Aerobasics – An Introduction to Aeronautics

7. Supersonic Aerodynamics

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Flight at supersonic speeds is accompanied by standing waves, a phenomenon absent in subsonic flows. The standing waves affect the flow radically and lead to wave drag and sonic booms. In this article, we explain the above phenomena and their effect on the performance of airfoils and wings in supersonic flows. In the light of the above, wing configurations suitable for supersonic airplanes are discussed.

Keywords
Standing waves, Mach waves, shocks, wave drag, sonic boom, supersonic airfoils, aerodynamic heating.

1. Introduction

A fluid flow is considered subsonic if the local flow velocity is less than the local speed of propagation of a small pressure disturbance (i.e., sound). Subsonic aerodynamics considered in Part 6 of this series applies to flows which are subsonic everywhere in the field of flow. As the stream velocity for the flow past a body is increased from zero, the flow is initially subsonic until a critical Mach number M_{cr} is reached. At a stream velocity larger than M_{cr}, the flow velocity in some region in the neighborhood of the body will be greater than the local speed of sound – the flow is locally supersonic. Such a flow wherein there are regions of supersonic and subsonic flow within the field of flow is said to be 'transonic'. As the free-stream velocity is further increased, the region of supersonic flow expands until it almost completely covers the field of flow and the name 'supersonic flow' applies to this. This happens for typical bodies at a free-stream Mach number greater than 1.2. As the free-stream velocity is further increased, there are no qualitative charges in the flow, but density and temperature variations within the flow field become very large (typically for M > 5) and the name 'hypersonic flow' applies to this. A body re-entering the earth's atmosphere after a mission outside it (in outer space) encounters such a flow situation. Airplane flight is generally limited to a Mach number less than 3. Typical combat aircraft are capable of maximum flight speeds in the range of M = 1.4 − 2.

In what follows, we study some simple supersonic flows to bring out their essential features like standing waves, limited domains of influence and wave drag. These features of supersonic flow are unrelated to the effects of viscosity. Thus we simplify the analysis by considering the fluid to be inviscid.

2. The Speed of Sound

To understand the key role of the speed of sound in high speed flows, we may consider a simple standing wave in a duct of uniform area as in *Figure* 1. A standing wave is a steady disturbance along the flow direction (fixed in space). The flow velocity V_1 changes to a velocity V_2 discontinuously on going through the station A, the front of the standing wave which is normal to flow direction. The corresponding pressures and densities on the two sides of the discontinuity surface are p_1, ρ_1 and p_2, ρ_2.. We first consider the case when $V_2 - V_1 = \delta V$ is infinitesimally small. Then the changes in pressure and density across the discontinuity, δp and $\delta\rho$, are also infinitesimally small and the equations of conservation

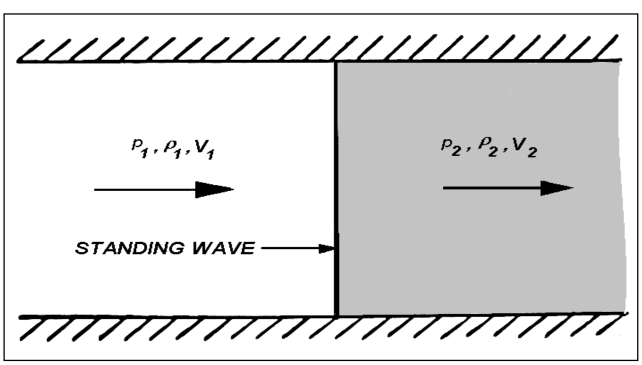


Figure 1. Standing wave in a duct: A simple standing wave consists of a surface fixed in space across which there is a small but discontinuous change in the flow variables like pressure and velocity. It can occur at only one flow speed in the duct. This is the speed of sound.

of mass and momentum across it lead to

$$V\,\delta V + \frac{\delta p}{\rho} = 0; \quad \text{(Momentum)} \tag{1}$$

$$\frac{\delta V}{V} + \frac{\delta \rho}{\rho} = 0. \quad \text{(Mass)} \tag{2}$$

Assuming isentropic changes across the discontinuity

$$\frac{\delta p}{p} = \gamma \frac{\delta \rho}{\rho}. \quad \text{(Isentropic flow)} \tag{3}$$

Using this in (1) we have,

$$V\,\delta V + \left(\gamma \frac{p}{\rho}\right) \frac{\delta \rho}{\rho} = 0\ ; \tag{4}$$

$$\frac{\delta V}{V} + \frac{\delta \rho}{\rho} = 0. \tag{5}$$

Equations (4) and (5) are simultaneous equations for $\delta\,V$ and $\delta\rho$ and being homogeneous, they have a nontrivial solution only when the determinant of the coefficients is zero.

$$\begin{vmatrix} V & \gamma \frac{p}{\rho^2} \\ \frac{1}{V} & \frac{1}{\rho} \end{vmatrix} = 0; \text{ this leads to } V^2 = \frac{\gamma\, p}{\rho}. \tag{6}$$

Equation (6) indicates that a stationary discontinuity is possible for the flow only at one speed, $V = \sqrt{(\gamma p/\rho)}$. Superposing a velocity $-V$ on the whole flow so as to make the fluid on the left stationary, we see that the discontinuity (which is now a traveling wave and is called a 'Mach wave') moves to the left at this speed which is the speed of sound, a. A Mach wave either compresses or expands the fluid it moves into ($\delta\rho$ and hence δp can be positive or negative). It is similar to a sound wave. A sound wave generally has a periodic variation of flow quantities while a Mach wave can have an arbitrary profile.

In the above, we considered only an infinitesimal discontinuity. A more general analysis on the same lines shows that, depending on its strength, a finite discontinuity travels faster than the speed of sound. Such a discontinuity is named a 'shock wave'. Unlike a Mach wave which can cause an increase or decrease in pressure, a shock wave always compresses the fluid (increases the pressure) into which it travels. Stationary expansion waves are always infinitesimal (finite expansion discontinuities are impossible due to thermodynamic considerations). Weak shock waves (with finite but small changes in density and pressure) are much like Mach waves and equations (3–6) are applicable to them with insignificant error. In what follows, we apply the above relations to weak shock waves also.

From the above it is clear that the fluid velocity component normal to a stationary wavefront is the speed of sound. Superposition of a constant fluid velocity component along the wavefront throughout the flow does not alter the above relations. Thus, equations (3–5) can be applied locally for the velocity components normal to a wavefront in a flow with standing waves. Such standing waves occur in supersonic flows past bodies. We consider some examples of this type later in this article.

3. Energy in a Wave

We consider a wave of unit length traveling along the x-axis as in *Figure* 2. We take the amplitude of the pressure change as δp over the whole length. Outside the region of the wave, the fluid is at rest and the pressure is uniformly p. The energy required to compress unit volume of the fluid in the region of the wave to the pressure δp above p adiabatically, is half the product of change in pressure and change in volume and is easily

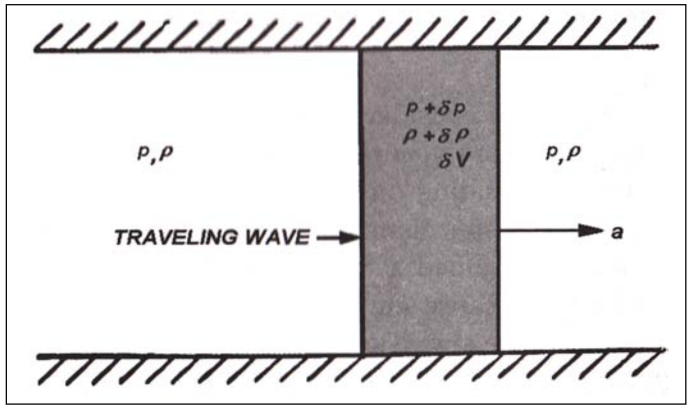


Figure 2. Energy in a traveling wave of unit length: The traveling wave indicated in this figure is a region of excess pressure δp over ambient and extends over unit length. Mechanical energy is required to create this wave. As the wave travels with the speed of sound, this energy stays with the wave but not the same fluid particles.

seen to be

$$E_1 = \frac{(\delta p)^2}{2}\frac{1}{\gamma p}. \tag{7}$$

In addition, the particles of fluid have a velocity δV related to δp through equations (3) and (5) as:

$$\frac{\delta V}{V} = -\frac{\delta \rho}{\rho} = \frac{-1}{\gamma}\frac{\delta p}{p}. \tag{8}$$

In the above, V is equal to the speed of sound as indicated by (6). We then have the kinetic energy of the fluid per unit volume in the wave, E_2, as:

$$E_2 = \frac{1}{2}\rho(\delta V)^2 = \frac{(\delta p)^2}{2}\frac{1}{\gamma p}. \tag{9}$$

Thus, the total energy in the wave per unit volume, E, is the sum of potential and kinetic energies and is:

$$E = (\delta p)^2\frac{1}{\gamma p}. \tag{10}$$

It is seen that the wave carries energy partly as kinetic energy and partly as potential energy to the extent of half each. It may also be noted that this energy, being the square of δp, is positive for both expansion and compression waves.

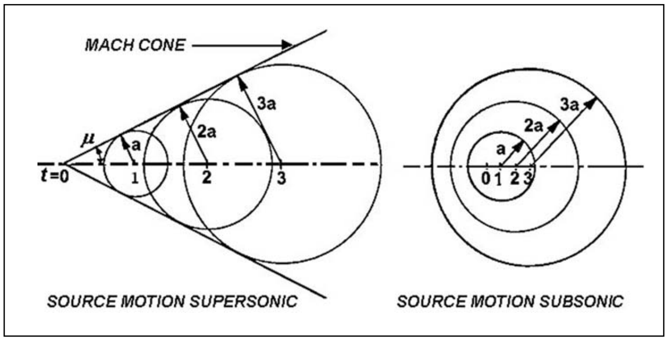


Figure 3. Domain of influence in supersonic flow: When a source of disturbance moves at a speed less than the speed of sound, the disturbance spreads in all directions and ultimately affects the whole space around the source. When the source moves at a speed larger than the speed of sound, the disturbance cannot propagate into some regions and its effect is confined to the region within the Mach cone with the source at its apex.

4. Domain of Influence in Supersonic Flow

When a body starts moving from rest in a fluid, the disturbance created by the body travels in all directions at the speed of sound. If the body moves slowly (below sonic speed) the influence of the body is ultimately felt throughout the field of flow. But, if the body moves faster than the speed of sound, the disturbance due to the body cannot catch up with the body. This results in a situation illustrated in *Figure* 3. Here, a body producing a small disturbance travels to the left at a speed V greater than the speed of sound a. The disturbances can only reach the region defined by the Mach cone of semi-vertex angle $\sin^{-1}(a/V)$. This is the Mach angle μ. The surface of the Mach cone is defined by a Mach wave which travels normal to the cone at the speed of sound.

A realistic body like an airplane creates a finite disturbance and the boundary of the disturbance is one of semi-vertex angle somewhat larger than the Mach angle as the shock wave travels faster than the speed of sound. However, at large distances from the body, the disturbances get weaker and the boundary of the region approaches a Mach cone.

5. Two-Dimensional Flows

Two-dimensional supersonic flows are particularly simple to analyze. As an example, we consider the two-dimensional flow past a wedge of a small ramp angle θ

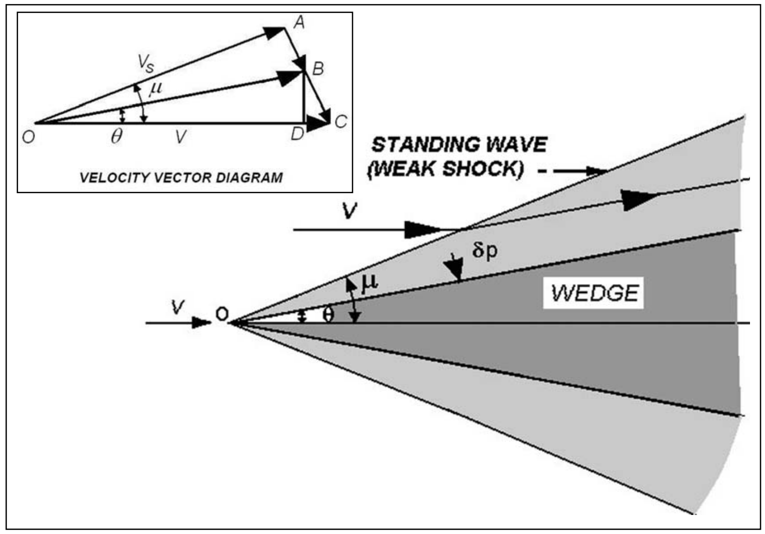


Figure 4. Flow past a two-dimensional wedge in supersonic flow: The free stream is unaffected in the region beyond the standing wave (which is a weak shock). The flow velocity abruptly changes direction on passing through the shock and becomes parallel to the wedge surface. There is a uniform increase in pressure on the wedge surface. The component of flow velocity normal to the shock decreases on crossing the shock.

as in *Figure* 4. The free stream approaching the wedge is unaffected till it crosses the Mach line drawn from the apex of the wedge. This line represents the position of the standing wave (a weak shock wave). Across the weak shock there is a small rise in pressure. The region between the shock and the wedge is of uniform pressure. The flow velocity in this region is also uniform and is parallel to the wedge surface. The component of free-stream velocity normal to the shock is the speed of sound a. On crossing the shock, the component of flow velocity normal to the shock is reduced and is indicated in the velocity vector diagram shown as an inset in *Figure* 4. One may relate the increase in pressure over the wedge to the ramp angle by using the standing wave relations of equations (3–5) as follows.

Figure 4 (inset) indicates the flow velocity components before and after the shock. The velocity vector before the shock, OC, is along the free-stream direction and is of magnitude V. Its component normal to the shock, V_{N1}, is the speed of sound a and is indicated by AC. The component along the shock, V_{S} is indicted by OA in the figure. It is unaltered on passage through the shock and is the same after the shock. The velocity vector after the shock is parallel to the ramp surface and is indicated by the line OB in the figure. Its normal component, V_{N2}, is indicated by AB in the diagram. Thus the change in the normal velocity component after passing through the shock is BC. This can be related to the known quantities, V/a (equal to the free-stream Mach number M) and the ramp angle θ, as follows:

$$V_{N1} - V_{N2} = \delta V_N = BC = \frac{BD}{\cos\mu} = \frac{V\theta}{\cos\mu}. \tag{11}$$

$$\frac{\delta V_N}{V_{N1}} = \frac{V}{a}\frac{\theta}{\cos\mu} = \frac{\theta M^2}{\sqrt{M^2-1}}. \tag{12}$$

$$\frac{\delta p}{p} = \gamma\frac{\delta V_N}{V_{N1}} = \frac{\gamma\vartheta M^2}{\sqrt{M^2-1}}. \tag{13}$$

Defining the pressure coefficient, $\delta p/q$, as the change in pressure divided by the dynamic pressure q, we finally have:

$$\frac{\delta p}{q} = \frac{2\theta}{\sqrt{(M^2-1)}}. \tag{14}$$

Equation (14) indicates that the pressure rise due to the ramp is directly proportional to the ramp angle θ. This flow can be used as a building block for constructing and studying more complex two-dimensional flows. We illustrate this by studying the flow past a double wedge in supersonic flow.

5.1 *A Double Wedge in Supersonic Flow*

A symmetrical double wedge as shown in *Figure* 5 consists of two wedges placed back to back. The double wedge has a thickness t and a chord length c. The ramp angles corresponding to the leading and trailing wedges are thus t/c and $-t/c$ respectively. The corresponding flow is constructed using the wedge flow studied above.

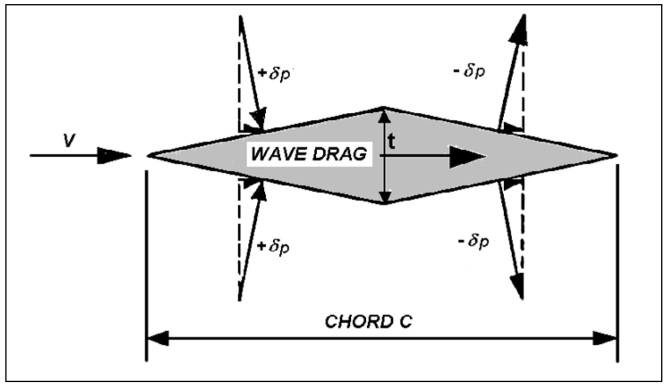


Figure 5. Flow past a double wedge: Two wedges placed back to back form the double wedge. There is increased pressure on the leading wedge and an equal decrease in pressure over the trailing wedge. The resultant force due to pressure changes on all the four surfaces of the double wedge is a drag which is designated by the term 'wave drag'. The total drag of the wedge includes drag due to viscosity effects which have been ignored in this analysis.

There is a pressure rise Δp on the leading wedge on both its faces as given by (14). There is an equal fall in pressure on both sides of the rear wedge. The resultant force on the double wedge is obtained by resolving the forces normal to the surfaces of the wedges and is easily seen to be a drag, D. Defining a dimensionless drag coefficient, C_{DW}, we can show using (14) that:

$$C_{\mathrm{DW}} = \frac{D}{qc} = 4\frac{(t/c)^2}{\sqrt{(\mathrm{M}^2-1)}} \quad . \tag{15}$$

C_{DW} is called the coefficient of wave drag. This drag is specific to supersonic flow. A body traveling at any supersonic speed creates waves which carry away energy. This energy is drawn from the body by offering resistance to its motion and hence the name 'wave drag'. This is easily demonstrated as below.

We now look at flow past the double wedge of *Figure* 5 from a different perspective as indicated in *Figure* 6. We consider the fluid to be at rest and the double wedge to move at the speed V along its chord direction. As the body moves, it creates a new region filled with waves of width $V \cos\mu$ per second. The region has a length equal to $c \sin\mu$ and consists of a zone of compression followed by one of expansion of equal magnitude. The total mechanical power required for creating this wave, P_{w}, is obtained by using (10). Waves are radiated from both sides of the wedge and this is taken into account

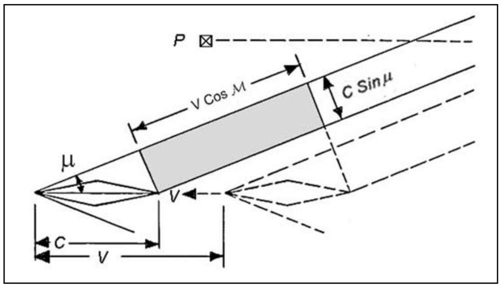


Figure 6. Wave motion due to a double wedge: Here, the double wedge moves into a static medium with a supersonic speed. New regions filled with waves are created at a uniform rate. The mechanical energy required to create these regions is drawn from the moving body by offering a resistance to its motion. The waves transport this energy to regions far from the body. When the waves reach a distant observer, she hears them as a sonic boom. The energy of the waves is ultimately dissipated due to the effect of viscosity.

in (16):

$$P_{\rm w} = 2\,\frac{(\delta p)^2}{\gamma p}\,c\sin\mu\;\;V\cos\mu \quad , \tag{16}$$

where $\sin\mu = 1/\mathrm{M}$ and $\cos\mu = \sqrt{(\mathrm{M}^2-1)}/\mathrm{M}$.

We may use (14) for the pressure rise δp in (16). We get after some simplification:

$$P_{\rm w} = 8\frac{(t/c)^2}{\gamma p}\,\frac{1}{\mathrm{M}^2\sqrt{\mathrm{M}^2-1}}q^2 c\,V. \tag{17}$$

Noting that

$$\frac{q}{\gamma p \mathrm{M}^2} = \frac{1}{2}\frac{\rho V^2}{\gamma\, p\,\mathrm{M}^2} = \frac{1}{2} \quad ,$$

we finally have

$$P_{\rm w} = 4\frac{(t/c)^2}{\sqrt{\mathrm{M}^2-1}}\,qc\,V. \tag{18}$$

It is easily seen that the power required for overcoming drag, $P_{\rm D}$, is the product of drag and flight speed and is given by:

$$P_{\rm D} = C_{\rm DW}qcV = 4\frac{(t/c)^2}{\sqrt{\mathrm{M}^2-1}}\,q\,cV. \tag{19}$$

Equations (18) and (19) show that the power expended by the body against wave drag is exactly equal to the power carried away by the wave.

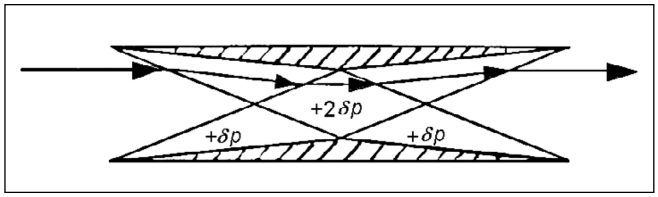


Figure 7. The Busemann Biplane at the design Mach number: The biplane configuration using two airfoils placed back to back at a suitable spacing can contain the standing waves completely between the biplane elements at one particular Mach number. As waves do not carry away the energy, there is zero wave drag in this condition. However, there will be wave drag at other Mach numbers and at nonzero incidence.

Wave drag can be eliminated if the waves are somehow prevented from carrying away the energy. This can be done under some conditions by a wave cancellation technique. An example of this is the Busemann biplane configuration shown in *Figure* 7. This configuration has zero wave drag at one Mach number at which the waves are completely contained between the biplane elements. However, the use of this method for drag reduction on any airplane has not been practical.

Box 1. The Sonic Boom

The sonic boom is a phenomenon specific to supersonic flight. Whenever a body moves at a supersonic speed, it creates waves traveling away from the body. These waves are confined to certain regions around the body, but extend over large distances. This is quite unlike flight at subsonic speeds. Pressure disturbances created by a body in subsonic flight decay rapidly and are mainly felt in a region around the body extending over only a few body lengths. We can consider the flow over the double wedge as an illustration. Referring to *Figure* 6, for a stationary observer located at P, as the wedge moves to the left, the pressure is ambient until the wavefront touches P. As the body moves further, the wave crossing the observer is felt as a sudden increase in pressure, δp, for a length of time equal to $c/2V$ and then as a sudden fall in pressure to $-\delta p$ (δp below ambient) for an equal time duration. The pressure then returns to ambient value. This fluctuation of pressure constitutes the sonic boom. It is heard by a human observer as an explosive sound whose strength depends on the magnitude of the pressure fluctuation. For a two-dimensional body in inviscid flow as in *Figure* 6, the sonic boom is of constant strength at all distances from the body. However for a three-dimensional body like an airplane, the boom gets weaker with distance away from the body due to the radial spreading of the waves. There is also a gradual dissipation of the boom due to the effect of viscosity. Still, the boom is strong at very large distances. For a typical supersonic plane like the Concorde flying at 60,000 feet, the boom is heard by an observer at sea level as a loud report and is often strong enough to break window glass on buildings subjected to the boom. For this reason, flight of supersonic civil aircraft over populated areas is not permitted.

6. Supersonic Airfoils

Wave drag is of major concern in designing aircraft for flight at supersonic speeds. It is seen from earlier examples that the wave drag increases as the square of the deflection angles and thus the ideal airfoil for supersonic speeds is a flat plate kept at very small incidence. *Figure* 8 shows the flow past such an airfoil. This flow is easily analyzed using the results of the flow past a wedge considered earlier. It is seen that there is a uniform increase of pressure on the bottom surface of the airfoil and an equal drop on the top surface. The resultant force is normal to the airfoil and acts exactly at the middle of the chord. The lift and drag of this airfoil are easily calculated using (14) as:

$$C_{\mathrm{L}} = \frac{4\alpha}{\sqrt{(\mathrm{M}^2-1)}}\ , \quad C_{\mathrm{D}} = \frac{4\alpha^2}{\sqrt{(\mathrm{M}^2-1)}}. \qquad (20)$$

Here C_{D} represents the wave drag associated with lift and increases as its square. One may compare (20) with the corresponding results for subsonic flow:

$$C_{\mathrm{L}} = \frac{2\pi\,\alpha}{\sqrt{(1-\mathrm{M}^2)}}\ , \quad C_{\mathrm{D}} = 0. \qquad (21)$$

In this case, the lift acts at the quarter chord point of the airfoil.

In the above, the effect of viscosity is not included. However as viscosity effects are similar in both subsonic and supersonic flows, (20) and (21) indicate that one may

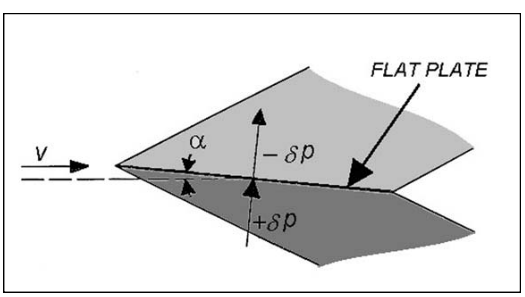
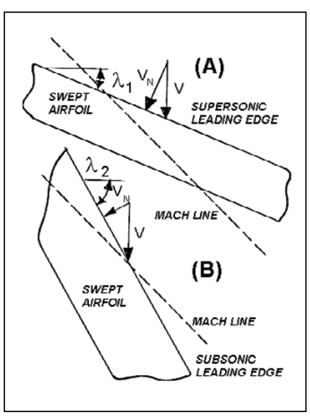


Figure 8. The flat plate airfoil in supersonic flow: There is a uniform increase in pressure on the bottom surface and a uniform fall in pressure on the top surface. The resultant force is normal to the airfoil surface and acts at its midpoint. The component of this force along the flow direction is the wave drag due to lift. It increases as the square of the incidence.

expect a substantial reduction of lift to drag ratio at supersonic speeds and it is indeed the case. Wave drag can be minimized by using very thin wings of nearly symmetrical cross-section. Typically wings of 3–5% thickness ratio are used at incidence values of only 2°–3°. One may compare this with thickness ratio up to 18% and incidence values of 10° or higher that are often used on subsonic aircraft

It may be noted that the lift in supersonic flow acts at the half-chord position on the airfoil while it acts at the quarter chord position in subsonic flow. This implies a large shift in the centre of pressure towards the rear when going from subsonic to supersonic flow. This is also true to some extent of an airplane wing. This shift is highly undesirable from the point of view of stability and control of an airplane. The rearward shift of the centre of pressure is less for a wing of delta plan form which is often used on supersonic aircraft. Airplanes designed for supersonic flight often manage the positions of their centre of gravity carefully by pumping fuel among several fuel tanks when changing from subsonic to supersonic flight or vice versa so as to keep their stability and control problems within manageable limits.

Figure 9. Effect of sweep in supersonic flow: The effect of sweep on an airfoil is to decrease the component of free-stream velocity in the direction normal to the airfoil axis. When the sweep angle is sufficiently large, this normal component of velocity can be less than the speed of sound. The airfoil under this condition performs as in subsonic flow and there is no wave drag.



6.1 *Effect of Sweep in Supersonic Flow*

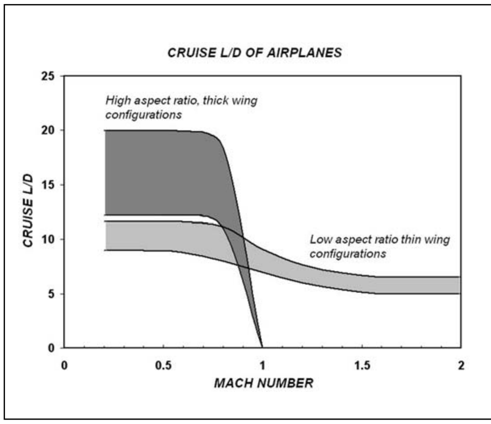
The effect of sweep on an airfoil in subsonic flow was considered in the previous article of the series (*Resonance*, Vol.14, No.1, 2009). There, it was shown to have a beneficial effect on flight speed due to a reduction in the velocity component normal to the airfoil axis. This argument is also valid in supersonic flow (*Figure* 9). Here we may consider two cases. In the first case, the angle of sweep is smaller than the Mach angle for the free-stream condition illustrated. The velocity component normal to the leading edge of the airfoil is larger than the speed of sound. Thus the flow over the airfoil cross-section in a plane normal to the airfoil axis is still supersonic, but at a lower Mach number compared to the free stream. In the second case, when the angle of sweep is larger than the Mach angle, the flow in the plane normal to the airfoil axis is subsonic. In this case, the wave drag associated with supersonic flow vanishes. Thus it is possible to reduce wave drag by using sweep. The result is true only for an airfoil. For a realistic airplane configuration which is three-dimensional and contains a central body (the fuselage) there is no possibility of eliminating wave drag completely. However, wing sweep is beneficial.

Airplanes designed for supersonic performance spend a fair part of their flight time at subsonic speeds and have to perform well in this speed range also.

7. Wings for Supersonic Airplanes

An airplane designed for supersonic flight has to take-off and land at relatively low subsonic speeds and thus the configuration of the wing has to meet the requirements for subsonic flight as well. Subsonic flight is at a relatively lower dynamic pressure and demands a wing of high aspect ratio for achieving a good lift-drag ratio. Supersonic flight is at a relatively larger dynamic pressure and a thin wing of smaller area is required for minimizing wave drag and wing weight. A thin wing is relatively weak and so the wing span should be reduced as much as possible to compensate. Thus the choice of a wing configuration for a supersonic airplane involves many compromises. Supersonic flight can get some benefit from the low air density at very high altitudes. For this reason, the supersonic civil airplane, Concorde, cruised at an altitude of about 20 km while subsonic civil aircraft generally cruise at an altitude around 11 km.

Figure 10 compares airplane configurations suitable for subsonic flight with those suitable for supersonic flight on the basis of their lift-drag ratio. The thick wing, large aspect ratio configurations achieve a good cruise lift-drag ratio up to a Mach number of about 0.8. The thin wing, small aspect ratio configurations are relatively worse in the subsonic range but have a cruise lift-drag ratio around 7–10 in the supersonic range. This is



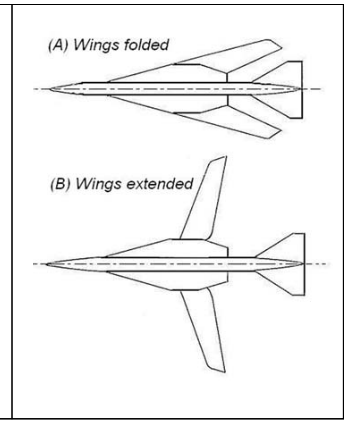


Figure 10 (left). Lift-drag ratio of practical airplane configurations: Airplanes with thick wings of large aspect ratio have a high lift to drag ratio, L/D, at subsonic Mach numbers. However, there is a rapid fall in L/D as the Mach number of unity is approached. At M = 1, L/D is practically zero. These airplanes cannot fly at supersonic speeds. Airplanes with needle-like bodies and very thin wings are required for efficient supersonic flight to minimize wave drag. Wing span is kept small for reducing wing weight. Such a configuration has a poorer L/D at subsonic speeds than thick, high aspect ratio wings. This L/D is further reduced at supersonic speeds due to wave drag.

Figure 11 (right). A configuration with a variable sweep wing: A wing configuration which has the benefit of high aspect ratio at low speeds and low wave drag at supersonic speeds is the variable sweep wing. The pivoted wing is swiveled into the low sweep position at low speeds for take-off and landing. The sweep can be increased to maximum for supersonic flight. The increase in structural weight due to the hinge and actuation devices detracts from its extensive use.

substantially lower than the cruise lift-drag ratio of current civil aircraft and represents a barrier for a supersonic civil airplane to cross to achieve cost effectiveness.

A concept which has achieved a limited success for supersonic airplanes is that of variable sweep. The wing configuration can be varied from one of high aspect ratio to one of low aspect ratio by rotating the outboard wing panels about suitable hinges as in *Figure* 11. The high aspect ratio configuration is suitable for subsonic flight while the high sweep configuration is better suited for supersonic flight. But experience seems to indicate that the aerodynamic benefits of variable sweep are severely compromised by the structural complexities and the concept is currently not favored.

8. Flight Speed Measurement in Compressible Flow

It was indicated in Part 4 of this series (*Resonance*, Vol.13, No.12, Dec 2008) that the pitot-static pressure difference, ΔP, is equal to the dynamic pressure, q, in incompressible flow. The flight speed of an airplane at a low Mach number can be calculated and indicated on an instrument (the air speed indicator) from a measurement of this pressure difference. The pitot-static pressure difference can also be used for calculating and indicating the flight speed of an airplane in flight at any Mach number if the effect of compressibility of air is taken into account. The required formulae can be derived starting from the compressible flow equations (1) and (3) as below.

Equations (1) and (3) can be integrated to get a relation between velocity and pressure. This is more conveniently done using p/ρ as the dependent variable. The term $\mathrm{d}p/\rho$ in (1) is easily seen to be $(\gamma/(\gamma-1))\mathrm{d}(p/\rho)$ and hence we get:

$$\frac{\gamma}{\gamma-1}\frac{p}{\rho}+\frac{V^2}{2}=\text{constant}. \tag{22}$$

This relation is the compressible flow equivalent of the Bernoulli law. Applying this to the flow at any two points indicated by subscripts 1 and 2, we have:

$$\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}+\frac{V_1^2}{2}=\frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2}+\frac{V_2^2}{2}. \tag{23}$$

We can choose the point 1 to be in the free stream with undisturbed static pressure p, and flow speed V. Point 2 is chosen to be the stagnation point (flow speed equal to zero) at the nose of the pitot tube with pressure p_{p}, we get:

$$\frac{\gamma}{\gamma-1}\frac{p}{\rho}+\frac{V^2}{2}=\frac{\gamma}{\gamma-1}\frac{p_{\mathrm{p}}}{\rho_{\mathrm{p}}}. \tag{24}$$

After some rearrangement, this reduces to:

$$\Delta P = p_{\mathrm{p}} - p = p\left[\left(1+\frac{\gamma-1}{2}\left(\frac{V}{a}\right)^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]. \tag{25}$$

This is the required relation between the flight speed (true air speed, TAS) and the pitot-static pressure difference ΔP. In this equation, the atmospheric pressure p and the speed of sound a depend on altitude. The relation is exact at all speeds below the speed of sound. At low Mach numbers, this relation can be shown to be consistent with the Bernoulli law. At supersonic flight speeds, a standing wave (bow shock) is formed ahead of the pitot tube and this affects ΔP. A new relation, the Raleigh supersonic pitot formula [1] consistent with (25) at M = 1 replaces it.

The air speed indicator is calibrated such that its reading V_{c}, (the calibrated airspeed) corresponds to this relation at sea level ISA conditions. Denoting the sea level pressure and sound speed respectively by the symbols p_0 and a_0, we get the relation:

$$\Delta P = p_0\left[\left(1+\frac{\gamma-1}{2}\left(\frac{V_{\mathrm{c}}}{a_0}\right)^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]. \tag{26}$$

Airspeed indicators are calibrated by applying a pressure ΔP and graduating the dial to read V_{c} as per (26).

9. Aerodynamic Heating

The airspeed indicator thus indicates correctly the flight speed under sea level ISA conditions all the way to Mach one and slightly beyond.

When a fluid element moving at a Mach number M is brought to rest without heat loss or addition, its temperature increases to the stagnation temperature, T_0, which is higher than its static temperature T_{s}. The relation between them is easily derived from (22). Replacing the quantity p/ρ in this equation by RT (using the perfect gas relation) and after some simplification we get:

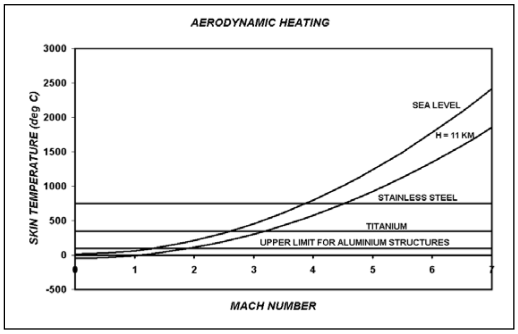


Figure 12. Aerodynamic heating: At high Mach numbers, aerodynamic heating is a major consideration. The airplane structure heats to a temperature which depends on the flight Mach number. As different structural materials begin to lose strength at different temperatures, a proper choice of structural materials is required for any specific flight condition.

$$T_0 = T_{\mathrm{s}} \left(1 + \frac{(\gamma - 1)\mathrm{M}^2}{2} \right). \tag{27}$$

T_0 represents the highest temperature possible in the flow. If a thermally isolated flat plate is placed aligned with the flow direction, its temperature rises to the recovery temperature, T_{r}, somewhat lower than T_0, and is given by

$$T_{\mathrm{r}} = T_{\mathrm{s}} \left(1 + r\frac{(\gamma - 1)\mathrm{M}^2}{2} \right). \tag{28}$$

In (28), r is a recovery factor. It depends on the nature of the boundary layer and is about 0.85 for the turbulent boundary layer. *Figure* 12 shows the variation of recovery temperature with Mach number based on (28).

Equations (27) and (28) indicate that bodies traveling at high speeds may experience high temperatures even though the ambient temperature T_{s} which depends on altitude is in the range of $-60\,^{\circ}\mathrm{C}$ to $+40\,^{\circ}\mathrm{C}$. Materials of construction like aluminum alloys which lose their strength at a relatively low temperature of about $100\,^{\circ}\mathrm{C}$ are only suitable for flight at Mach numbers up to 2. Titanium alloys can be used up to about $350\,^{\circ}\mathrm{C}$ and are suitable for higher Mach numbers up to about 3.5. Stainless steel can withstand up to about $750\,^{\circ}\mathrm{C}$ and can be used at higher Mach numbers up to about 4.5. Often, bodies meant for flight at very high speeds are protected by suitable heat shields made of ceramics and other materials.

Suggested Reading

[1] H W Liepmann and A Roshko, *Elements of Gas Dynamics*, John Wiley and Sons, New Jersey, 1960.

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