Aerobasics – An Introduction to Aeronautics

4. Fluid Flow Fundamentals

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A study of air flow past simple bodies provides a background for the understanding and interpretation of aerodynamic forces on airplanes. In this article some simple flows are studied by treating air as a viscous and compressible fluid subject to the laws of mechanics. The importance of two dimensionless parameters, the Reynolds number and the Mach number, respectively representing the viscous and compressibility effects is clearly brought out. The different flow phenomena associated with specific ranges of the Reynolds and Mach numbers are indicated and explained.

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1. Properties of Air

Understanding and interpreting aerodynamic forces – the forces exerted by moving air on the surfaces of an airplane – is critical to the study of airplane motion. In this context, air can be treated as an ideal gas following the gas law:

$$P = \rho \mathrm{RT}. \quad (1)$$

In the above relation, the pressure P, the density ρ, and the temperature T define the state of the air. R is the gas constant (R = 287 Nm/kg.K). It is clear from the above that, at constant temperature, a change in pressure of air leads to corresponding change in density. As will be seen later, a typical pressure change at any point on a wing moving at a flight speed V, is related to the dynamic pressure q, defined by:

$$q = \frac{1}{2}\rho V^2 \ . \quad (2)$$

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Thus at a typical flight speed of about 50 m/s, the dynamic pressure is about 1500 N/m^2 and represents a 1.5% change in

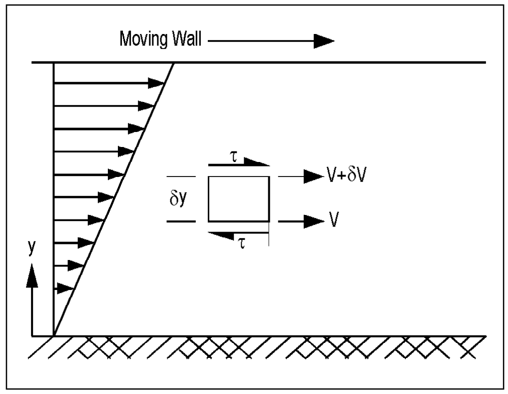

Figure 1. Shear stress in a moving fluid. The stress is proportional to the rate of shear deformation unlike in a solid where the stress is proportional to the deformation itself.

relation to atmospheric pressure and a comparable change in density. One may neglect this change of density under such conditions and assume air to be incompressible. We consider the effect of change in density in more detail in Section 4.

All fluids are viscous in that a shearing motion of the fluid is resisted by a shear stress τ, as in *Figure* 1

$$\tau = \mu \frac{\delta V}{\delta y}, \tag{3}$$

where μ is the index of viscosity. For air, μ increases with temperature. A typical value for μ of air at room temperature is 18 $\times 10^{-6}$ Pa s. This shear stress is in general very small. Even in the neighbourhood of a body near which velocity gradients are large (in the boundary layer), the shear stress is generally small compared with the pressure differences. One may thus neglect the effect of viscosity and consider air, as a first approximation, to be an ideal fluid – incompressible and inviscid. The theory of ideal fluids is well developed and many of the results can be applied to the motion of air. However, the ideal fluid theory cannot explain some aspects of airflow and the results of ideal fluid theory need to be applied with caution.

Another property of air of interest in fluid flow is the thermal conductivity of air – defined as the heat flow per unit area per unit temperature gradient. The value of the thermal conductivity for air is about 0.025 W/m.K. Heat conductivity is important in high speed flows where there is a substantial rise in temperature of the air due to pressure changes and due to frictional heat generation near surfaces. We shall not consider these special problems here. We neglect the effect of heat transfer within the fluid and consider the flow as isentropic.

2. Motion of Ideal Fluids

An ideal fluid is one which is incompressible and inviscid. In the motion of such fluids, fluid particles are driven entirely by pressure differences. One may apply the basic conservation laws (mass, momentum and energy) to a steady flow of an ideal fluid along a stream tube (a closed space bounded by streamlines on the sides and areas normal to flow directions at the ends) as shown in *Figure* 2. Conservation of mass implies the mass flux through the tube is a constant along the tube. In particular, mass flow entering at A must equal the mass flow leaving at B. As the density of the fluid is constant, we have,

$$A_1 V_1 = A_2 V_2 . \tag{4}$$

Also, on application of Newton's second law to a fluid material volume at a distance S along the tube, we have

$$-A\,\delta P = \rho A\,\delta S\, a. \tag{5}$$

Here $A\,\delta P$ is the net force along S due to the pressure difference δP across the fluid element. $\rho A\,\delta S$ is the mass of the element; a is the acceleration of the fluid element and is

$$a = \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}S}\frac{\mathrm{d}S}{\mathrm{d}t} = V\frac{\mathrm{d}V}{\mathrm{d}S} . \tag{6}$$

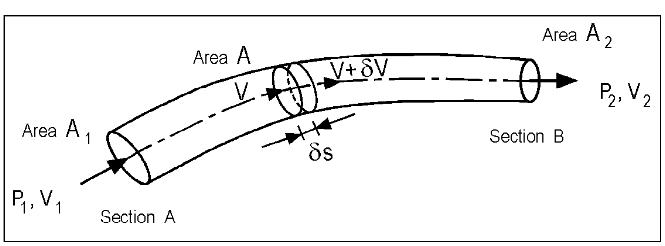

Figure 2. Flow of an ideal fluid along a stream tube. The fluid element is accelerated by the force due to the pressure difference between its ends. Viscous forces are absent.

From equations (5) and (6) we have

$$-\delta P = \rho V \frac{\mathrm{d}V}{\mathrm{d}S} \delta S \,. \tag{7}$$

This gives on integration along S

$$P + \frac{1}{2}\rho V^2 = P_o \,. \tag{8}$$

Equation (8) is the Bernoulli law and is a useful relation between pressure and velocity along a streamline. P_o is called the total pressure along the streamline while the pressure P is qualified as the static pressure for clarity. The quantity $\rho V^2/2$ as defined already, is the dynamic pressure. Equation (8) implies that at the

Box 1. Measurement of Static and Total Pressure

The static pressure inside the body of a fluid is not easily measured without disturbing the flow. However, a small hole on the surface of a body flush with the body surface connected to a pressure sensor (a transducer or a manometer) provides an indication of pressure at that location on the body as in *Figure* A. The stagnant fluid inside the pressure hole transmits the static pressure of the moving fluid just outside it.

The total pressure at any point in the fluid can be measured by introducing a 'pitot tube' at that location (*Figure* A). The pitot tube consists of a tube open at its end and placed facing the flow direction. The tube is connected at its other end to a pressure measuring instrument. The fluid velocity at B, the stagnation point just ahead of the pitot tube, is zero and the Bernoulli law indicates that the pressure here must be the total pressure corresponding to the flow along the streamline AB. Thus, a differential pressure indicator connected between a pitot tube and a static hole will indicate the dynamic pressure. The speed of flow can be easily calculated from this indication knowing the fluid density.

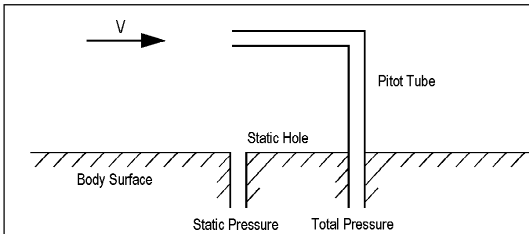

Figure A. Measurement of static and total pressures. The fluid flow past a small hole in a wall slips past leaving the fluid in the hole at rest. The pressure in the moving fluid is transferred to the fluid in the hole and this is conducted to the measuring instrument. But in the case of the pitot tube, the fluid decelerates ahead of the opening and comes to rest at the opening. The pressure indicated is the total pressure.

ends of the stream tube in *Figure* 2,

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 \; . \qquad (9)$$

Equation 9 could also be given an interpretation in terms of conservation of mechanical energy. The fluid is considered incompressible and thus external pressure does not interact with the internal energy of the fluid. Further, as the viscosity is assumed to be zero, there is no dissipation of mechanical energy inside the

Box 2. The Airspeed Indicator

A pitot tube can also be used to measure the flight speed of an airplane. To this end, an airplane is provided with a pitot tube located near the front of the airplane. The static pressure is obtained from a static hole on the surface of the airplane. The position is carefully chosen such that the static pressure at this position is very close to the pressure of atmosphere just outside the airplane for all flight speeds and orientations of the airplane (incidence and sideslip). There will still be a small error in the static pressure which is called the position error of the static tube and corrections can be applied for this.

The pressure difference between the pitot tube and the static tube is conveyed by tubes to a pressure sensing device called the airspeed indicator directly calibrated in terms of speed. Ignoring position error and compressibility effects, the pressure difference is equal to the dynamic pressure for the flight condition and depends on the air density at the flight altitude. Thus, it is possible to calibrate the airspeed indicator to read the flight speed correctly for only one value of air density and this is chosen by convention to correspond to the sea level, ISA condition (see Part 2 of this article in *Resonance*, Vol.13, p.971, 2008). The speed indication thus obtained is the calibrated airspeed (CAS). Thus the speed indicated by the airspeed indicator on an aircraft is the flight speed (true airspeed) only at ISA sea level condition in the absence of position error. The actual indication on the instrument is referred to as the indicated airspeed (IAS).

The airspeed indicator reading, the IAS, is generally in nautical miles per hour (Knots). While the IAS is significantly different from the true airspeed (TAS) except at sea level ISA conditions, it is still a useful flight instrument. The lift of a wing is directly dependent on the dynamic pressure and is thus directly related to IAS. An airplane will be able to take off and land safely at all altitudes at the same IAS values. The true airspeed is only required for navigational purposes.

In the above, we have not considered compressibility effects. It is possible to calibrate the airspeed indicator taking compressibility of air into account and this is normally done. In this case, the relation between CAS and TAS is modified to some extent at the higher Mach numbers and this will be indicated in a later section.

fluid. Thus, the flux of mechanical work entering section A in *Figure* 2 must be leaving at B. Equating the two, we have

$$A_1V_1\left(P_1+\frac{1}{2}\rho V_1^{\,2}\right) = A_2V_2\left(P_2+\frac{1}{2}\rho V_2^{\,2}\right). \qquad (10)$$

Here $P_1A_1V_1$ is the energy flux due to pressure at section A and $A_1V_1(\rho V_1^{\,2}/2)$ is the flux of kinetic energy of the fluid at A. The other terms are similar quantities at B. Using (4), (10) reduces to the same form as (9).

The energy interpretation of the Bernoulli law clearly indicates that, in a real fluid, P_o will generally decrease downstream along a streamline due to viscous dissipation.

3. Viscous Flows

All aspects of air flow around bodies cannot be explained by the ideal fluid theory and it is necessary to consider the effect of viscosity, however small, for a full understanding. To illustrate, we may consider a flow through a long, smooth circular pipe of diameter D as in *Figure* 3. For an ideal fluid, the flow is uniform over the area of the pipe and without friction at the walls. Thus there is no resistance to flow (drop of pressure) over the length of the pipe. Effect of viscosity is felt as friction at the walls leading to a parabolic variation of flow velocity over the cross-section of the pipe. There is a linear drop of pressure over the length of the pipe given by the analytically derived Hagen–Poiseuille law,

$$\frac{P_1-P_2}{L} = 32\mu\frac{V}{D^2}, \qquad (11)$$

where V is the mean velocity of flow and μ is the index of

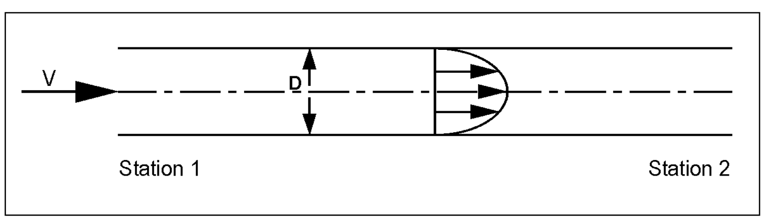

Figure 3. Viscous flow along a long pipe. The viscous forces on the walls of the tube are balanced by the force due to the pressure difference between the ends.

viscosity. Equation (11) can be represented in a dimensionless form useful in engineering applications by defining a resistance coefficient λ, as

$$\lambda = \frac{P_1 - P_2}{L} \frac{D}{\frac{1}{2}\rho V^2}. \tag{12}$$

Then, (11) becomes

$$\lambda = \frac{64}{\mathrm{R}}. \tag{13}$$

Here, R is the Reynolds number defined as

$$\mathrm{R} = \frac{VD\rho}{\mu}. \tag{14}$$

Experiments confirm the validity of (13) for values of R below 2000. However, at higher Reynolds numbers, above about 4000, the resistance coefficient λ follows an experimentally determined formula due to Blasius:

$$\lambda = 0.3164\ \mathrm{R}^{-1/4}. \tag{15}$$

These formulae for frictional resistance of smooth pipes are shown in *Figure* 4. It may be noted that for Reynolds numbers in

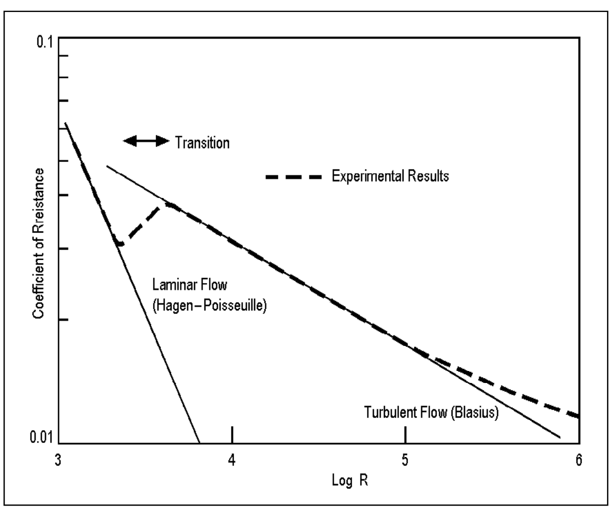

Figure 4. Frictional resistance of smooth pipes. At a Reynolds number below 2000, the flow is laminar. At a Reynolds number over 4000, the flow is turbulent and the resistance is much larger than in laminar flow. There is a small range over which the transition occurs.

the range of 2000 to 4000, the resistance coefficient follows neither equation (13) nor (15).

Visual observation of flow of water in a smooth pipe (made visible by injecting a dye), indicates that for R less than 2000, the flow in the pipe is smooth with parallel streamlines all along the pipe and is said to be laminar. For R greater than about 4000, the streamline flow is replaced by a chaotic motion, steady only in the mean. This flow is said to be turbulent. In the range $2000 < R < 4000$, the flow is said to be in transition. It may be noted that turbulent flow corresponds to a much higher resistance to flow as compared to laminar flow.

The flow in a smooth pipe considered above illustrates one aspect of viscous flow involving laminar and turbulent motion. Flows over bodies of practical interest involve further complexities. We consider the flow over a curved wall as in *Figure* 5. The wall could be considered a part of a body extending in both directions from the indicated surface. The fluid flow approaches parallel to the wall on the left and continues on the surface. The velocity of the fluid in contact with the wall is zero and increases away from the wall quickly approaching the value corresponding to ideal flow for the same body. The thin layer in which the velocity varies rapidly, while the pressure remains nearly constant, is called the boundary layer. The momentum of fluid in the boundary layer decreases downstream as its motion is opposed by pressure rise along the flow due to external flow as well as due to the wall friction. As a consequence, the thickness of the boundary layer

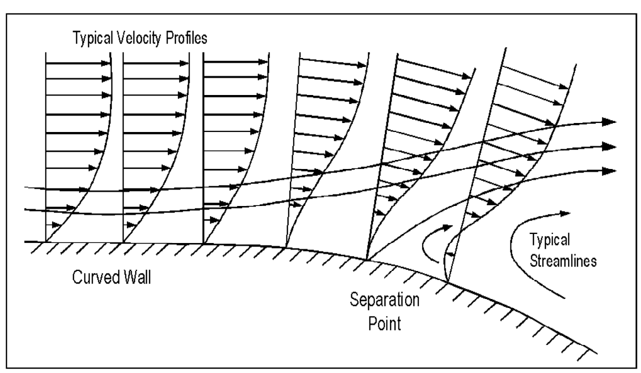

Figure 5. Flow separation along a curved wall. The flow near a wall is slowed down by the viscous shear stress as well as pressure rise along the flow direction. The fluid close to the wall decelerates until all its momentum is lost. This is the separation point. Beyond the separation point, there is flow reversal.

increases. Further, the fluid very close to the wall loses all its momentum and reaches zero velocity at some point called the 'separation point'. Beyond the separation point, there is flow reversal and the flow is locally from right to left. The flow coming from the left cannot continue further; it therefore leaves the wall and moves into the interior of the fluid. Thus the separation of flow due to viscous effects in the boundary layer alters the flow drastically as compared to ideal fluid flow. This leads to formation of wakes which are regions of low energy fluid downstream of bodies.

The phenomena of laminar flow, transition, turbulent flow, boundary layer and flow separation are all viscous effects which modify the flow of an ideal fluid past a body to an extent depending on the body shape and Reynolds number. These complexities and their role in affecting the aerodynamic forces on bodies will be indicated in specific cases later.

4. Compressibility Effects

The compressibility of air was considered briefly in Section 1. It was noted that air is a perfect gas following equation (1). One may further suppose that in flows past bodies, there is no addition of external heat and the flow can be considered isentropic and the relationship between pressure and density is then given by

$$\frac{P}{\rho^{\gamma}} = K \, . \tag{16}$$

Here, K is some constant in the whole field of flow, γ is the ratio of specific heats for air and is nearly equal to 1.4. Thus changes in density are related to changes in pressure by

$$\frac{\delta\rho}{\rho} = \frac{1}{\gamma}\frac{\delta P}{P} \, . \tag{17}$$

The Bernoulli law, equation (8), clearly shows that pressure changes in a flow are proportional to the dynamic pressure q of equation (2). Thus, a typical change of density in the flow is given by

$$\frac{\delta\rho}{\rho} = \frac{1}{\gamma}\frac{\delta P}{P} = \frac{1}{\gamma}\frac{1}{2}\frac{\rho V^2}{P} = \frac{\rho}{\gamma P}\left(\frac{V^2}{2}\right) . \quad (18)$$

We know that $\gamma P/\rho$ is the square of velocity of sound a, and thus we have

$$\frac{\delta\rho}{\rho} = \frac{1}{2}\frac{V^2}{a^2} = \frac{M^2}{2} . \quad (19)$$

In the above, V/a, the ratio of the speed of flight to speed of sound is defined as M, the Mach number. Equation (19) clearly shows that density changes in a flow are proportional to the square of the Mach number. The speed of sound in air varies from about 340 m/s at sea level to about 300 m/s at 10 km altitude. The effect of compressibility of air results in only a 0.5% change in density at a Mach number of 0.1. This corresponds to a speed of 30–34 m/s. The effect increases rapidly with Mach number and needs to be considered at higher speeds.

As there are local changes in the flow properties in a field of flow, it is useful to consider M as the local Mach number in that it is the ratio of the local flow speed to the local speed of sound. The flow is said to be subsonic if $M < 1$. The flow is supersonic for $M > 1$. It may be noted in a largely subsonic flow, there may be local regions of supersonic flow. This happens when the Mach number in the free stream (far ahead of the body) is moderately high, typically $M > 0.7$. Such flows are said to be transonic.

Subsonic flows show the influence of Mach number only from small to moderate extent and may be considered incompressible to a first approximation. Supersonic flows are dominated by compressibility effects to such an extent that these have to be considered as fundamental. As the Mach number increases, the compressibility effect results in a rise in temperature in the flow particularly close to the body surface near a leading edge. At a Mach number of 5 or higher, this temperature rise is of major concern. The flow is then said to be hypersonic.

Suggested Reading

[1] **L Prandtl and O G Tietjens, *Applied Hydro- and Aeromechanics*, Dover Publications, New York, 1957.**

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