Hubble, Hubble's Law and the Expanding Universe

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Hubble's name is associated closely with the idea of an expanding universe as he discovered the relation between the recession velocity and the distances of galaxies. Hubble also did a lot of pioneering work on the distribution of galaxies in the universe. In this article we take a look at Hubble's law and discuss how it relates with models of the universe. We also give a historical perspective of the discoveries that led to the Hubble's law.

1. Hubble's Law

Edwin P Hubble is best known for his discovery of the relationship between the distance and radial velocities of galaxies. All models of the universe are based on this relationship, now known as 'Hubble's law'. Hubble found that the rate at which galaxies are receding from us is proportional to the distance, $V \propto r$, and used observations to determine the proportionality constant. This constant is now called the 'Hubble's constant' in his honour.

$$V = H_0 r.$$

This form for the relationship has important implications (see *Box* 1).

We step back a little and fill in some background before continuing with our discussion of the Hubble's law.

The early part of the 20th century saw considerable discussion and activity focused on understanding the structure of our own galaxy. Eventually it was understood that our galaxy is a fairly large system with around a

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Box 1. Hubble's Law and the Cosmological Principle

The Hubble's law is written as

$$V = H_0 r,$$

with V as the radial component of the velocity. The reason for writing down only the radial component is that this is the only component of the motion that we can observe through the shift of spectral lines. However, it is implicit in the form of the Hubble's law that the rate of expansion is independent of direction. In other words, the expansion is isotropic around us*. Clearly, the Hubble's law is *consistent* with a vectorial relationship between velocity and distance.

$$\mathbf{V} = H_0 \mathbf{r}.$$

In this form, it is easy to see that the Hubble's law retains its form if we shift the origin: as seen from the origin, the galaxy at $\mathbf{r}_1$ recedes with velocity $\mathbf{V}_1$. If we now try to rewrite the recession law in the frame of this galaxy, we get:

$$\mathbf{r}' = \mathbf{r} - \mathbf{r}_1,$$

$$\mathbf{V}' = \mathbf{V} - \mathbf{V}_1 = H_0 (\mathbf{r} - \mathbf{r}_1) = H_0 \mathbf{r}'.$$

As claimed, the Hubble's law retains its form in the frame of any other galaxy as well. Thus expansion appears the same in every direction, and from every place in the universe. Unless we are observing at a special moment in the history of the universe, this also means that the universe is homogeneous and isotropic. The cosmological principle [1] elevates and encapsulates this idea, and it is noteworthy that a homogeneous and isotropic model of the universe allows us to define a cosmic time. Most models of the universe are based on this principle.

* Exceptions are anisotropic models [2] where galaxies recede from us at different rates in different directions. However observations restrict the level of deviations from isotropy and one needs to construct models carefully in order to match observational data.

hundred billion stars. Our galaxy, or the Galaxy, is nearly 80,000 light years across. Astronomers use a different unit, a parsec (1 parsec = 3.26 light years) and the Galaxy is around 25 kilo parsecs across. The Galaxy is shaped like a disk with stars highlighting spiral arms in the disk; there is also a spheroidal bulge near the centre of the Galaxy[1]. The disk is surrounded by a faint halo of stars and globular clusters; each globular cluster is a tight group of stars and these may contain $10^3 - 10^6$ stars each. The Sun is around 8 kpc from the centre in the disk.

[1] An *appam* is a good description of the shape of disk and bulge, though not in proportion.

Many galaxies have been known for a long time. Hubble provided the first reliable determination of distances to these galaxies and convincingly proved that these are large systems of stars in their own right.

Many other galaxies have been known for a long time. However it was not very clear whether these are a part of our own galaxy or are similar systems located very far away. Hubble provided the first reliable determination of distances to these galaxies and convincingly proved that these are large systems of stars in their own right.

Now we revert to our discussion of the Hubble's law. In the velocity–distance relation, velocities are measured in km/s, distances in millions of parsecs (Mpc) and for this reason the Hubble's constant is written in complicated looking units of km/s/Mpc even though it has dimensions of inverse time. We can recast the Hubble's law and write it in terms of direct observables. We do this step by step. We first rewrite the recession velocity in terms of the redshift of spectral lines that is determined directly from spectra.

$$z = \frac{V}{c} = \frac{r}{cH_0^{-1}} \quad .$$

The speed of light is denoted by the usual symbol c. Here z is the Doppler redshift; note that this definition of redshift is valid only for $|\mathbf{V}|/c \ll 1$. Distances are often measured using reference stars, or other objects that are known to have a given luminosity (see *Box* 2). In such a case, the flux observed from the reference object is related to the luminosity and the distance. The energy emitted in unit time is radiated uniformly in all directions and eventually spreads out in a spherical shell of radius r. The energy observed per unit area, per unit time can then be written as

$$S = \frac{L}{4\pi r^2} \quad .$$

Here S is the observed flux and L is the luminosity. If we observe a number of light sources then the redshifts and observed fluxes are expected to have the following

Box 2. Distance Ladder

Measuring distances to other galaxies is a challenging task as there is no direct method of ascertaining the distance. There are two basic methods that are combined for measuring distances to galaxies.

- Parallax: We measure the parallax of nearby stars across Earth's orbit around the Sun. The parallax angle is $1''(3.08 \times 10^{16}\mathrm{m}/r)$ *. Observations from the Earth can give reliable parallax measurements of up to $0.1''$.
- Standard Candle: If there is a source with known luminosity (energy output per unit time), then the observed flux from such a source can be used to compute the distance if the radiation from the source has not been absorbed by intervening gas and dust. Luminosity L, flux S and distance r are related as $S = L/(4\pi r^2)$, assuming that the source radiates uniformly in all directions.

There are no standard candles where the luminosity is known *a priori*, therefore one needs to do a calibration. In the absence of such a calibration we can only measure relative distances and not absolute distances. Calibration is done by matching with the distance to a group of stars measured using some other method, either parallax or some other standard candle. A chain of standard candles is used and calibrated against each other, with the nearest ones calibrated using the parallax method. This sequence of distance measurement methods is often referred to as the 'distance ladder'. Each step in the distance ladder involves cross-calibration and introduces errors. The *Hipparcos* space mission reduced errors by a significant amount by providing accurate parallax measurements of up to $0.001''$, increasing the number of stars with known parallax distances by a significant factor [3]. The upcoming Gaia mission [4] is likely to make further progress in this direction.

* The distance at which we get a parallax of $1''$ is called a parsec: 1 parsec= 3.08×10^{16} m.

relationship:

$$\log z = \log \frac{r}{cH_0^{-1}} = \log \sqrt{\frac{L}{4\pi S}} \frac{1}{cH_0^{-1}} \quad .$$

In short, $\log z \propto -0.5 \log S$, where the constant of proportionality depends on the luminosity of the source and the value of the Hubble's constant. Astronomers use an inverse logarithmic scale called 'magnitudes' to quantify observed fluxes. These are defined as:

$$m = -2.5 \log S/S_0 \quad .$$

Here, S_0 is a reference flux. Thus the relationship between redshift and magnitude for a standard candle is:

$$\log z \propto 0.2m \qquad \Rightarrow \qquad m \propto 5 \log z \ .$$

The full relation, with constants, can be written as:

$$m - M = 5 \log \frac{r}{10 \text{ pc}} = 5 \log \frac{r}{1 \text{ Mpc}} - 25$$

$$= 5 \log z + 5 \log \frac{cH_0^{-1}}{1 \text{ Mpc}} - 25 \ .$$

Here M is the absolute magnitude that is defined as the magnitude if the source is located at a distance of 10 pc. Clearly, this is related to the luminosity and we note that unless we know the luminosity of the source we cannot use the two observables (magnitude m and redshift z) to determine the Hubble's constant.

Hubble used a variety of ways to determine distances to galaxies [5]. Using the 100″ Hooker telescope, he was able to find what are called 'Cepheid variables' in nearby galaxies. Cepheid variables are stars whose luminosity varies with time in a predictable manner and the time period of variation is related to the average luminosity. This allowed Hubble to show that these galaxies were very far away, certainly at distances that are much larger than the size of our own galaxy. This was the first convincing determination of distances to other galaxies and this is amongst Hubble's most significant scientific discoveries. *Box* 3 illustrates the difficulties involved in measuring distances to galaxies and the measurement of Hubble's constant.

Hubble then looked for potential standard candles in order to be able to determine distances to more distant galaxies. He used the brightest stars, the brightest globular clusters, and many other sources to determine distances to tens of galaxies. Redshifts, and hence recession velocities of a number of galaxies were already

Box 3. Why is Measuring H_0 Difficult?

The main challenge in measuring the Hubble's constant is in the accurate determination of distances to galaxies*. This in turn requires us to use the distance ladder: the progression of standard candles and parallax based methods of distance measurement through cross-calibration. Each step in the distance ladder involves some uncertainty and hence adds to the error in our knowledge of the luminosity of the standard candle that is finally used. If each step in the distance ladder introduces a few percent error, and the standard candle used to determine distances to galaxies requires five steps of cross-calibration then one can see that the error in luminosity of the standard candle adds up to around ten percent.

Another source of error are the observational uncertainties. If we are observing a star in a distant galaxy, then it is very difficult to check if another star happens to be in the same direction. Finite resolution of imaging devices, the large surface density of stars in galaxies, and the large distances to galaxies makes *blending* of stars a very common phenomena. The problem becomes more acute for distant galaxies as the projected density of stars becomes large. This introduces errors in the measured flux, and hence in the measured distance.

Lastly, galaxies are not merely receding from us due to expansion of the universe; they move around in the gravitational field of other galaxies. This component of motion is called 'peculiar velocity'. The total velocities are thus:

$$\mathbf{V} = H_0\mathbf{r} + \mathbf{v}.$$

Peculiar velocities are not expected to have any average value, these are expected to be random. If peculiar velocities have a typical magnitude σ_v, then these motions introduce an error of order $\sigma_v/(\sqrt{3}H_0 r)$ in the determination of the Hubble's constant, where the factor of $\sqrt{3}$ arises from our use of only one component of the peculiar velocity. In order to appreciate the impact of this effect, let us consider some numbers. In our universe, $\sigma_p \sim 300$ km/s and $H_0 \simeq 70$ km/s/Mpc. Thus the factor $\sigma_v/(\sqrt{3}H_0 r) \sim 25\%$ at a distance of 10 Mpc, and the distance to the nearest large galaxy is less than 1 Mpc. One can reduce this factor by measuring distances to a large number of galaxies but this in itself is a fairly difficult and challenging task. Therefore it is essential to use very distant galaxies for an accurate determination of Hubble's constant. On the other hand use of more distant galaxies increases errors due to the first two factors mentioned above.

* A noteworthy aside: It is apparent from the form of Hubble's law that it is possible to verify the relationship without any knowledge of the value of Hubble's constant.

known from the work done by V M Slipher [8]. The stage was now set for the discovery of the distance–velocity relation. Hubble approached this issue three years after his paper giving distances to galaxies, and showed that the relationship was linear. By this time, at least two relativists had already used the data published by Hubble and Slipher to arrive at the same conclusion (see *Box* 4 for details). However, it is easy to see that Hubble approached the problem from a different perspective and that his discovery was made independently. The reason we associate this discovery with Hubble more than anyone else is that he continued to work on the problem and refine measurements in order to improve determination of distances to other galaxies and to make a convincing case that the expansion was cosmological in origin

Box 4. Who Discovered Hubble's Law?

The velocity distance relation encapsulated in Hubble's law [6] is expected in all relativistic cosmological models, the sole exception being Einstein's static model. Dynamical models with expansion were discovered by Friedmann, Lemaître and Robertson in the decade preceeding Hubble's discovery. At the time when Friedmann [7] published his solutions of Einstein's equations, the size of the Galaxy was not known conclusively, nor was it known whether other galaxies are a part of the Galaxy or are similar systems situated at large distances. The issue was finally settled by Hubble [5] who used Cepheid variables to measure distances to nearby galaxies and showed that these are independent systems in their own right and lie at very large distances. The recession velocities of a large number of galaxies had been measured painstakingly over the years by V M Slipher [8]. Thus the Hubble's law could have been discovered at any time after 1926. Lemaître [9, 10] and Robertson [11] discovered cosmological solutions at this time; both realised that the recession of galaxies constitutes an observational evidence of models of an expanding universe. Both used the data from Slipher and Hubble to verify the velocity–distance relation and determine the proportionality constant. To them the form of the velocity–distance relation was natural and hence they did not highlight it in their papers. In an almost parallel effort, observers were trying to make sense of relativistic models. Lundmark [12] decided to fit a quadratic relationship between velocity and distance, postulating that there must be finite maximum recession velocity. Hubble's paper [6] appears to be an attempt to demonstrate that the quadratic term is either not required or that its coefficient must be very small. By the time Hubble followed up this work with more data [13], the language and the underlying paradigm underwent a significant shift: while the initial work by Hubble as well as Lundmark's work uses methods common in stellar and galactic astronomy, there was a sudden realisation of the cosmological scenario in all later work.

Hubble did indeed discover the Hubble's law, and did so independently, but he was not the first one to get there.

[13,14]. On the other hand, Lemaître [9,10] and Robertson [11] were checking whether the velocity–distance relationship expected from theoretical models was seen in nature or not.

The value for Hubble's constant determined by Hubble and others was around 500 km/s/Mpc. In the context of cosmological models, this indicated an age of the universe around 2 billion years[2]. However, radioactive dating showed that some rocks on Earth were much older than this. This led to a crisis as the universe cannot be older than all its contents. The problem was related to cross-calibration and incorrect identification of some standard candles. It required painstaking work over the next quarter of a century to understand these issues and get a better value for the Hubble's constant. At present the value of Hubble's constant is determined [15] to be close to 70 km/s/Mpc and the corresponding age of the universe is 13.6 billion years.

[2] $1/H_0$ is a useful order of magnitude estimate of the age of the universe. A more precise value requires knowledge of other parameters but it can be shown that the order of magnitude estimate is accurate to better than 50%.

The current challenge is to extend the redshift–magnitude relationship to larger distances as we can find out more about the universe. Indeed, this aspect does not require determination of the Hubble's constant and hence the errors due to cross-calibration are not relevant. *Figure* 1 shows the Hubble diagram for supernovae of type Ia, the brightest standard candle known to us. We see clearly that at low redshifts ($z \ll 1$), the data satisfies the Hubble's law ($m - M \propto 5 \log z$) very well. At larger redshifts, the effects of space-time curvature modify this relationship and hence we see some deviations[3]. For these objects, we do not have a good calibration of the luminosity and therefore we cannot use this data to determine Hubble's constant, but we can verify the Hubble's law with the data.

[3] These deviations from Hubble's law at high redshift are used to constrain the contents of the universe. The data shown here can be used to demonstrate the existence of an exotic component called dark energy that has negative pressure, and is leading to an accelerated expansion of the universe.

With this we end the story of Hubble and Hubble's law. What we have outlined here is only one aspect of Hubble's contribution. He contributed directly to many

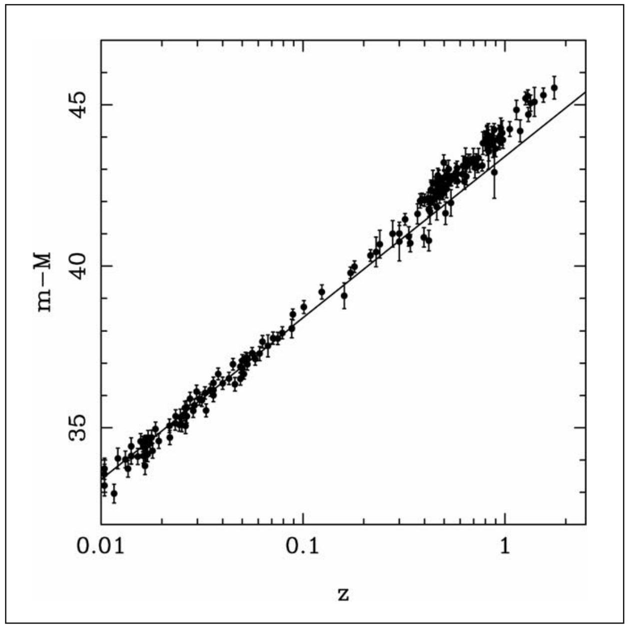

Figure 1. Hubble diagram for supernovae of type Ia. The data shown here corresponds to the Gold+Silver sample [16]. The line corresponds to m – M∝ 5 log z, the relation expected from Hubble's law.

aspects of extra-galactic astronomy; indeed he started this entire field. Hubble convinced his employers about the need for larger telescopes and the 100″ Hooker telescope and the 200″ Hale telescope at the Palomar observatories that were set up primarily for extra-galactic work have been used for research in all areas of astronomy. I can only refer you to other articles in this volume for some details of other contributions made by Edwin P Hubble.

Suggested Reading

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