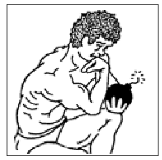
Think It Over



This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

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A Surprising Result on Correlation Coefficient

Let n be a positive integer with $n \geq 3$. Let π be a random permutation of the integers $1, 2, \ldots, n$. The elements of π are denoted by $(\pi(1), \pi(2), \ldots, \pi(n))$. By a random permutation we mean a permutation selected from $n!$ possible permutations with equal probability $(1/n!)$. If $\pi(j) = i$, then $\pi^{-1}(i) = j$, i.e., $\pi^{-1}(i)$ represents the position of the integer i in the permutation π.

We associate two random variables with the selected random permutations as follows:

$$X = \text{Max}(\pi^{-1}(1), \pi^{-1}(n)),$$

$$Y = \text{Min}(\pi^{-1}(1), \pi^{-1}(n)).$$

We note that $2 \leq X \leq n, 1 \leq Y \leq (n-1)$ and $Y < X$. Compute the correlation coefficient between X and Y. It is a surprising fact that this correlation coefficient does not depend upon n.

Keywords
Correlation coefficient, random permutation.