Fluid Mechanics of Fish Swimming

1. Lift-based Propulsion

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Fish swim by coordinated motion of their body and fins. This article discusses the lift-based propulsion adopted by fast swimmers like dolphins.

Introduction

Many of us would have marveled at the seemingly effortless and graceful way in which fish swim, turn, accelerate, and brake. From a mechanics perspective, questions are often asked how good they are as swimmers.[1] Are they more efficient than a propeller driven under-water vehicle? Where does the energy expended by the muscles go? What is the drag force due to the highly unsteady flow on the undulating fish body? Is the wake of a whale quieter than that of a submarine? Answers to many of these questions are not known and are topics of current research. In this article some basic issues related to self propelling systems are discussed, and we will look at in some detail a type of propulsion adopted by many fast swimming fish, which is essentially based on the 'lift' forces generated by their flapping tails. Anguilliform swimming, seen in eels, where the whole body seems to take part in propulsion will be discussed in a later article. Fish swimming is generally at high Reynolds numbers (>1000), implying that pressure and inertia forces dominate (see *Box* 1).

Figure 1 shows the main parts of fish that are relevant to swimming. All of these features will not exist in all the species. Motion is achieved by coordinated movement of the body and some or all of the fins.

Fish that generate thrust principally via body and/or

Keywords
Propulsion, lift, drag, self-propelling bodies.

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[1] Lighthill has made major contributions to the understanding of many aspects of fish locomotion using his unique ability of combining physical insight into mathematical models.

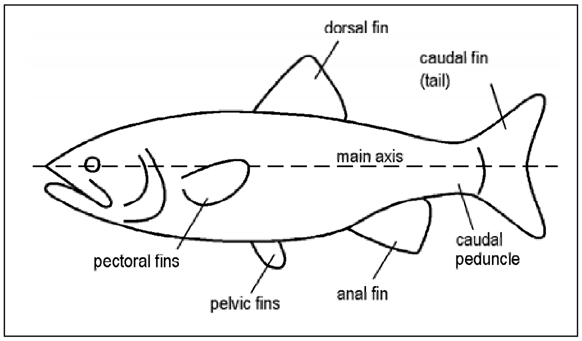

Figure 1. Morphological features of a fish. Pectoral and pelvic fins are paired. Anal and dorsal fins are median fins.

caudal tail fin (BCF) motions are known collectively as BCF swimmers. The swimming motion of BCF swimmers may be further classified based on the type of movement of the body and the tail (*Figure* 2). *Anguilliform* locomotion involves undulation of the entire body, with amplitude growing toward the tail. As one goes from anguilliform to thunniform modes the body undulation reduces. Typical carangiform swimmers (jacks, mackerel, and snapper) have a narrow peduncle and a tall forked caudal fin. These are among the swiftest of swimmers. The fastest are *thunniform* swimmers. These fish, including tuna and some sharks, have very low-drag body shapes, narrow peduncles, and tall lunate (crescent-shaped) caudal fins. In *anguilliform* locomotion, adopted by eels for example, the entire body is the propulsor; thrust is produced by undulation, i.e., passing a transverse wave from head to tail. In thunniform locomotion, almost all of the thrust is by the caudal fin.

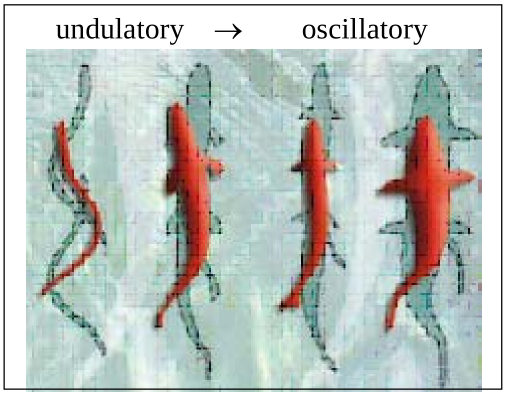

Figure 2. Different modes of BCF swimming.

A – Anguilliform. (e.g., eel)

B – Subcarangiform. (e.g., trout)

C – Carangiform. (e.g., Euthynnus affinis)

D – Thunniform. (e.g., shark)

Redrawn from Review of Fish Swimming Modes for Aquatic Locomotion, *IEEE Journal of Oceanic Engineering*, Vol.24, No.2, pp. 237–252, 1999, D M Lane, M Sfakiotakis and J B C Davies, Heriot-Watt University.

Box 1. Reynolds Number and Strouhal Number

Reynolds number and Strouhal number are two important non-dimensional numbers in relation to swimming of fish. Reynolds number is defined as

$$\mathrm{Re} = \rho \frac{U l}{\mu} = \frac{U l}{\nu}.$$

ρ is fluid density, μ is fluid dynamic viscosity, and $\nu = \mu/\rho$ is kinematic viscosity. For water, $\rho = 1000$ kg/m^3, $\mu = 1 \times 10^{-3}$ N.s/m^2, and $\nu = 1 \times 10^{-6}$ m^2/s; for air, under standard atmospheric conditions, $\rho = 1.2$ kg/m^3, $\mu = 1.8 \times 10^{-5}$N.s/m^2 and $\nu = 1.5 \times 10^{-5}$ m^2/s. U is a reference velocity, e.g., the forward velocity of a fish, and l is reference length, e.g., the length of the fish body. Reynolds number is a relative measure of the importance of inertial forces to viscous forces. Thus in a high Reynolds number flow, the inertial and pressure forces dominate and viscous forces may be expected to be negligible. However, as Prandtl showed, the effect of viscosity can be large even when its absolute value is very small. For low Re (< 1), viscous forces dominate. In swimming creatures, low Reynolds numbers are obtained for extremely small sizes.

For most flows in water and air, the Reynolds numbers tend to be high. For a blue whale with a length of 30 m and swimming at 10 m/s the Reynolds number is 300 million; for a 30 cm long mackerel swimming at 3 m/s, the Re is about 1 million. At such high Reynolds numbers, at least part of the boundary layer on the body will be turbulent, and the wake will definitely be turbulent. In contrast, for a sea urchin sperm with a length of 0.15 mm and speed = 0.2 mm/s, the Re $\simeq$ 0.03.

A commonly used non-dimensional measure of frequency for periodic flows is Strouhal number, defined as

$$\mathrm{St} = \frac{f H}{U}.$$

For a fish, f is the frequency of flapping, of for example, the tail; H is a length scale, usually the amplitude of the tail motion; U is the forward velocity. Another commonly used non-dimensional frequency is the reduced frequency,

$$\sigma = \frac{2\pi f l}{U}.$$

Although the caudal fins are used in most of fish species, there are many types of fish, known as Median and/or Paired Fin (MPF) swimmers, that generate thrust using principally median (e.g., dorsal and anal) and paired (e.g., pectoral) fins. BCF swimmers also may use MPF modes for manoevring and stabilization.

Forces on Bodies Moving with Constant Velocity

It is useful to look at some general principles with regard to 'self-propelling' bodies (see *Box* 2). A person walking, riding a bicycle or monocycle (*Figure* 3a), a motor vehicle, an airplane, a fish are examples of self-propelling bodies.

In this article we will confine ourselves to fish moving with constant speed in a straight line. It must be mentioned that even in this case the oscillatory motions of the body and fins result in small periodic variations of speed and lateral position, and the motion is steady only in an average sense.

Application of Newton's 2nd law in the vertical and horizontal directions to a body moving with constant speed in a straight line, i.e., a non-accelerating body gives

$$L - W = 0,$$

$$T - D = 0.$$

The weight (W) has to be supported by an upward force (L), and any frictional force (D) has to be overcome by a forward force (T). In addition, the net moment in each direction has to be zero.

For an aircraft flying level with constant speed, L is the aerodynamic lift force which supports the weight, T is the engine thrust which overcomes the drag or frictional

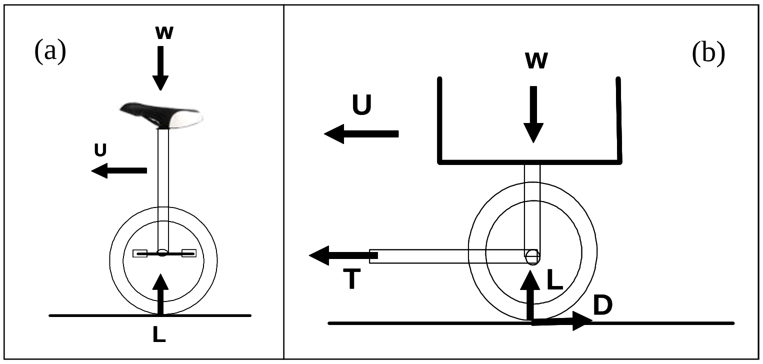

Figure 3. Examples of bodies moving with constant speed on level ground.
a) Monocycle; b) cart being pulled. For the monocycle (a), there is no force in the horizontal direction.

Box 2. Self-Propelling Bodies

For a self-propelling body moving with constant velocity, the net force on the body in the vertical and horizontal directions is zero.

In natural systems, like a human walking or a fish swimming, there is always unsteadiness. Even during steady walking, the body is not really moving with a constant velocity; there is acceleration and deceleration during each cycle. When we put a foot down on the ground, friction acts in a direction opposite to the direction of motion; when we raise a foot, friction on the rear foot is in the direction of motion. These two opposing frictional forces cancel over one cycle.

Analysis of walking, though extremely interesting, is complicated. A cart being pulled with constant velocity (*Figure* 3b) is relatively easier to understand. A horizontal force (T) has to be applied to overcome the wheel (rolling) frictional force (D). Similiarly in a bicycle, the ground at the rear wheel applies a forward force to overcome the rearward frictional force being applied by the ground on the front wheel. But a monocycle (*Figure* 3a), which would seem to be conceptually simple, is more complicated. The total weight is supported by a normal component of the reaction force from the ground. Is there a frictional force applied by the ground in the horizontal direction? What is its direction? The person is certainly doing work; where is the energy going? The reader can think about answers to these questions.

Aircraft and fish propelled primarily by their caudal fins are somewhat like the bicycle. The thrust, which is from the engine or the fish tail, and the frictional force (the drag), which is on the main body, are acting at distinct points. However, in the case of certain other fish, like eels, and birds the situation is more like the monocycle: it is difficult or impossible to distinguish between forward and rearward forces.

In self-propelling bodies moving with constant velocity, there is no gain in mechanical energy, potential or kinetic, as would happen when climbing a mountain or when accelerating. All the work goes into heat energy through non-elastic deformation of surfaces and viscous dissipation in the surrounding fluid.

force (D) of the air (*Figure* 4a). On a swimming fish, buoyancy force (F_{B}) and any lift force (L) generated by the fins counteract the weight (*Figure* 4b).

As mentioned above, in most of the systems found in Nature, the velocity is not really constant and $T = D$ only when averaged over one cycle of the generally periodic motion. Also, there is a subtle difference between natural swimmers and flyers and man-made ones like

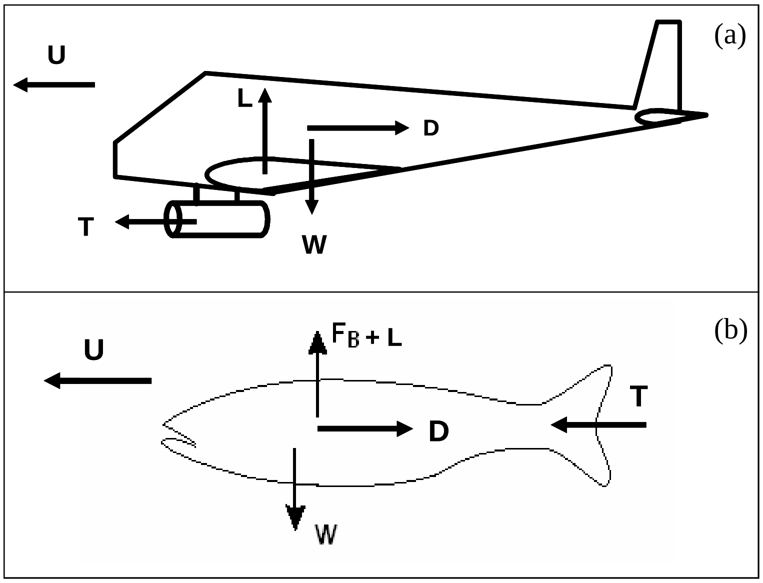

Figure 4. a) An aircraft flying with constant velocity. Thrust (T) from the engine balances the drag (D); lift (L) mainly from the wings balances the weight (W). b) For the fish shown here the thrust is mainly from the flapping tail. Lift from fins and buoyancy (F_B) balance the weight.

aircraft. In the case of an aircraft the thrust producing portion (propeller or jet engine) and the drag producing portion (primarily the airplane body and the wings) are separate entities. In the case of fish (especially anguilliform swimmers where the whole body undulates and produces 'thrust') both thrust and drag are produced by the same moving surface. Bird and insect flight is even more complicated; lift, thrust and some of the 'drag' are generated from the flapping wings.

Propeller-type or Lift-based Propulsion

The airfoil shape (*Figure* 5a) – long and thin with a rounded nose and sharp trailing – is a remarkable geometry that produces a large lift force (L) and experiences very low drag force (D). (This shape works for subsonic, high Reynolds number flows. Supersonic airfoils have pointed leading edges to minimize shock produced drag. At low Reynolds numbers the force generation mechanism is viscous and shape is not so important.) Lift is *perpendicular* to the *relative velocity* between the fluid and the airfoil. Drag, like friction, is opposite in direction to the relative motion.

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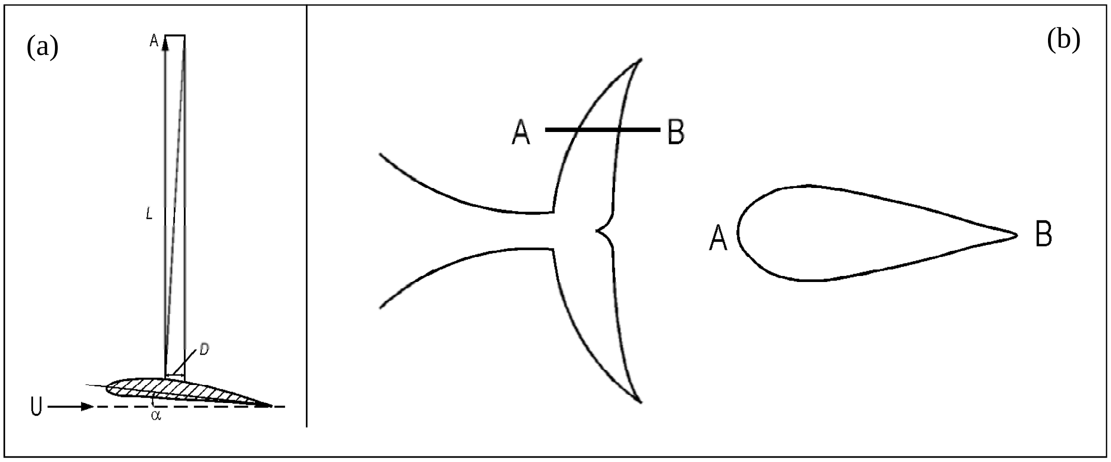

Figure 5. a) The airfoil shape with rounded leading edge and sharp trailing edge gives a large lift perpendicular to the direction of motion, and experiences a relatively small drag opposing the motion. U is the relative velocity between the fluid and the airfoil.
b) Fish tails have symmetrical airfoil cross-sections.

The airfoil forms the basis for a large number of surfaces where lift or thrust needs to be produced efficiently. Wing of an aircraft is the most visible application of airfoil. But wings of birds, marine propeller, steam turbine and helicopter rotor blades, tails of fish (*Figure* 5b) all have cross sections with an airfoil shape.

The lift force on an airfoil of planform area A moving with velocity U in stationary fluid is usually written as

$$L = C_{\mathrm{L}} \left(\frac{1}{2}\right) \rho U^2 A. \tag{1}$$

For symmetrical airfoils of the type found in fish tails, the lift coefficient

$$C_{\mathrm{L}} \approx 2\pi\alpha. \tag{2}$$

The angle of attack, α, between the airfoil and the relative velocity[2] direction usually is less than about 15° to prevent stall (*Box* 3). The maximum lift coefficient for steady flow is typically around 1.5. Note that lift varies linearly with density and area, and as the square of velocity. The above relations are useful to estimate lift forces. A 1 m^2 airfoil with a lift coefficient of 1, moving at 1 m/s in water generates a lift of 500 N or about 50 kg. In air with 1000 times lower density, to generate the same force would require a velocity of about 30 m/s.

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[2] For generation of fluid mechanical forces what matters is the relative velocity between the fluid and the body. Velocity of the fluid *U* past a stationary airfoil or the airfoil with velocity *U* in a stationary fluid will produce identical lift forces.

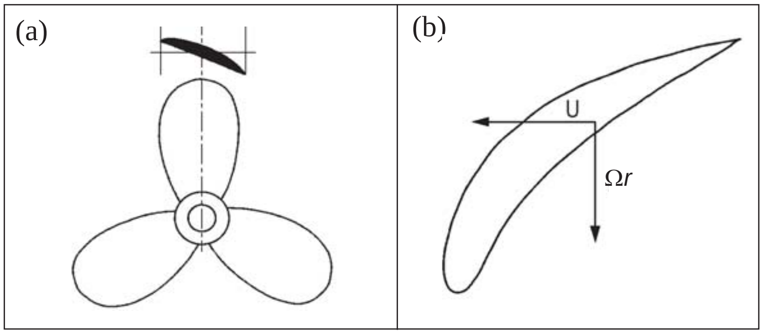

Figure 6. a) Propeller blades have curved airfoil cross-sections.
b) The relative motion of any section of the blade consists of forward velocity U and velocity Ωr due to the blade rotation.

The lift can also be written in terms of the circulation around the airfoil, $L = \rho U \Gamma$. Circulation (Γ) is defined as a line integral of fluid velocity around a closed loop, $\Gamma = \oint \overrightarrow{U}.\mathrm{d}\overrightarrow{S}$, where S is distance measured along the loop. Γ is one measure of the strength of a vortex. Many of the vortical structures (trailing vortices and starting vortices) have roughly same values of circulation as around the wing.

The thrust from a propeller (*Figure* 6a) is essentially from the lift generated by the rotating blades. Consider the case of a propeller moving with constant velocity U and producing thrust, T. At any radial cross-section the airfoil will see a relative velocity due to the forward velocity U and the rotational velocity Ωr (*Figure* 6b). Ω is the rotational velocity in radians/s and r is the radial distance of the airfoil cross-section from the axis of rotation. The lift and drag forces can be decomposed into a component in the forward direction contributing to the thrust and tangential component contributing to the torque required to turn the propeller (*Figure* 7).

The thrust from the flapping tail (the caudal fin) of a fish is very similar to that from a rotating propeller blade. Both are based on lift force generated from airfoil type cross-sections. The caudal fin oscillates laterally as the fish moves forward. It has two oscillatory motions – heave or lateral motion with amplitude H, and a rotary or pitching motion around an axis within the airfoil. The heaving motion and the constant forward velocity

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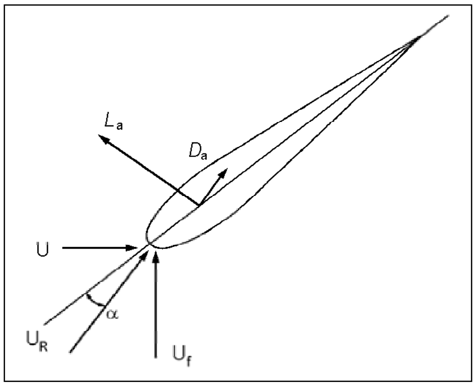

Figure 7. For a propeller blade or a fish tail, the forward velocity U and the airfoil velocity U_f due to either blade rotation or tail flapping gives a resultant velocity U_R. Lift L_a is perpendicular to U_R and drag D_a is in the direction of U_R. The resolution of L_a and D_a in the forward direction contributes to thrust.

results in wave-like path of the fin (*Figure* 8a); the pitching motion ensures a proper angle of attack of the airfoil at each location. *Figure* 8b shows the heave and pitch motions of the tail if one rides with the fish. Again *Figure* 7 shows how thrust is generated. Unlike in the propeller case, U_f varies periodically with time.

An important parameter is the frequency of oscillation of the caudal fin, f. Optimal thrust generation seems to occur when the non-dimensional frequency, Strouhal number, St $= fH/U$ is in the range 0.25–0.4 (see also *Box* 1). Higher forward speed is achieved by a higher frequency of flapping, quicker beating of the tail.

Wake Structure

The disturbance created behind a moving body in a stationary fluid is known as the *wake*. *Figure* 9 shows the

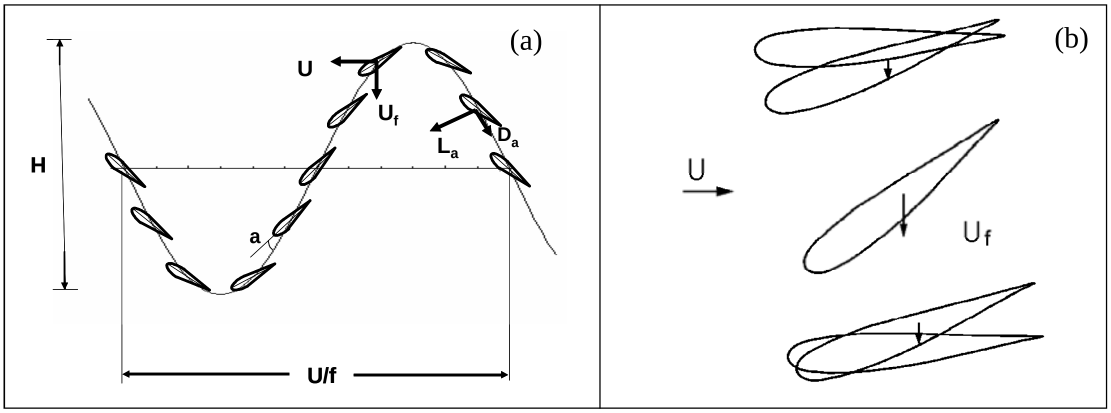

Figure 8. a) Motion of a tail airfoil section consists of forward motion and a up and down heaving motion with amplitude H and frequency f. The tail continuously re-orients or pitches to maintain a proper angle of attack. The wavelength of motion in the direction of motion is U/f. b) The heaving and pitching motions of the airfoil during ½ a cycle. The airfoil quickly re-orients at the two extreme points.(Adapted from Lighthill [1].)

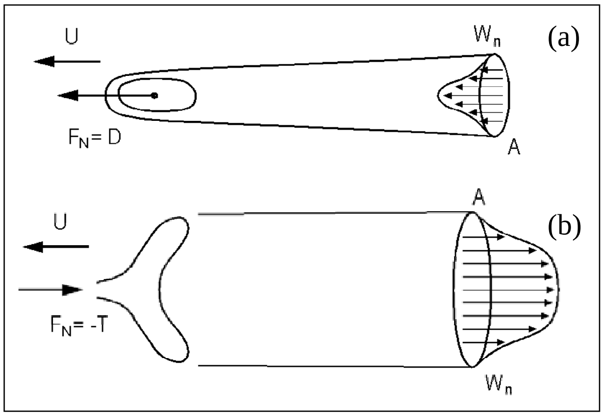

Figure 9. a) A drag producing body drags the fluid in the forward direction in the wake.
b) A thrust producing body pushes the fluid behind. In both cases, the bodies are shown moving in a stationary fluid.

fluid being dragged along behind a (drag producing) body; the fluid in front and sides is pushed away by essentially a non-viscous mechanism. This phenomenon is readily experienced as a gust of air on the side of a road as a fast moving bus passes us. A thrust producing body (*Figure* 9b) pushes the fluid in a direction opposite to direction of motion.

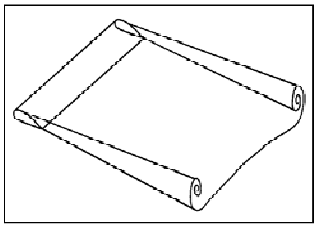

Figure 10. The two opposite signed trailing vortices from the tips of a lift generating wing. The circulation around the vortices is proportional to that around the wing. The flow behind the wing is downward. The kinetic energy in the vortices is related to the induced drag, which goes like the square of the lift.

Depending on the type of body, a wake can have interesting structures. Two cases are relevant to fish propulsion. A lifting body, as a wing, moving with constant velocity produces a pair of trailing vortices (*Figure* 10). The induced drag (see *Box* 3) is related to the energy carried by these vortices and is a penalty for generating lift. The propeller blades (since they individually generate 'lift') also leave behind trailing vortices. However, since the tip is rotating, the trailing vortex from each rotating blade of the propeller traces a helical path (*Figure* 11).

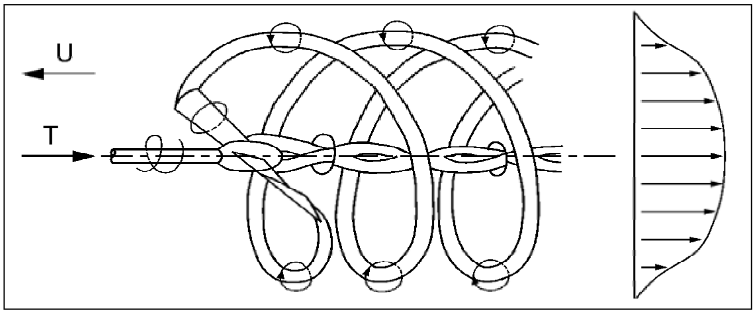

Figure 11. Trailing vortices of the type seen in Figure 10 trace helical paths from each rotating propeller blade.

Box 3. Lift and Drag *

Any body moving steadily forward with speed U experiences a force (which in general can be time varying) that is usually decomposed into drag force (D) opposite to the direction of motion, and lift force (L) normal to the direction of motion. (We are not concerned with the lateral component of the force.) These forces are due to viscous stresses, which are generally tangential to the body surface, and fluid pressure, which is normal to the surface. Integral of the stresses and pressure over the body surface area gives the force acting on the body. At high Reynolds numbers, lift is mostly due to pressure and drag is due to both viscous stresses and pressure.

The lift force is usually written in terms of the lift coefficient, C_L,

$$L = C_\mathrm{L}\left(\frac{1}{2}\rho U^2\right)A.$$

For symmetrical airfoils of the type found in fish tails, the lift coefficient $C_\mathrm{L} \approx 2\pi\alpha$. The angle of attack, α, between the airfoil and the relative velocity direction usually is less than the stall angle, which typically lies between 10 and 15 degrees. Stall happens due to flow separation and results in sudden loss of lift and increase in drag; stall is to be generally avoided. The lift coefficient is generally less than about 1.5 under steady conditions, though much higher lift coefficients (and higher stall angles) are obtained under unsteady conditions, as on a flapping fish tail.

Lift generation can be explained by the Bernoulli principle: relatively higher fluid speeds and lower pressure on the upper surface of an airfoil compared to those on the lower surface. In terms of circulation around the airfoil, lift can be written as $L = \rho U \Gamma$.

An important feature of a wing that generates lift are two trailing vortices (*Figure* 10) roughly in line with the tips of the wing; these are sometimes visible as white trails (condensed water vapour in the low temperature cores of the vortices) in the sky behind an aircraft. The strength of these vortices is proportional to the lift. These vortices represent lost energy left behind and cause an additional drag called induced drag.

Drag force is generally due to contributions from fluid pressure and tangential (viscous) stress acting on the surface of the body. For streamlined bodies, pressure drag is negligible compared to the skin friction drag; for bluff bodies skin friction drag is negligible compared to the pressure drag.

For lifting bodies, like a wing, it is common to write another component of drag called the induced drag, D_i, which may be considered to be a penalty for producing lift.

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[*] See article by S P Govinda Raju, *Resonance*, Vol.14, No.1, pp.19–31, 2009.

continued...

Box 3. continued...

The presence of the trailing vortices can be explained in two different ways. It is due to a relatively higher pressure existing on the lower surface of the wing compared to that on the upper surface. The fluid from the lower surface thus tries to move towards the upper surface around the two corners of the wing creating the wing tip vortices. The strength of these vortices is directly related to the pressure difference and thus to the lift force,

$$D_{\mathrm{i}} = \frac{L^2}{\rho U^2 b^2},$$

where b is the span of the wing. Thus wide wings, as found in gliders, have low induced drag.

Like lift, drag is usually written in terms of a drag coefficient C_{D},

$$D = C_{\mathrm{D}} \frac{1}{2} \rho U^2 A.$$

The reference area is proportional to wetted area for streamlined bodies, and the frontal area for bluff bodies. C_{D} is typically 0.01 for streamlined bodies and around 1 for bluff bodies.

The second case is related to the fact that a vortex is also produced whenever the lift around a body is changed. Lift around an airfoil can change due to a change in the relative velocity or a change in angle of attack (see equations (1) and (2)). An airfoil suddenly started from rest produces a distinct vortex termed as the starting vortex (*Figure* 12). The circulation around the starting vortex is the same as that around the airfoil.

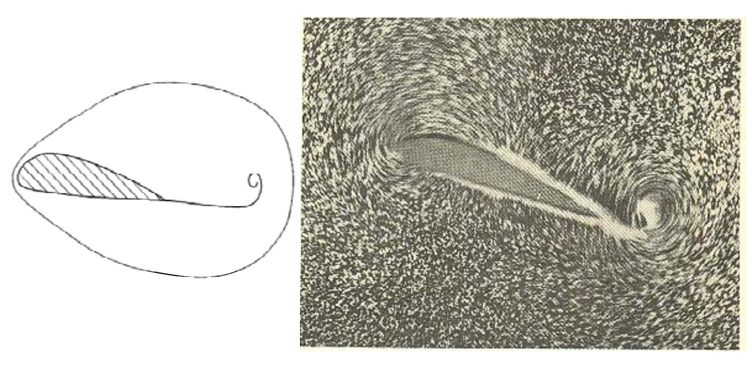

Figure 12. An impulsively started airfoil creates a starting vortex behind it. Any change in lift either due to change in angle of attack or change in speed creates such vortices. The flow visualization picture is from Prandtl and Tietjens [2].

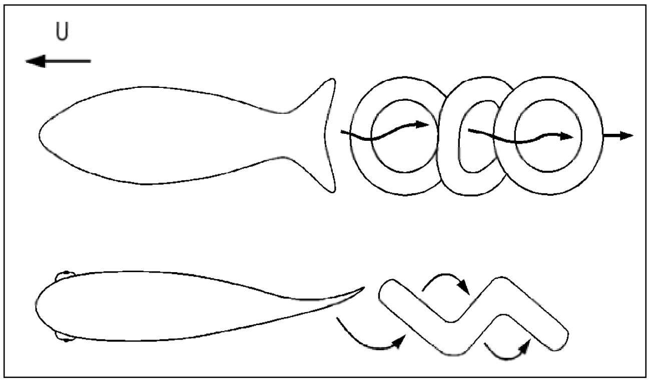

Figure 13. The wake of a flapping fish tail consists of interconnected, alternating signed vortex rings. The flow created by these rings is an undulating backward jet.

Both the angle of attack and the magnitude of the relative velocity between the tail and fluid change cyclically in time. Thus the lift on the tail (which contributes to the thrust) changes cyclically; in fact, the direction of lift with respect to the airfoil changes sign during one cycle. The cyclical change in lift usually results in two opposite signed vortices being shed from the trailing edge of the tail fin in each cycle. These together with the tip vortices (the equivalent of the helical vortices from a propeller) form continuous vortex loops (*Figure* 13). The vortex loops are essentially a series of interconnected inclined vortex rings. (Vortex rings in common parlance are known as 'smoke' rings.) The axial component of the impulse in each ring is connected with the thrust generated by the tail.

Momentum and Energy in the Wake

The starting vortices and the tip vortices together form a series of interconnected vortex rings.

Earlier, we had commented that a drag producing body pulls the fluid along with it in the wake and a thrust producing one pushes the fluid in the backward direction (*Figures* 9, 11). From momentum principle the velocity in the wake can be related to the force on the body,

$$F_{\mathrm{N}} \approx \rho U \int w_n \mathrm{d}A. \qquad (3)$$

w_n is the velocity component in the forward direction in the wake and the integral is over the cross-sectional area of the wake. For drag producing body $F_{\mathrm{N}} = D, w_n > 0$;

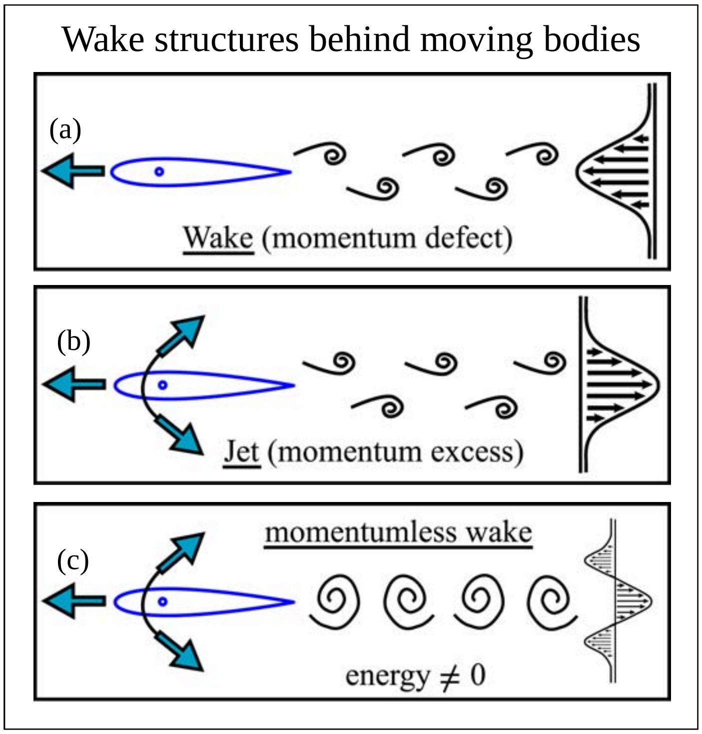

Figure 14. The three types of wakes behind an airfoil: a) drag producing with a Karman vortex street; b) thrust producing obtained from appropriate oscillatory motion of the airfoil with a reverse Karman vortex street; c) oscillation with a Strouhal number ~ 0.3 produces zero force on the airfoil, a momentum less wake and vortices aligned on the line of motion.

for thrust producing body $F_{\mathrm{N}} = -T, w_n < 0$. Thus *a self-propelling body moving with constant velocity should leave behind a zero momentum wake,* $\int w_n \mathrm{d}A = 0$.

A simple oscillating airfoil, depending on the frequency of oscillation, can produce all three types of wakes (*Figure* 14). For two-dimensional bodies the drag producing wake generally produces a Kármán vortex street (*Figure* 14a, 15a); vortices are aligned such that they produce a forward velocity in the wake. The so-called reverse Kármán vortex street results from a thrust producing body (*Figure* 14b); the mid-plane cross-section of vortex loops (*Figure* 13) would show a reverse Kármán street structure. In a zero-momentum wake, one possible configuration is for the vortices to be aligned on the centreline (*Figures* 14c, 15). For three-dimensional bodies like a propeller or fish tail, as we have seen, the vortex structure is more complex, though the basic principles would be the same.

Suggested Reading

[1] **James Lighthill, *An informal introduction to theoretical fluid mechanics*, Oxford University Press.**
[2] **L Prandtl and O G Tietjens, *Fundamentals of Hydro- and Aero-mechanics*, Dover Publication.**
[3] **John J Videler, *Fish Swimming*, Chapman Hall.**
[4] **Jaywant H Arakeri and P N Shankar, Ludwig Prandtl and Boundary Layers in Fluid Flow, *Resonance*, Vol.5, No.12, 2000.**
[5] **Raghuraman N Govardhan and O N Ramesh, A Stroll down Kármán Street, *Resonance*, Vol. 10, No.8, 2005.**

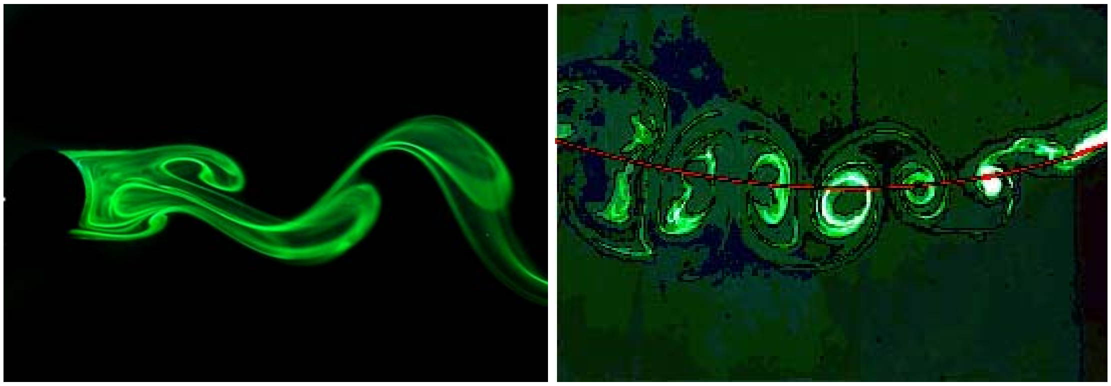

Figure 15. a) A Karman vortex street behind a stationary circular cylinder; flow is from left to right [5]. b) A momentum less wake from an oscillating airfoil moving left to right in a slightly curved path. The vortices are aligned on the path of motion. Picture taken by Sachin Shinde, PhD student, Department of Mechanical Engineering, IISc.

What about the energy in the wake? Vortices carry kinetic energy with them, and thus even in a zero momentum wake, the energy left behind is non-zero. In the case of a propeller, the mechanical work done on the propeller by the engine (torque $\times\Omega$) will go into producing useful work (TU) in propelling the airplane forward and some into energy in the wake. Simliarly, part of the work done by the fish tail goes into pushing the fish body against fluid resistance, part goes into kinetic energy associated with the vortex rings and some is lost in viscous dissipation within the boundary layers on the tail surface.

Conclusion

Although fish propelling themselves primarily with oscillation of their tails is similar to propeller based propulsion of submarines and aircraft, there are important differences. The motion of a fish is inherently unsteady even when it is swimming 'steadily'. The drag and lift forces due to such unsteady flows is not as well understood as for steady motions. Another complication in the fluid mechanics is due to the high flexibility of fish bodies and fins. Finally, it may be noted that all self propelling bodies moving with constant velocity, be it walking or swimming do no useful mechanical work; all the work goes into heat!

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