Yoichiro Nambu: Seer* of Modern Physics

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*I have taken this term of reverence from Madhusree Mukerjee's excellent profile of Nambu that appeared in the February 1995 issue of *Scientific American*. I urge all readers of *Resonance* to read this not only to know about Nambu the scientist, but also about Nambu the human being. I have also, inevitably, used other parts of this profile in preparing my own article.

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In this article I try to explain, in as non-mathematical a manner as possible, the path-breaking contributions of Yoichiro Nambu who was awarded the Nobel Prize in Physics for 2008. It can easily be said that the three pillars of modern high energy physics are: The Electroweak Theory, Quantum Chromodynamics and String Theory. Nambu made fundamental contributions in all these areas.

1. Introduction

The basic theme that underlies the 2008 Nobel Prize in Physics is *Symmetry*. While the work of Kobayashi and Maskawa concerned *broken symmetries*, Nambu (*Figure* 1) was rewarded for elucidating the subtle notion of *spontaneously broken symmetries*, and for succesfully applying these to some outstanding problems of elementary particle physics.

Let us first try to understand the notion of symmetry in physics, and understand its extreme importance. Everyone has an intuitive idea of what symmetry is from our day to day experiences. Symmetry signifies orderliness and is often equated with beauty and aesthetics. We see this order in a variety of things around us. Let us take symmetry in flowers for example. If we look at the beautiful flower in *Figure* 2 it is intuitively obvious that its shape has a lot of regularity or order in it. We would say that its shape is *symmetrical*. But in physics this notion, in addition to having aesthetic value, also turns out to be very powerful. For that, we need to make this notion more precise, to the point of being able to build *mathematical structures* around it.

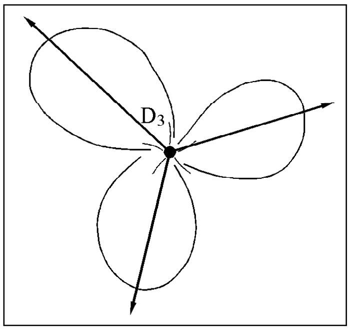
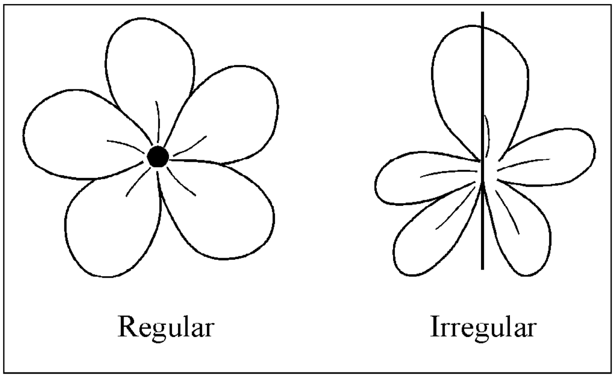


Figure 1 (left). Yoichiro Nambu: For the discovery of the mechanism of Spontaneous Broken Symmetry in subatomic physics. Figure 2 (center). Floral symmetry. Figure 3 (right). D-3 symmetry.

To appreciate how one goes about it, let us consider a simpler situation depicted in *Figure* 3. If we draw lines starting from the centre of the flower and along its petals, there is a three-fold symmetry. What we really mean by this is that if we rotate the pattern by 120° it remains the same. There are two notions that one should clearly distinguish here; one is the notion of rotating the pattern by 120°, which can be performed on any pattern whether it is symmetric or not. We denote this as *a transformation*. The other is the notion of certain patterns being left *unchanged* by transformations. In the latter situation we say that the pattern has certain symmetry and the particular transformation can be called a *s*ymmetry transformation of the pattern.

Transformations can be studied on their own without reference to any patterns that may be symmetric under them. For example, one can study the *composition* properties of transformations. If one transformation is a rotation, say, by 15° and another by 22° and we follow the first transformation by the second we will get a third transformation which is rotation by 37°. This leads to the important mathematical notion of *groups of transformations*. In the particular example considered, the order of composing the transformations was irrelevant and groups of such transformations are called *Abelian*. On the other hand, if we were to consider rotations in three dimensions as the transformations, the situation is completely different. Imagine taking a book and defining the X- and Y-axes to be in the plane of the pages and the Z-axis to be normal to that plane. Then first rotating the book around Z-axis by 90 degrees and then rotating it by 90 degrees around X-axis would give a different orientation were we to do these operations in the other order. We say the different operations do not *commute* and the group of transformations would be *non-Abelian.*

Figure 4 . Broken symmetry.



Often in physics, one is more interested in transformations that leave unchanged the mathematical form of some laws. For example, the Newton's laws of gravitation do not change their form when all vectors in question are rotated in the same manner; they are also unchanged if we change the sign of the time variable, etc. The form of Maxwell's equations are unchanged if the space-time coordinates, the electric and magnetic fields are transformed according to special relativity. The same Maxwell's equations when written in terms of *vector potentials* are also unchanged when one performs the *Gauge Transformations*. But it should be emphasized that gauge transformations are in a totally different class from symmetry transformations mentioned before.

Apart from signifying symmetry in physical laws, these considerations also lead to powerful consequences. For example, invariance of laws under rotations implies that *angular momentum* is conserved, that invariance under certain phase transformations of charged fields implies conservation of charge, etc.

The notion of *broken symmetries* plays as important a role as the notion of symmetry itself. *Figure* 4 exemplifies broken D-5 symmetry. When the departure from symmetry is small, it is meaningful to talk of a particular symmetry as broken, and the mathematical machinery found so useful in describing symmetries can nevertheless be exploited efficiently. The work of Kobayashi and Maskawa was concerned with the broken symmetry between matter and antimatter (more precisely, the CP-symmetry of elementary particle physics).

2. Spontaneously Broken Symmetry (SSB)

Nambu's work for which he was awarded the Nobel Prize concerned the rather subtle notion of *spontaneously broken symmetries* which I shall explain in some detail and generality before turning to Nambu's work in particular. This notion is subtle because one can say that the symmetry is both there and not there! That the symmetry is broken, and yet not broken! It turns out that the notion is so universal that it embraces widely different physical systems, from the behaviour of magnets to the mechanism of mass generation for subatomic particles!

Let us consider a pencil that is perfectly symmetric about its long axis, and which has been sharpened to a point. Now if we balance this pencil on its sharpened point, we have a situation of *unstable equilibrium*. Even a slight disturbance would make the pencil fall. In the (unlikely) situation of absolutely no disturbance, the pencil would remain in this state of unstable equilibrium, at least classically. Quantum mechanically, the situation is of course very different owing to the uncertainty principle. The pencil has some probability of being in the position of unstable equilibrium and some for falling. But in which horizontal direction will the fallen pencil face?

Let us confine ourselves to classical physics and analyse this interesting problem. Let us say that the disturbance is a weak breeze in the northerly direction. Then the pencil will fall facing north and no one will be surprised. Even though the pencil and the gravitational potential both were perfectly symmetric, the *perturbation* broke the symmetry. No matter how weak the symmetry-breaking perturbation, the perturbed state fully remembers the perturbation. A somewhat similar situation occurs in the case of degenerate perturbation theory in quantum mechanics.

Now let us consider the case where the breeze is totally random (the reader may rightly wonder whether anything in nature is truly random! Of course, quantum phenomena are indeed truly random). This means that the breeze is as likely to blow in one direction as any other. More precisely, the *probability distribution* for the direction of the breeze is *rotationally symmetric*. Since things are random, there is no way of predetermining the direction in which it will blow at first, even though one knows for sure that it will blow in *some* direction. Then the pencil will fall in that direction, and having fallen in a particular direction will break the rotational symmetry! So in this situation, there is rotational symmetry in the pencil and the breeze angle distribution, yet the direction of fall of the pencil does not have this symmetry. Equally, the final direction of the pencil represents *broken symmetry* but the symmetry is unbroken by the pencil and the distribution.

A more folkloric example is a hungry dog presented with two equally tasty bowls of food. In this case the relevant symmetry is called Z_2 (two-fold symmetry) and the final choice breaks this *spontaneously*. Another example is a circular dining table with each plate accompanied by a fork. Each diner has no particular preference for picking the fork to either the left or the right so there is an inherent Z_2 symmetry. Yet the random choice made by the first diner forces everyone else to make the same choice.

A noteworthy feature of these cases is that if one were to perform these 'experiments' many times over, the directions of broken symmetry would themselves be distributed uniformly, thus 'restoring' the symmetry in a statistical sense.

Real experts would object to calling these instances as examples of *spontaneous breakdown of symmetry* on the ground that in *finite* systems no such symmetry breakdown is possible. This is of course true but it would require a considerable amount of technical *tour de force* to both explain as well as understand it. Instead let us turn to the classic case of *ferromagnets* where, even by the standard of diehard experts, there is genuine spontaneous symmetry breakdown.

3. A Ferromagnet

Let us consider a bar magnet, which is an example of a ferromagnet. What is happening on an atomic scale is this: Each atom is like a tiny magnet (you can learn more about this aspect of atoms from any good book on atomic physics) and in a bar magnet all these atomic magnets (a very very large number of them) are more or less aligned so that the 'magnetic power' of all these add up to give the magnetism of the bar magnet. This is called the *ordered state* of the ferromagnet and is shown in the left of *Figure* 5.

We also know that when a bar magnet is heated it loses its 'magnetism'. What is the explanation of this on an atomic scale? It is that the elementary atomic magnets are all oriented randomly cancelling out each others' magnetism. This

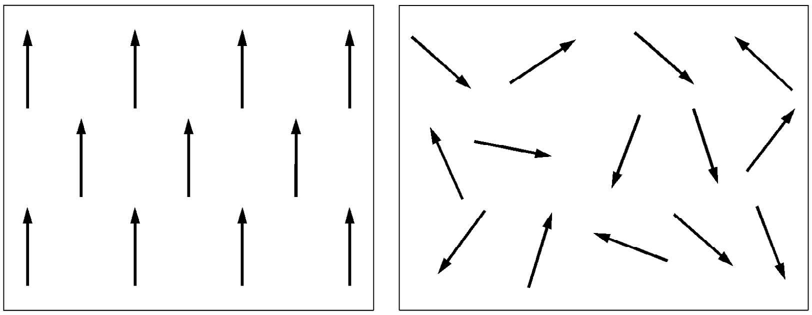


Figure 5. Ordered and disordered states of a ferromagnet.

state, shown on the right of *Figure* 5, can be called the *disordered state* of the ferromagnet. There is a mathematically precise way of characterizing the degree of this *order*, and in the particular case of the ferromagnet can be done through the amount of magnetism per unit volume, or more precisely the *magnetisation*. This is an example of the important notion of an *order parameter*. As we cool the ferromagnetic system from high to low temperatures, this order parameter changes from zero to non-zero values. In this case, the transition happens rather abruptly. We say that the system undergoes a *phase transition* at this temperature. Other examples of phase transitions are water-to-ice, water-to-steam, normal-to-superfluid, etc.

Let us examine the connection of all this to spontaneously broken symmetries, and in particular identify the relevant symmetry. Let us consider a single isolated atomic magnet, symbolically denoted by one of the arrows in *Figure* 5. Its energy does not depend on its orientation and we have a fully rotationally (in the sense of three-dimensional rotations) invariant system. If the elementary magnet is in *uniform external magnetic field*, the energy of the magnet depends on its orientation relative to this magnetic field. The atomic magnet system is no longer invariant under three-dimensional rotations (it is nevertheless still invariant under rotations about the direction of the external magnetic field).

Instead of the elementary magnet being in an external magnetic field, let us consider it in the presence of another elementary magnet. For example, consider any two arrows in the disordered state of *Figure* 5. Denoting the elementary *magnetic moments* by $\vec{\mu}_1, \vec{\mu}_2$, the interaction energy of the system can be written as

$$E = -\vec{\mu}_1 \cdot \vec{\mu}_2 \quad . \tag{1}$$

If we hold the first magnet fixed, the energy of the second magnet is no longer fully rotationally invariant, but only invariant under rotations about the direction of the first magnet. However, the energy of the system is fully rotationally invariant if we rotate *both* the magnets the same way. Thus the total system is rotationally symmetric. Now under the assumption that all the magnets are located at the sites of a lattice (atoms of a crystal are indeed arranged so), and that only *neighbouring* magnets influence each other (not true of real-life magnets, but nevertheless a reasonable idealization and what's more, real-life magnets do not change the essential features like spontaneous symmetry breaking, etc.,) the energy of the entire ferromagnet can be represented as

$$\mathcal{H} = -J \sum \vec{S}_i \cdot \vec{S}_j \quad , \tag{2}$$

where the sum is taken over all sites of the lattice, and for each site the sum is taken over all directions. This is called the Heisenberg Ferromagnet problem when $J > 0$ (the system is an *antiferromagnet* when $J < 0$). This system is also invariant under the simultaneous rotation of all the spins.

So how does the system pass from its disordered state to its ordered state? Heisenberg solved this problem for the first time in 1928. Each elementary magnet is in an environment of its neighbouring magnets and depending on the latter, the elementary magnet will have a probability distribution for its orientation. But the big question is to how they conspire to orient in a particular direction. The probability for that to happen seems extremely tiny. It is in fact the collective dynamics of the entire assembly of spins that results in this. Heisenberg's work was the first and explicit realisation of what is, in modern terminology, called *Dynamical Spontaneous Symmetry Breaking*. This is one of the most important and fascinating physical systems for the reader to study and understand.

The final orientation of the magnets breaks the rotational invariance though there is no source of symmetry breaking in the interactions. The symmetry is in fact *spontaneously* broken. How any particular direction gets chosen by all the spins is somehow coded in the initial configuration and its thermal fluctuations, but this coding is not something easy to decipher! All one can say is that if many many samples of the ferromagnet are cooled below the transition temperature, each one will end up getting its bulk magnetisation in a different direction with equal probability.

3.1 *Spin Waves*

It is obvious that when all the spins are parallel the energy of the system is the lowest (as $J > 0$). It is said to be in its ground state. It is interesting to now look at the nature of *excited* states of this system. Consider a configuration of the type shown in *Figure* 6. We can consider the angle between every neighbouring pair to be the same. This excitation is called a spin wave for obvious reasons. Smaller the angle between neighbouring spins, longer the wavelength, and lower is the energy of the excitation. The angle between a neighbouring pair of spins and the wavelength of the spin wave are related as $\theta = \frac{2\pi a}{\lambda}$, which is just the wave number times the lattice spacing a. Thus excitations of arbitrarily low energy can exist corresponding to arbitrarily low wave vectors (momentum). They are the *massless* excitations. One also says that there is no *mass gap* in the spectrum. *It is worth emphasizing that these excitations can be massless only if the symmetry*

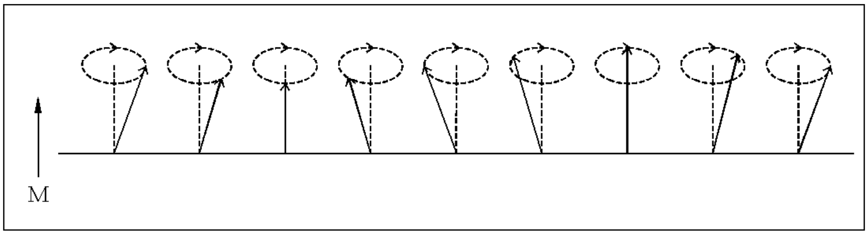


Figure 6. Spin wave in a ferromagnet.

that is spontaneously broken is continuous. For example, if the spontaneously broken symmetry is *discrete* then it is not possible to make adjacent configurations arbitrarily close and the energy of excitation cannot be made arbitrarily low. These massless excitations are called Nambu–Goldstone (NG) modes for reasons to be explained shortly.

3.2 *SSB in Field Theory*

To illustrate how these concepts arise in *Field Theory*, consider the simplest type of a field, namely a scalar field $\phi(x)$. From our previous discussions it is clear that the necessary ingredients for realising SSB are: (i) An *unstable* state that is explicitly symmetric. (ii) A set of *degenerate* states of lower energy than the unstable state each of which explicitly breaks the symmetry. (iii) A transformation that carries one of the degenerate states to any other degenerate state. Let me clarify the choice of words used here with the pencil example discussed right in the beginning. The symmetric unstable state there was the rotationally symmetric state of the pencil balancing in a perfectly vertical direction. All the horizontal states, each of which explicitly breaks rotational symmetry, are *degenerate* in energy in the sense that their energies are all equal. The transformation that carries any one of the degenerate horizontal states to any other is of course a rotation about the vertical axis. An example of a potential energy function that has these properties is the *Mexican Hat Potential* shown in *Figure* 7. The following explicit formula for the potential (note that this does not represent the full Mexican Hat potential, but only a vertical section of it) also has these properties:

$$V(\phi) = \lambda(\phi^2 - \mu^2)^2 \quad , \qquad \lambda, \mu > 0 \quad . \tag{3}$$

This potential is symmetric under the transformation $\phi \to -\phi$ and represents the *discrete* two-fold Z_2 symmetry. It is easy to verify that the stationary points (the maxima and minima) of this function are at $\phi = 0$ (maxima), and $\phi = \pm\mu$ (minima) with the values $V(0) = \lambda\mu^4$ and $V(\pm\mu) = 0$. It should be noted that the unstable state $\phi = 0$ is symmetric under $\phi \to -\phi$ while the two degenerate stable states go into each other under this transformation.

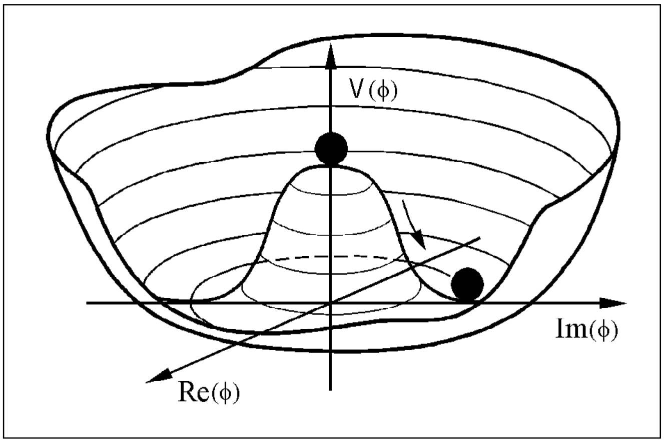


Figure 7. The Mexican hat potential.

Now let us examine the nature of the excitations around one of the degenerate ground states, say, $\phi = \mu$. The way to address this is to write the full field ϕ as

$$\phi = \mu + \phi_{\text{exc}} \quad . \tag{4}$$

Then it is an elementary exercise to check that

$$V(\phi_{exc}) = 4\mu^2\lambda\phi_{\text{exc}}^2 + 4\mu\lambda\phi_{\text{exc}}^3 + \lambda\phi_{\text{exc}}^4 \quad . \tag{5}$$

All these excitations are therefore *massive*. The thumb rule to tell when excitations are massive or not is to look at the quadratic term in the potential. If it is *positive*, the excitations are massive.

Thus we have explicitly verified that in this example, where the spontaneously broken symmetry is discrete, namely Z_2, there are no NG modes.

Now let us consider the situation represented by the full Mexican hat potential of *Figure* 7. The scalar field now has two components and it is convenient to think of it as a *complex scalar field*. The explicit form of the potential is now

$$V(\phi) = \lambda(|\phi|^2 - \mu^2)^2 \quad , \tag{6}$$

where λ, μ are real and positive. The stationary points are: (i.) $\phi = 0, V(0) = \lambda\mu^4 > 0$ (local maxima) and (ii.) $|\phi|^2 = \mu^2$ with $V = 0$ (degenerate minima). Notice that the degenerate states, i.e., with the same value of V, lie on a circle of radius μ. The transformation that connects them is *rotation* in the ϕ-plane expressed by $\phi \rightarrow e^{i\theta}\phi$. Not surprisingly, this rotation is also a *symmetry* of the Mexican hat potential.

After SSB, the system picks one point on this circle of degenerate states. It can pick any point and the true content of the symmetry is that physically all of them are equivalent. Let us say that the point chosen is $\phi = \mu$. The nature of excitations is now given by

$$V(\phi_{\text{exc}}) = 4\lambda\mu^2(\text{Re}\phi_{\text{exc}})^2 + 4\lambda\mu(\text{Re}\phi_{\text{exc}})|\phi_{\text{exc}}|^2 \quad . \tag{7}$$

This shows that $\text{Re}\phi_{\text{exc}}$ (Re stands for real) behaves like a *massive* excitation, while $\text{Im}\phi_{\text{exc}}$ (Im stands for imaginary) behaves like a *massless* excitation, confirming our earlier expectation that when the symmetry that is spontaneously broken is *continuous* there should not be a mass gap. The fact that it is the imaginary part of ϕ_{exc} that became massless has simply to do with the special choice $\phi = \mu$ that was made for the spontaneously broken state; if that had been chosen to be the point $\phi = \mu e^{i\delta}$, then the imaginary part of $e^{i\delta}\phi_{\text{exc}}$ would have been the field that would have become massless and the real part, the massive field with the same value for the mass that we found earlier. We shall see in the next section that the phenomenon of *superconductivity* can be understood in terms of a *complex scalar field order parameter*.

4. A Superconducting Interlude

Superconductivity is the amazing phenomenon by which certain materials completely lose their electrical resistance when cooled below a certain critical temperature. They have many amazing properties which will not be discussed here. The phenomenon was first discovered by Kamerlingh Onnes who observed that the electrical resistance of mercury suddenly disappeared when cooled below 4.2 °C. An effect that will have a major bearing on our discussion of Nambu's work is the *Meissner Effect*. This effect was discovered by Meissner and Ochsenfeld in 1933. Consider a superconducting sample above the critical temperature placed in an external magnetic field. The field penetrates the sample as shown in the left of *Figure* 8. But as the temperature is lowered below the critical temperature the field is almost wholly expelled from the sample. This is the Meissner Effect. In reality the field is not totally expelled from the sample. As analyzed by the brothers Fritz and Heinz London, the field decays *exponentially* within the sample. It is as if the electromagnetic field becomes *massive* within a superconductor.

4.1 *Understanding Superconductivity: The Ginzburg–Landau (GL) Theory*

It took a long time to arrive at a theoretical understanding of superconductivity. An important intermediate step was the *Ginzburg–Landau* theory formulated in

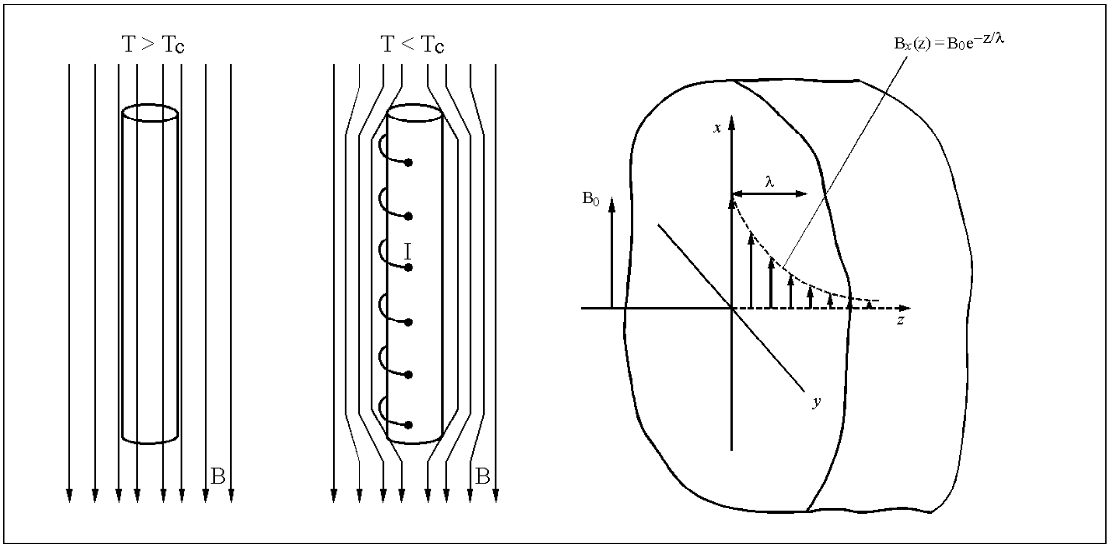


Figure 8. Meissner effect and penetration depth.

1950. It is important to understand the nature of this theory as Nambu's own approach to the complex problems of elementary particle physics was in this spirit. The GL theory of superconductivity was not a *microscopic* theory in the sense of a quantum mechanical theory of the constituents of the superconducting material. Instead, it was a theoretical construct to understand the broad features of superconductivity based on general thermodynamic principles. Such theories can also be called *phenomenological* or *effective*. Experience in several areas of physics has shown that such effective descriptions can be very powerful. In particular, GL theory introduces a complex scalar field order parameter ψ which behaves as if it carries *twice* the electronic charge. The free energy was taken to be

$$F = F_n(T) + \alpha(T)|\psi|^2 + \frac{\beta(T)}{2}|\psi|^4 + \frac{1}{2m}|(-i\hbar\nabla - 2e\mathbf{A})\psi|^2 + \frac{|\mathbf{B}|^2}{8\pi} \quad , \qquad (8)$$

where F_n is the free energy(density) of the *normal*, i.e., non-superconducting state. The last term is the energy density of a magnetic field. The term before that should be familiar from Schrödinger equation for a particle of charge $2e$ in a magnetic field. It should be noted that the remaining terms are functionally like the Mexican hat potential. Actually one requires $\frac{\alpha(T)}{\beta(T)}$ to be *negative* to get the shape of a Mexican hat. Now according to the GL theory, $\alpha(T)$ changes sign as we pass the critical temperature. Above T_c its sign is such that μ^2 in the Mexican hat potential would be negative and it is easy to see that the degenerate stable points disappear and the symmetric unstable point becomes stable corresponding to the normal state. Below T_c however the Mexican hat form obtains. It is very important to note that the underlying symmetry in GL and Mexican Hat case is the same, i.e., phase rotation symmetry. Thus the superconductor corresponds to a situation of SSB. We will have more to say on this shortly. Note that in the superconducting state where $|\psi|^2 = -\frac{\alpha}{\beta}$, the **A**-dependent terms look as if the magnetic field has become massive. That is indeed the Meissner Effect.

But the deep question is, if the superconductor realises SSB of a continuous symmetry, where are the massless spin-wave like excitations?

4.2 *Understanding Superconductivity: BCS Theory*

The microscopic understanding of superconductivity came with the Bardeen, Cooper, Schrieffer (BCS) theory in 1957 which is one of the finest accomplishments of the human mind! The physical basis of this theory is that *lattice vibrations* (phonons) induce a mild *attraction* between electrons which gives rise to pairing of electrons (Cooper pairs) and the complex order parameter of GL theory is essentially an *effective field* for these pairs. The Hamiltonian underlying the BCS theory is worth exhibiting explicitly as we shall be referring to its chief characterstics often:

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} - g^2_{\text{eff}} \sum c^\dagger_{\mathbf{k_1+q},\sigma_1} c^\dagger_{\mathbf{k_2-q},\sigma_2} c_{\mathbf{k_1},\sigma_1} c_{\mathbf{k_2},\sigma_2} \quad . \qquad (9)$$

The reader should keep in mind that this is a Hamiltonian for a *Quantum Field Theory*. The $c_{\mathbf{k},\sigma}$ are the *annihilation operators* for electrons with momentum $\mathbf{k}$ and spin σ while $c^\dagger_{\mathbf{k},\sigma}$ are the corresponding *creation* operators. Suppose our initial (quantum) state is an electron with $\mathbf{k}, \sigma$; the $c_{\mathbf{k},\sigma}$ in the first term would annihilate this state giving rise to vacuum state, but the $c^\dagger_{\mathbf{k},\sigma}$ in the term with the same values of $\mathbf{k}, \sigma$ recreates the original state. In effect, the particle continues to remain in the same state. In other words, the first term describes a *free* particle.

Now let us look at the second term. For it to be operative there must be at least two electrons in the state (why?). Let the momenta and spins of the two electrons in the initial state be $\mathbf{k_1}, \mathbf{k_2}, \sigma_1, \sigma_2$. *Pauli Exclusion Principle* says that two sets of values cannot all be equal. For example, if $\mathbf{k_1} = \mathbf{k_2}$, then $\sigma_1 = -\sigma_2$. The second term in the BCS Hamiltonian, called the *interaction* term, is such that the two c's annihilate the initial state but the $c^\dagger$'s create a different (in general) two-particle state. This amounts to *scattering* of the electrons. The overall coefficient of this term being *negative* means that the scattering interaction is *attractive.*

Note however the very important point that all the terms in the microscopic BCS Hamiltonian *conserve* the number of electrons. More precisely the Hamiltonian is *invariant* under the continuous phase rotations $c_{\mathbf{k},\sigma} \to e^{i\delta} c_{\mathbf{k},\sigma}$.

4.2.1 ***BCS Theory: Main Results***: I shall simply enumerate the main results of BCS theory. It is important to introduce the idea of *Quasiparticles*, first introduced by the celebrated Russian physicist Lev Landau. The microscopic constituents, in our case electrons, can under the right dynamical circumstances behave collectively, whereby many particles conspire to behave as a single 'particle'. You may have sometimes noticed a swarm of insects moving collectively like a ball. If you had focussed on the motion of the individual insects it would look very complex, though the collective motion as a ball often has a simple behaviour (like moving in a regular path). This is the best analogy I can think of for quasiparticles.

The energy spectrum in terms of these quasiparticles has the following simple features: (i) There is a *mass gap*; this explains the complete absence of resistivity when the superconductor is in its ground state. Resistivity arises out of scattering and with a finite mass gap it takes a lot of energy for scattering to be effective. (ii) The other important consequence of BCS theory was a microscopic explanation of the *Meissner effect*. The most distinguishing feature of the ground state BCS wavefunction is that it is a *superposition* of states with differing electron numbers.

5. Nambu and Superconductivity

Nambu, though a particle physicist by conventional point of view, got deeply involved in the problem of superconductivity. The dramatic circumstances are best understood by his own words: "An event happened, in the meantime, for which my long-standing interest in condensed matter dating back to my Tokyo days proved useful. One day, before the publication of the BCS paper (8 July 1957), Bob Schrieffer, still a student, came to Chicago to give a seminar on the BCS theory in progress. I was also very much disturbed by the fact that their wave function did not conserve electron number."

We have already mentioned this fact about the BCS wavefunction not conserving electron number. But what precisely was its significance to Nambu? From quantum mechanics we know that given two bona fide physical states, their complex linear superposition is also a bona fide physical state. So for example, we can form a superposition of states of *different angular momenta*. But a more sophisticated analysis shows that there are quantum numbers for which it is meaningless to form superpositions of states with different values for these quantum numbers. This is called a *super-selection rule* and electron number and electric charge obey this. In effect what it means is that the BCS wave function somehow violated *gauge invariance* which is one of the sacred principles! This is what disturbed Nambu so deeply. It was like presenting a theory that violated energy conservation.

5.1 *Nambu, BCS and Gauge Invariance*

To understand the origin of this apparent violation of gauge invariance, Nambu carefully analysed the Meissner effect, in particular the London equation. It is to be emphasized that the gauge non-invariance that he was concerned with had nothing to do with the way the London equation is often presented:

$$\mathbf{j}_{\mathrm{s}} = -\frac{n_{\mathrm{s}}e^2}{mc}\mathbf{A} \quad , \tag{10}$$

where $n_{\mathrm{s}}, \mathbf{j}_{\mathrm{s}}$ refer to the superconducting density and superconducting current density. This equation, because of the explicit appearance of the vector potential $\mathbf{A}$ appears gauge non-invariant. The above equation is usually qualified to be valid only in the *London gauge*. In any case it can be recast in the gauge-invariant form:

$$\nabla \times \mathbf{B} = -\frac{n_{\mathrm{s}}e^2}{mc}\mathbf{B} \ . \tag{11}$$

So this is not what Nambu was concerned with. The gauge invariance issues that occupied him were far deeper and subtler. He realised that lack of gauge invariance manifested itself explicitly through the fact that the current corresponding to the longitudinal component of the electromagnetic field did not vanish in the original BCS formulation. To understand this point better, consider static electromagnetic phenomena. The current conservation equation, which is equivalent to gauge invariance, reads $\nabla \cdot \mathbf{j} = 0$ or in terms of Fourier components, $\mathbf{k} \cdot \mathbf{j}(\mathbf{k}) = 0$. This means that the component of the current *longitudinal* to the direction of $\mathbf{k}$ vanishes. More importantly Nambu, through very careful calculations, showed that gauge invariance could be restored if collective excitations coupled to the longitudinal component were properly taken into account. What is more, he showed that these collective excitations were massless. What Nambu had shown was that the superconductor realises SSB *dynamically*, and as expected there were the massless excitations Nambu–Goldstone modes).

But he found something even deeper; he demonstrated that when the role of long-range gauge fields like the electromagnetic fields is examined properly, the

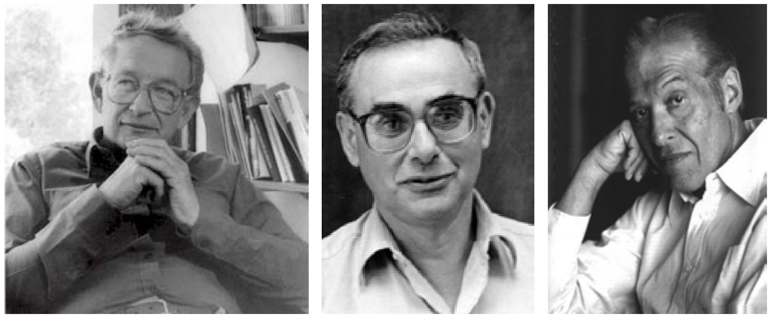


Figure 9. P W Anderson, J Goldstone and Julian Schwinger

collective massless excitations cease to manifest physically. Instead they combine with the massless electromagnetic fields to make them *massive*! Let us explain this important point in some more detail. A massless *vector field* like the electromagnetic field has *two* transverse degrees of freedom. These could be the two states of circular polarisation for a given wave vector or two mutually orthogonal states of linear polarisation, etc. What Nambu found was that the NG mode in the superconductor combined with these two to provide a longitudinal degree of freedom. Altogether now there were three degrees of freedom characteristic of a massive spin-1 particle! This then was the explanation of the Meissner effect! These results were published by Nambu in July 1959.

The famous condensed matter physicist, P W Anderson, too had realised the gauge invariance difficulties of the BCS theory and, in a paper published on 27 January 1958, barely a few months after the BCS paper, had shown that coherent excitations would resolve these difficulties and that electromagnetic gauge fields would quench these massless excitations. There were others like G Rickayzen who also published identical results in 1958. These works appeared more than a year earlier than Nambu's 1959 paper. The following quote by Nambu is an interesting insight into his personality and style of scientific creativity: "But my major concern was in the purely theoretical issues like gauge invariance, and it took me two years before I was able to convince myself of its resolution and write a paper. While in the meantime experts like N N Bogoliubov and P W Anderson were refining the BCS theory, I deliberately tried to keep independence".

5.2 *Nambu–Goldstone Particles*

In 1961 Jeffrey Goldstone, a student of Bethe and at the time a postdoc at CERN, gave a proof based on quantum field theory which has since then become a milestone result in theoretical physics called the *Goldstone Theorem*. In this work he showed that massless excitations like spinwaves in ferromagnets, and the collective excitations found by Anderson and Nambu in superconductors, are not specific to these systems. Rather, they are *generic* to systems in which a continuous symmetry is spontaneously broken. So strong was the 'belief' in the infallibility of this result that later when Higgs claimed a way out of it he was ridiculed by some prominent physicists to the point of being called 'the idiot who thought he could get around the Goldstone Theorem'! Of course, we have already seen that both Anderson and Nambu, long before Higgs and others, had shown that the massless excitations did not always manifest themselves. It was of some disappointment to Nambu that he did not show the generality of the results earlier as indicated by this quote: "As for the last point, I had been debating with myself how to write a paper addressing it as a general phenomenon, so I felt as if a prize catch had been stolen from under my nose". Soon afterwards these particles were aptly named Nambu–Goldstone modes.

6. Lessons from Superconductivity for Elementary Particle Physicists

Nambu's focus had by now shifted to the realisation that these ideas developed in the context of superconductivity could be relevant for some fundamental problems in elementary particle physics. One of these was the vexing issue of the masses of elementary particles like protons, neutrons (considered elementary around late 50's) and pions. One view of pions was that they were bound states of protons and antiprotons. Yet, the pion mass was incredibly low compared to that of protons. A bit of historical background is helpful at this stage.

It was in 1928 that Heisenberg had developed the theory of ferromagnets. This work explicitly realised SSB dynamically. He was certainly aware of the massless spinwaves in these systems. Based on many striking similarities between protons and the just discovered neutrons (discovered in 1932), he introduced the notion of *Isotopic Spin* in the same year. According to this, protons and neutrons are to be viewed on the same footing, and more precisely as two states of the new quantum number which was postulated to have the same mathematical properties as spin-$\frac{1}{2}$ angular momentum. It is said that he wanted to view the pions as the NG modes of spontaneously broken isospin rotations. We now know that such a scheme would have serious problems like *parity violations* in *strong interactions* which is not observed in nature. But what is of significance is that Heisenberg was thinking seriously about applying ideas from SSB to the problem of elementary particle masses.

Subsequently Heisenberg and Pauli initiated their program of *Unified Theory of Elementary Particles*. This was a non-linear field theory invariant under special

Figure 10. Heisenberg and Pauli.

relativistic transformations. In mid-fifties they planned a publication of their work. In 1958 Pauli withdrew from the collaboration primarily because of criticism from younger physicists and the paper remained unpublished. In 1959 Duerr, Heisenberg, Mitter, Schlieder and Yamazaki published an extended version of this work containing all the essentials of this theory. It is again significant that the Heisenberg–Pauli work as extended by the five authors explicitly discussed using SSB for addressing problems of elementary particle physics.

6.1 *Mass Generation*

It should be appreciated that SSB had shown two distinct paths to the problem of mass generation for elementary particles. The first one would be as in a superconductor *without* the massless mode having been quenched; the energy gap would correspond to particles becoming massive accompanied at the same time by a massless particle. This was the path Nambu chose to explain the nucleon mass and a very light pion. This was the Nambu–Jona-Lasinio (NJL) Model. Nambu was not alone in pursuing this direction. Independently, similar conclusions about mass generation had also been reached by Vaks and Larkin in 1960. Their work was in fact very close to the NJL model. It is a historical coincidence that Heisenberg, Nambu, Vaks, and Larkin all presented their ideas at the 1960 Rochester meeting!

The other path is to exploit the analogy with Meissner effect to make the *gauge bosons* massive but not have the massless NG in the physical spectrum. This is now called the *Higgs effect* and is the crux of the highly succesful Glashow–Salam–Weinberg model of the unification of electromagnetism and weak interactions, called simply the *Standard Model*. Let us examine the NJL model first. But before doing so we shall explain the very important notion of *chirality* that is at the heart of both the approaches.

6.2 *Chirality and Chiral Invariance*

The strategy for the NJL model was to start with a *massless* nucleon and no pion in the symmetric phase, and end up with massive nucleons and massless pions in the spontaneously broken phase. In the approach based on the Higgs effect, one starts with massless fermions and gauge bosons in the symmetric phase and ends up with massive fermions and massive gauge bosons (except for the photon) and *no* NG bosons in the SSB phase.

So in both approaches the starting point has massless spin-$\frac{1}{2}$ particles. An intermediate concept to understanding *chirality* is *helicity*. This is defined as the component of angular momentum along the direction of motion. Consider a massive particle whose spin angular momentum is along the z-axis and so is its momentum. The helicity is then positive. Now we can view the particle from another inertial frame which is also moving along the z-axis with a velocity *higher* than that of the particle. In this frame the helicity of the particle is negative. Thus for massive particles helicity is not a *Lorentz Invariant* notion. On the other hand, for massless particles, no matter how fast the second inertial frame is moving, the massless particle still appears to be moving in the same direction and of course the same velocity (that of light). Therefore helicity has a Lorentz Invariant meaning.

A massless particle with spin therefore can be thought of as an independent collection of *left-handed* (positive helicity) and *right-handed* (negative helicity) particles. Now let us revisit Heisenberg's Isotopic spin concept. We can now think of a separate *left-isotopic spin* (I^{L}) and a *right-isotopic spin* (I^{R}). Then each of these isotopic spins behaves quantum mechanically like angular momenta:

$$[I_i^{\mathrm{L}}, I_j^{\mathrm{L}}] = i\hbar\epsilon_{ijk}I_k^{\mathrm{L}}; \qquad I_i^{\mathrm{R}}, I_j^{\mathrm{R}}] = i\hbar\epsilon_{ijk}I_k^{\mathrm{R}}\ ; \qquad [I_i^{\mathrm{L}}, I_j^{\mathrm{R}}] = 0\ . \qquad (12)$$

For the moment let us consider the strong interaction world of only protons, neutrons and pions. Pions are anyway spin-0 particles. Though the left-handed and right-handed particles can be thought as independent, the *parity*-transformation maps them into each other. So as far as strong interactions are concerned, which is known experimentally to be parity invariant, everything must be symmetric with respect to interchanging left and right.

In particular, the left-handed proton and the right-handed proton must belong to the same *representation* of the group $SU(2)$. Hence, as far as strong interactions are concerned, it makes sense to consider the combinations $V_i = I_i^{\mathrm{L}} + I_i^{\mathrm{R}}, A_i =$

$I_i^{\rm L} - I_i^{\rm R}$ yielding

$$[V_i, V_j] = i\hbar\epsilon_{ijk}V_k \quad ; \quad [A_i, A_j] = i\hbar\epsilon_{ijk}V_k \ . \tag{13}$$

The V_i's are associated with isotopic spin rotations for the *entire* massive field while A_i's are associated with *chiral rotations*.

In the standard model on the other hand, there is no parity invariance. This followed pathbreaking developments coming from the discovery of parity violations in nuclear beta decay by T D Lee, C N Yang and C S Wu on the one hand, and the $V - A$ theory discovered by E C G Sudarshan and Robert Marshak on the other. Because of parity violation, the left-handed and right-handed fields no longer need to be in the same representation.

6.2.1 *Dirac Equation*: Keeping in mind that Nambu's work concerned elementary particles and that the behaviour of atomic and subatomic particles are governed by quantum mechanics the reader who wishes to understand the issues in some depth must gain familiarity with quantum mechanics. Furthermore, the description of elementary particles must also obey special relativity. Such a *relativistic quantum mechanical* description of electrons was given by Dirac in 1928. Only the equation will be written down here and some notations explained.

$$-i\hbar\gamma^\mu\partial_\mu\psi + mc\psi = 0 \ . \tag{14}$$

Here ψ is the relativistic wavefunction of the electron; γ^μ are four 4×4 matrices satisfying

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu} \ , \tag{15}$$

where the four-dimensional indices μ, ν take values $0, 1, 2, 3$ and $\eta^{\mu\nu}$ is the *Minkowski metric*. Its nonvanishing elements are $\eta^{00} = -1, \eta^{11} = \eta^{22} = \eta^{33} = 1$. From the four gamma matrices a fifth can be formed $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, which is of fundamental importance in generating chiral transformations.

7. The Nambu–Jona-Lasinio (NJL) Model

Soon after his superconductivity work Nambu began in earnest to work out their implications for some of the pressing problems in elementary particle physics, in particular those centred on the origin of elementary particle masses. He realized that similarities notwithstanding, there were important differences between the description of superconductivity and that of elementary particles. One such important difference was that the elementary particle physics had to be relativistic.

So he began looking for relativistic quantum field theories where he could investigate SSB dynamically. His first choice was a theory like *Quantum Electrodynamics*. But that, he said, might be too hard. Instead, he chose the Heisenberg–Pauli theory as a possible testing ground. To quote Nambu: "So I chose a second model, namely, a nonlinear spinor field theory which was a straightforward relativistic version of the BCS theory. To tell the truth, I did not like the fact that it belonged to the type of Heisenberg's nonlinear theory, for I had never believed that a theory like that, with no elegance at all, should be the ultimate law of physics. Nevertheless it compelled me to study Heisenberg's papers seriously. I was duly impressed when I found the concept of spontaneous breakdown invoked there. I adopted the nonlinear model because it was mathematically simple and clean provided that one did not resort to gimmicks like indefinite metric."

This quote is once again a beautiful commentary on Nambu's dislike for mathematical formalism for its own sake, and for the emphasis he laid on elegance of fundamental scientific theories. It is also a testimony to his pragmatism that he was willing to work with a theory he did not like as long as it met certain reasonable physical criteria.

The *Lagrangian* chosen for the NJL model was:

$$L = -\bar{\psi}\gamma_\mu\partial_\mu\psi - g[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2] \quad . \tag{16}$$

The first term is the familiar Lagrangian for Dirac equation and it represents on its own a free relativistic massless spin-$\frac{1}{2}$ particle. The second is of the Pauli–Heisenberg nonlinear theory type and it represents an *attractive* interaction among the fermions. Both these features bear close analogy to those of the BCS Hamiltonian presented earlier except that the NJL model is relativistically invariant. The third feature of the NJL model to focus on is its *symmetry content*. Apart from invariance under relativistic transformations, it is separately invariant under Left-SU(2) and Right-SU(2) groups of continuous transformations. This, like rotations in three dimensions is a non-Abelian symmetry. But the essence of the NJL model is retained if we consider its *Abelian* version. Then the theory is invariant under the continuous rotations

$$\psi \to \mathrm{e}^{i\theta_1}\psi \quad , \quad \psi \to \mathrm{e}^{i\theta_2\gamma_5}\psi \quad . \tag{17}$$

This simplified version has a symmetry closer to that of the BCS model. We need a scalar order parameter; $\bar{\psi}\psi$ is certainly a candidate. This is invariant under $\psi \to \mathrm{e}^{i\theta_1}\psi$ but *not invariant* under $\psi \to \mathrm{e}^{i\theta_2\gamma_5}\psi$ so that the phase in which this order parameter is *nonzero* would violate the γ_5-invariance. Therefore the γ_5-invariance plays the role of gauge invariance in BCS theory. In principle we could have also considered $\bar{\psi}\gamma_5\psi$ as an order parameter. But a nonzero value for this would have violated parity and would be unsuitable for describing strongly interacting particles.

Without solving the NJL model in detail, we can anticipate many of its key features based on the experience with BCS theory. In the NJL model the fermion (nucleon) will turn out to be *massive* in the SSB phase even though the theory is γ_5-invariant! If this seems surprising (i.e., contradicts your intuition), recall that in the BCS theory the photon becomes massive even though the BCS theory is gauge invariant! The nucleon mass in NJL actually corresponds to the energy gap in BCS theory.

Exactly as in BCS theory, in NJL there is a collective excitation coupling to the longitudinal axial current that is massless, and identified with the light pion. In the NJL theory with full non-Abelian symmetry, the pion emerges as an isotopic spin-1 particle, which is the way it is in nature! On a more technical note, the coupling of the pion to the conserved axial current (conserved because of γ_5-invariance) is precisely such as to yield what is famously known as the *Goldberger–Treiman* relation in particle physics. Of course in the real world the pion has a very small mass but is not strictly massless. Even in the NJL model the pion can be made light by a small explicit breaking of the γ_5 invariance. NJL showed that their model has in addition several *meson states*. In particular, there is a *scalar-isoscalar meson* of *twice* the fermion mass. The last state is what could be called the *Higgs Particle* today.

7.1 *Some Fallout of the NJL Work*

Subsequently Nambu, with several collaborators, developed the *soft pion emission* theories. These showed that *chiral symmetry* is really an example of what may be called a *dynamical symmetry*. A prime example of such a symmetry is gauge invariance in Quantum Field Theory (QFT) as opposed to its role in Classical Electromagnetic Field Theory. Given the probability amplitude for a reaction

$$A \quad \rightarrow B, \tag{18}$$

where A and B are arbitrary states, gauge invariance in QFT enables the prediction of the probability amplitudes for

$$A \quad \rightarrow B + \gamma \tag{19}$$

when the emitted photon carries very low momentum and hence low energy. Likewise, the work of Nambu and coworkers showed that given the amplitude for $A \to B$, chiral symmetry predicts the amplitudes for the processes $A \to B+\pi_a+\pi_b$ when the emitted pions carry very low energy and momentum. This reasoning can be recursively extended. Later Julian Schwinger and Steven Weinberg showed how to encode this body of results in the language of effective field theories. These *Chiral Lagrangian* methods have now been refined and extended by a whole community of workers into an activity called *Chiral Perturbation Theory* whose agreement with experimental data is impressive.

7.2 *Why is NJL Model so Successful?* The reader may be puzzled as to how the NJL model could have explained several key features of strong interaction physics long before the *Quark Model*, which in its early form came out only in 1964, and the proper theory of strong interactions based on the quark model called *Quantum Chromodynamics* which was formulated only in 1971! The answer to this is again provided by going back to the Ginzburg–Landau theory of superconductivity which too was very succesful even though it was not a microscopic theory. Soon after the formulation of the NJL model, J J Sakurai in a discussion of the model had rightly emphasised that the crucial ingredients for the success of the NJL model were its symmetry content along with some minimal, but important dynamical input like the attractive nature of the interactions. That was also true of the GL model. In fact this is true of all *effective* theories that we talked about earlier. Their simplicity is also what gives them their power! This is a very important message for the young readers who wish to embark on a career in physics.

8. The Other Path: Higgs Effect

Now let us consider the other path to the problem of mass generation, but again inspired by superconductivity. That is to make use of the analogue of the Meissner effect. This is what one obtains when the superconductor is coupled to a massless gauge field like the electromagnetic field. As we have already seen, in addition to the energy gap the gauge field also becomes massive. But the massless NGL mode does not manifest physically. Instead, it becomes a part of the degrees of freedom of the massive vector field.

Even though these results were already part of the earlier works of Anderson and Nambu, Anderson carried out a careful analysis of this very important problem in 1963. The inspiration for this analysis came from a very influential paper in 1962 by Julian Schwinger called 'Gauge Invariance and Mass.' In this work Schwinger showed that contrary to popular belief then (and as still taught in countless courses on Quantum Field Theory even to this day!), gauge invariance in itself does not preclude the possibility of a vector field from being massive. Though it was shown in the context of a *two-dimensional theory*, it nevertheless surprised many. From a technical point of view also, this work was special as it showed how to perform calculations in QFT without resorting to perturbative methods.

Anderson considered a model with SSB and the consequent NG modes and showed that coupling this system to massless gauge fields resulted in their becoming massive even though the system had *exact* gauge invariance. This subsequently became famous as the *Higgs effect* of particle physics. But Anderson's work did not have the kind of impact on particle physicists that it ought to have. It is instructive to consider what P W Higgs had to say on this: "However, since he had neither found an error in the proof of the Goldstone theorem nor discussed explicitly any relativistic model, Anderson's remark was disbelieved at the time by those particle theorists that read it, myself included!" This shows the magical spell that the Goldstone theorem had cast on theoretical physicists! In 1964 Abraham Klein and Benjamin Lee made a careful analysis of field theoretical issues concerned and showed that indeed the Goldstone theorem could be evaded. Subsequently the flood gates opened in this area spearheaded by the works of Brout, Englert, Guralnik, Hagen, Higgs and Kibble. The rest, as they say, is history!

Figure 11. Robert Brout, F Englert, P W Higgs, C R Hagen, Guralnik and Kibble.

8.1 *Some Reactions to The Higgs Effect*

P W Higgs narrates these amusing incidents about some early reactions to his work: "At tea before my Princeton talk the axiomatic field theorist Klaus Hepp told me that there must be an error in my work, since Kastler, Robinson and Swieca had just proved the Goldstone theorem by C^*-algebraic methods – the ultimate in rigor! Nevertheless, I survived the questions by the Princeton axiomatists." "Years later, when I met Sidney Coleman, he told me that he and his colleagues 'had been looking forward to some fun tearing this idiot who thought he could get round the Goldstone theorem'. Well, they did have some fun, but I had fun too!"

9. Two Postscripts

I would like to add two postscripts to this fantastic interplay of ideas between condensed matter physics and elementary particle physics. An interplay in which the greatest minds on both sides took part, enriching each other and enriching human experience in turn. I hope this will be a revelation to my young readers that true joy is not in compartmentalising science but to take part in it as one grand adventure of the human mind.

9.1 *A Condensed Matter Postscript*

Even in BCS systems there is a state analogous to the Higgs particle. While the NG mode that got eaten up to yield a massive photon, equivalently Meissner Effect, corresponded to the phase fluctuations, there are fluctuations in the magnitude of the order parameter as well! Condensed matter theorists refer to this as the *Amplitude Collective Modes*. This was experimentally discovered in Raman scattering experiments on NbSe2 by Sooryakumar and others only in 1981, nearly a quarter century after the BCS theory! A theoretical analysis of these modes was given by Littlewood and Varma in 1982. In that conceptual sense, condensed matter physicists have beaten the particle physicists in detecting the Higgs-like particle of SSB without having to build a formidable apparatus like the LHC (Large Hadron Collider)! But in fairness to the particle physicists, the system in which they are trying to observe the Higgs is very different and very challenging not to mention the fact that the manifestations of the Higgs particle in their case is more subtle and technologically far more challenging to detect!

9.2 *A Particle Physics Postscript*

In the NJL model no Higgs field was explicitly introduced as is done in the

Standard Model of particle physics. Instead, both SSB and the Higgs are realised entirely by the dynamics. There is a parallel in the superconductivity case also; in the BCS model no order parameter was introduced. The dynamics generated such a field on its own while in the Ginzburg–Landau theory the order parameter field was introduced by hand. In that sense the Glashow–Salam–Weinberg model is more like the GL model though it is also a microscopic theory! Now there are many attempts to realise SSB dynamically even in the Standard Model. Mass generation would then be realised without explicitly introducing a fundamental scalar Higgs field. At a different level, fundamental scalar fields are believed to be problematic for a variety of theoretical reasons. Recall that the NJL model yields a 'Higgs Particle' of close to twice the fermion mass. In the models of dynamical SSB in the standard model the relevant fermion masss is the *top quark mass*. The test of these ideas is of course whether a Higgs particle would be found at the predicted mass. In the context of present day searches for a Higgs particle, say at the LHC, it is worth pointing out that Relativistic Quantum Field Theory does not preclude the possibility of a SSB wherein mass generation for particles takes place but there exists no single particle excitation corresponding to a Higgs particle.

10. Nambu and QCD

Murray Gell-Mann and George Zweig independently discovered the *Quark Model* in 1964. This model postulated that all particles thought to be elementary like protons and neutrons are composed of *quarks* which are also spin-$\frac{1}{2}$ particles but with charges in units of $\frac{e}{3}$! At the time of its introduction the Quark Model was not much more than a book-keeping device for various properties of elementary particles much as the *Periodic Table* was for elements in chemistry. Even at this early stage it was realised that consistency with the Pauli Exclusion Principle for the wave function of the proton meant that the quarks had to be in a state of nonzero orbital angular momentum. This was not satisfactory as states with such angular momentum are higher in energy. In other words there would be states lower in mass than protons but none had been seen. It might have also implied that protons are *unstable*, an even more disastrous result. Shortly after the quark model was proposed, Nambu in 1965 introduced the idea of an additional quantum number for quarks, *Colour*, to circumvent the difficulties with the statistics of quarks. Mathematically speaking the relevant group was conjectured to be SU(3). This idea is at the very heart of Quantum Chromodynamics (QCD) which is now accepted as the correct theory of strong interactions.

With Han, Nambu developed the Han–Nambu model wherein quarks had integer charges unlike the quarks of the Gell-Mann–Zweig model. They would also be observable. This version of QCD has however not been very popular mainly because of the absence of any decisive experimental test that would distinguish it from the conventional QCD.

In 1966, again long before the development of QCD in the 1970's, Nambu made a crucial observation which he presented as a contribution to the Weisskopf Festschrift. The idea was that just as the massless photons mediate interactions between charges, massless particles (now called *gluons*) would also mediate the interaction between quarks. Nambu's observation was that it would be energetically favourable if there were eight gluons transforming as the *adjoint representation* of SU(3). This was a great step forward towards creating the correct theory, but it went largely unnoticed. The following quote from Gell-Mann makes this clear: "Since I was always convinced that quarks would not emerge to be observed as single particles ('real quarks'), I never paid attention to the Han–Nambu Model. However, it is a pity that I missed a follow-up article by Nambu. It appeared in the 1966 *Festschrift* volume devoted to my old teacher, Weisskopf. As one of his students, I should, of course, have contributed, but my habit of procrastination proved to be an obstacle. If I had seen Nambu's article, I might have concluded that his color octet vector interaction should be utilized without the Han–Nambu idea."

This quote touched on another issue central to the construction of QCD. It was that the gluons should be *vector fields* exactly like the photons. While one might have thought of it in analogy with photons, the deeper reason for wanting them to be so has to do with the fact that vector interactions preserve chiral symmetry. This, if you recall, was the symmetry content of the NJL model. Thus we see that Nambu's thinking influenced all the major ingredients that created QCD.

11. Nambu and String Theory

By the early sixties a deep pessimism had set in regarding the efficacy of QFT in addressing problems relating to elementary particles. Focus had shifted in many places to *S-matrix theory* whose ambition was to use minimal structures like analyticity, crossing symmetry and unitarity, along with some empirical input to arrive at the S-matrix. S-matrix is the totality of probability amplitudes for all possible processes. A very important concept that emerged from this approach was that of *Duality*. In 1968 Veneziano proposed a very simple formula for 2-

particle scattering that incorporated all these concepts. Immediately, this formula was generalised to n-point amplitudes. Nambu made a very systematic study of these n-point amplitudes and found them to have several remarkable properties called *factorizability*. He also found that the states had exponentially increasing density. He was very surprised to find infinitely many harmonic-oscillator type structures underlying these amplitudes. Nambu immediately recognised that this could be interpreted as modes of a *Relativistic Quantum Mechanical String*. He subsequently proposed the famous *Nambu-Goto Action* for strings in analogy with invariant length actions for point particles. Leonard Susskind and Holger Bech Nielsen had independently proposed strings as an 'almost physical' interpretation of the dual amplitudes. Nambu presented these ideas in 1969 at a meeting at Wayne State University organised by Ramesh Chand, and later at a meeting at Copenhagen in 1970. Holger Nielsen presented his ideas in an unpublished Conference presentation for the 1970 Kiev meeting. The Nambu-Goto action played a crucial role in the early developments of string theory. In 1979, in a remarkable paper Nambu established a formal correspondence between Wilson loops of non-Abelian gauge theories and string theory. This work has recently gained prominence in the context of QCD-strings and is the first manifestation of a kind of Gauge-String duality which is currently one of the most remarkable developments emerging from string theory. Nambu also developed the point of view that many such considerations are universally applicable to a diverse variety of physical systems like flux tubes, vortices, etc.

12. And Much Much More....

In this article I have given an account of Nambu's landmark contributions. Of course a full account of Nambu's important contributions does not end here. Even contributions that may have seemed minor to him were nevertheless substantial. I'll just cite two such examples. He was the first to write down the *Bethe–Salpeter Equation*. This is one of the equations that has had tremendous impact on the field and literally countless number of papers have been written on it and a countless number of people have made their careers out of it. But Nambu did not think it was so important a contribution. He was the first to propose the Ω-meson but did not publish the ideas as it met derision at the hands of physicists like Feynman. Feynman is supposed to have shouted "In a pig's eye"(an American slang for a poor, highly unlikely idea) at the conference presentation by Nambu!. The irony is that it was discovered some six months later and the discovery is rated as one of the milestones of high energy physics!!

Suggestions on how to read this article:

Interested readers can approach this article, whose subject matter is intrinsically quite technical, at various levels. They can, I hope, get an intuitive feel by reading the article very carefully. They may need to read more than once. Those wanting to go further should focus on keywords and concepts, often shown in italics and study them in any manner they can. They can start with web searches on Google, Wikipedia, etc. and follow the references. Nothing can of course replace discussions with experts in the field whom the readers should approach without inhibition. I, for one, will be more than happy to answer their questions!

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