ is largest. Thus the best flight speed for maximum range is higher than the flight speed for maximum endurance. It can be shown that the

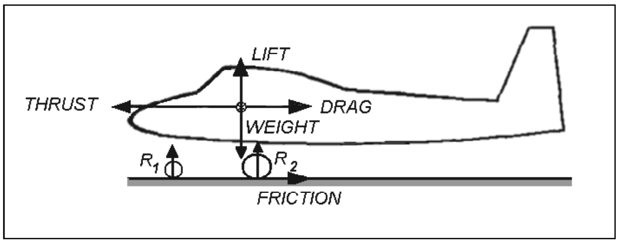

Figure 7. Forces on an airplane during take-off. The airplane accelerates on the runway to the take-off speed under the influence of engine thrust. There is a resistance to this motion due to aerodynamic drag and ground friction. As these are relatively small compared to thrust, use of an average value for these forces leads to an insignificant error in the calculation.

ratio is about 1.3. But due to practical considerations, airplanes do not always fly at these best speeds and the penalty for flight at other speeds is accepted.

Equations (12) and (13) are called the 'Breguet formulae' for endurance and range of airplanes. For a typical subsonic civil transport, $V = 1000$ km/hr, $L/D = 18$, $W_i/W_f = 1.3$, $C = 0.6$ kg/kgf.h. For this condition, the flight time is about 8 hours and the range is about 8000 km.

4. Accelerated Flight

In order to perform a useful mission, an airplane has to take off from an airfield, climb to cruise altitude, perform maneuvers like turns as required and finally descend and land at an airfield. In addition combat airplanes have to make rapid pull-ups from dives as dictated by the requirements of air combat. During these phases of flight, the airplane is in accelerated flight. We shall consider a few of these flight cases in some detail here.

4.1 *Take-off*

During take-off, an airplane at rest at the beginning of the runway accelerates under full engine power until it reaches the take-off speed at which it lifts off to climb. During the take-off roll, the airplane motion is resisted by ground friction and aerodynamic drag as in *Figure* 7 and the equation of motion along the horizontal direction can be written as

$$m\frac{\mathrm{d}V}{\mathrm{d}t} = T - D - F \ . \qquad (14)$$

Here, the acceleration of airplane, $\mathrm{d}V/\mathrm{d}t$ is due to forward thrust of the airplane reduced due to the resisting forces of drag D and ground friction F; m is the mass of the airplane at take-off.

Equation (14) can be integrated twice to get the ground roll required to achieve the take-off speed V_{TO} (which is about 1.2 times the stall speed of the aircraft at the take-off condition). The quantities T, D and F vary during the ground roll, but an average value of $T - D - F$ (corresponding to its value at 0.7 of take-off speed) can be used for calculation purposes. The resisting force due to friction is the product of ground reaction $(R_1 + R_2)$ and the coefficient of rolling friction μ. The value of μ depends on the nature of the runway and is about 0.02 for smooth concrete and is about 0.1 for an unprepared grass surface. The ground reaction can be easily calculated using the equation of equilibrium in the vertical direction. Equation (14) finally leads to an expression for ground roll S_{GR}, as

$$S_{\mathrm{GR}} = \frac{m V_{\mathrm{TO}}^2}{2\,(T - D - F)_{\mathrm{av}}} \,. \tag{15}$$

If V_{TO} is assumed to be 1.2 times the stall speed of the airplane, then

$$V_{\mathrm{TO}} = 1.2\sqrt{\frac{mg}{(\rho S_{\mathrm{w}} C_{\mathrm{L,max}})}} \,. \tag{16}$$

Thus

$$S_{\mathrm{GR}} = \frac{1.44\ g\ m^2}{(2\,\rho\, S_{\mathrm{w}}\, C_{\mathrm{L,max}}\ (T - D - F)_{\mathrm{av}})} \,. \tag{17}$$

Typical ground roll varies from about 100 m for small airplanes to 2000 m for large airplanes. We may define the take-off distance S_{TO} as the distance on the ground required for clearing an obstacle of fifty feet height at the end of the runway. S_{TO} then includes some flight distance and is larger than S_{GR} by a factor of about 1.7. The length of a practical runway for the take-off of a twin-engined airplane must consider the possibility of one engine failure during take-off when the available thrust is much reduced. This will be considered in a later article.

4.2 *Landing*

An aircraft may be assumed to touch down onto a runway at about 1.3 times the stall speed. Then it rolls to a stop under the combined decelerating force due to aerodynamic drag (enhanced by using spoilers in the case of large aircraft), engine thrust (either zero or negative if thrust reversers are employed) and ground friction (enhanced by using wheel brakes to a friction coefficient of about 0.4 on dry runways). The stall speed in the landing configuration is reduced by using flaps to the maximum extent as the increase in the drag due to this is not a disadvantage as in take-off.

As in take-off, one may derive the landing distance based on (15) as

$$S_{\mathrm{GR}} = \frac{m\,V_{\mathrm{L}}^2}{(2\ (T-D-F)_{\mathrm{av}})} \tag{18}$$

$$V_{\mathrm{L}} = 1.3\sqrt{\frac{mg}{\rho S_{\mathrm{w}}\,C_{\mathrm{L,max}}}}\ . \tag{19}$$

Safety regulations specify a landing distance as the horizontal distance covered by the airplane when landing from a height of 50 feet. This distance is larger than S_{GR} by a factor of about 1.7.

4.3 *Pull-ups and Turns*

An airplane descending rapidly can arrest the descent by a pull-up. During this maneuver, the airplane flight path is locally a circular arc of radius R as in *Figure* 8. During the maneuver, the lift has to overcome the sum of the weight and centrifugal force as in *Figure* 8. We may define a total load factor n as the

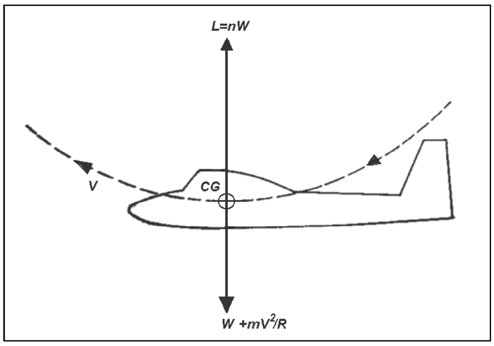

Figure 8. Forces on an airplane in a pull-up. During a pull-up, the wing lift is larger than airplane weight due to inertia force. As the flight path is curved, there is a centripetal acceleration V^2/R and hence the inertia force mV^2/R. Thus the lift is the sum of airplane weight and the inertia force mV^2/R.

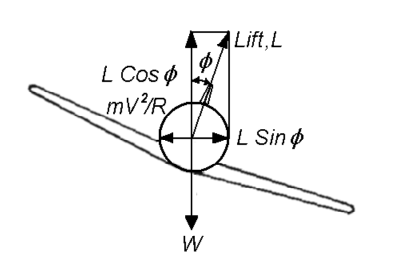

Figure 9. Forces on an airplane in a turn. During a turn, there is a centrifugal force mV²/R acting outwards from the centre of rotation. The lift L balances the resultant of airplane weight and the centrifugal force.

ratio of lift to weight, L/W. We then have an expression for the angular velocity in the vertical plane, the pitch rate ω, as follows:

$$F = mV^2/R = W(n-1)\ . \tag{20}$$

Hence

$$R\ =\ \frac{mV^2}{W\ (n-1)}\ , \tag{21}$$

and

$$\omega\ =\ \frac{V}{R}\ =\ \frac{g(n-1)}{V}\ . \tag{22}$$

In a turn, an aircraft follows a circular path of radius R in the horizontal plane. The wing lift thus has to balance the resultant force due to gravity and centrifugal force as in *Figure* 9. The bank angle ϕ is such as to tilt the lift vector into line with the resultant force. The radius of turn, R, can be calculated as follows. Defining the total load factor, $n = L/W$, we have for equilibrium,

$$F = W\tan\phi = W\sqrt{n^2-1}\ . \tag{23}$$

But, the centrifugal force F is also $F = mV^2/R = W\sqrt{n^2-1}$.

Thus

$$R\ =\ \frac{V^2}{\left(g\sqrt{n^2-1}\right)}\ . \tag{24}$$

The angular velocity of the airplane during the maneuver (the rate of turn), ω, can then be written as

$$\omega\ =\ \frac{V}{R}\ =\ \frac{g\sqrt{n^2-1}}{V}\ . \tag{25}$$

Rapid pull-ups and sharp turns are maneuvers performed by a combat airplane particularly during an intercept mission while chasing or being chased by an enemy airplane. During these maneuvers, the load on the wing structure is larger than in level flight by the factor n which often reaches values as large as 10. The wings of a combat airplane have to be designed for withstanding these large maneuver loads without failure. Pilots of combat aircraft also suffer these g-loads and often use special g-suits to minimize the physiological effects associated with high-g maneuvers.

The need for sharp pull-ups and turns is absent in the case of a civil airplanes. Hence, these aircraft are designed for a much smaller maneuver load factor of 3.0 to 3.8. This reduces the strength requirement of the structure and the structure weight is reduced. This makes the airplane more economical.

Suggested Reading

[1] Courtland D Perkins and Robert E Hage, *Airplane Performance Stability and Control*, John Wiley & Sons, Inc., 1958.

[2] Martin E Eshelby, *Aircraft Performance – Theory and Practice*, Butterworth-Heinmann, first reprint 2001

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