Black-Body Radiation

G S Ranganath

Towards the end of the nineteenth century, it became clear that within the frame work of classical physics, it was not possible to understand the phenomenon of Black-Body Radiation. In 1900 Max Planck came up with a theory that at one stroke accounted successfully for all its observed features. This theory has been presented against a backdrop of the major discoveries that led to it.

G S Ranganath was formerly at the Raman Research Institute, Bangalore. He is fan of Alexandre Dumas, Conan Doyle and Bernard Shaw. He is interested in classical Karnatak Music.

Introduction

On 27th April 1900 Lord Kelvin gave a lecture at the Royal Institution of Great Britain on '*Nineteenth-Century Clouds over the Dynamical Theory of Heat and Light*'. The two clouds that he referred to were the Black-Body Spectrum and the result of the Michelson–Morley experiment. He said that the 'beauty and clearness of theory' was overshadowed by these 'two clouds'. It is now a part of history how the study of the first led to the birth of Quantum Mechanics and that of the second resulted in the development of the Theory of Relativity. Thus Black-Body Radiation occupies a central position in the history of modern physics. We shall look at the early attempts to understand it and also Max Planck's non-classical theory of the black-body radiation. An attempt has been made to present the subject in its historical perspective.

Keywords
Black-body radiation, thermal radiation, heat, electromagnetic radiation, Stefan's Law, Stefan–Boltzmann Law, Wien's Law, Rayleigh–Jeans Law, black-body spectrum, ultraviolet catastrophe, zero point energy, photon.

Early Studies on Radiation

Nature of Thermal Radiation

We know that heat transfer takes place through the processes of conduction, convection and radiation. The

The famous British astronomer William Herschel, the discoverer of the planet Uranus, was probably the first to demonstrate that heat is a form of light beyond the red end of the visible spectrum.

first two processes can take place only in a medium. The process of radiation does not need a medium. It is through this process that heat from the Sun reaches the Earth. The famous British astronomer William Herschel, the discoverer of the planet Uranus, was probably the first to demonstrate that heat is a form of light beyond the red end of the visible spectrum. In an experiment carried out in 1800, he stacked up an array of thermometers behind a prism when the sunlight was incident on the other side of it. There were thermometers covering not only the visible part of the spectrum but also beyond. After a few hours of exposure each thermometer around and beyond the red region indicated a rise in temperature. To his surprise he found that as he moved away from the red end into the invisible (infrared) region, the temperature gradually increased. He published three papers on this subject in the *Philosophical Transactions of the Royal Society* of London. In his second paper he concluded that heat and light were a part of the same spectrum. It became clear from later studies that heat is an electromagnetic radiation with a wavelength ranging from 1 to 100 micro meters or microns i.e., larger than 0.7 microns corresponding to the red end of the visible spectrum. Later investigations established that the Sun emits radiation not only in the visible and the infrared regions but also in the ultraviolet region.

Heat is an electromagnetic radiation with a wavelength ranging from 1 to 100 micro meters or microns i.e. larger than 0.7 microns corresponding to the red end of the visible spectrum.

Black-Body Radiation

To quantify such a spectrum, we have to measure the amount of radiation energy appearing at different wavelengths. For this we need a detecting material that absorbs the radiation at all wavelengths completely. Gustav Robert Kirchhoff, a German physicist and mathematician, interested himself in this problem. His studies in 1860 led him to look at the properties of such a material which he termed as a perfect absorber. He pointed out that when such a material is heated, it will emit radiations of all wavelengths i.e. it will be a perfect emitter. Such a body is referred to as a black body. A material that appears black behaves very nearly like a black body. But how to get a true black body? The answer to this question was provided by Wilhelm Wien and Otto Lummer in 1895. They showed that a hollow body whose walls are at the same temperature behaves like a black body. They pointed out that it will be a perfect emitter emitting through a tiny hole in its surface, radiations of all wavelengths. Further, any radiation entering such a cavity will undergo infinitely many reflections inside and will lose energy at every reflection. Thus no incident radiation will emerge out of the tiny hole. In other words, it is also a perfect absorber. Thus such a cavity, which can be easily constructed, is almost a perfect black body. The amount of radiation emitted by a black body can be measured using a bolometer, a thermopile or a radiometer.

How to get a true black body? The answer to this question was provided by Wilhelm Wien and Otto Lummer in 1895.

Stefan's Law

Scientists had studied radiation even before the concept of a black body had emerged. For example, the French physicist Pierre Louis Dulong and the French mathematician Aléxis Thérèse Petit had published way back in 1817 results from what they considered to be purely radiation heat transfer between a spherical bulb and a spherical chamber. Various gases filled the gap between the two and they had measured the rate of change of temperature of the bulb over a range of pressures. They extrapolated their results to zero pressure hoping that only the radiative process remained. They concluded from their studies that the total energy E emitted per unit area per second is related to absolute temperature T of the body. They even got an empirical relationship between E and T.

Scientists had studied radiation even before the concept of a black body had emerged.

In 1879, the Austrian physicist Josef Stefan reassessed this work of Dulong and Petit. Stefan from his experi-

Using his own law Josef Stefan arrived at the important result that Sun is at a temperature of about 6,000 K.

ments on gases had shown that the thermal conductivity of gases was not pressure dependent. Hence, Dulong and Petit, by their extrapolation to zero pressure had eliminated convection but not conduction. Therefore, Stefan corrected for this and calculated the pure radiation component of heat transfer between the bulb and the chamber. He discovered that a relation of the type

$$E = \sigma T^4$$

matched better the experimental results of Dulong and Petit. Thus was born his famous law. The constant of proportionality σ is called the Stefan Constant. This law was indeed a very important discovery. Unfortunately it did not attract the attention of many physicists till Lummer took notice of it. In 1889, Lummer, along with Ernst Pringsheim and Ferdinand Kurlbaum experimentally measured the amount of radiation emitted by a cavity and succeeded in substantiating Stefan's law. They got the following value for Stefan's constant:

$$\sigma = 5.70 \times 10^{-5} \text{ erg cm}^{-2}\text{sec}^{-1}\text{deg}^{-4}(10^{-8}\text{Wm}^{-2}\text{K}^{-4}).$$

Stefan himself indicated the importance of his law. By that time, the amount of solar radiation that reaches the Earth, had been measured and was found to be 1360 W/sqm. From astronomical observations, the Earth–Sun distance and the Sun's diameter were known. With all this data available to him, and assuming the Sun to be a black body, Stefan could easily calculate its E. Then using his own law he arrived at the important result that Sun is at a temperature of about 6,000 K. This was indeed a significant result since until then no one was sure of the Sun's temperature. Temperatures anywhere between 2,000 K and 10,000 K were being quoted by physicists. We can also apply Stefan's law to work out the temperature of the Earth. We know the amount of solar radiation reaching the Earth. About 30% of this incoming radiation is reflected back into space by the

We can also apply Stefan's law to work out the temperature of the Earth.

Earth and the rest heats up the Earth. Soon the Earth starts emitting radiation as a black body. It reaches a steady state when it starts emitting as much as it receives. It is easy to work out this problem and arrive at the Earth's temperature. It turns out to be about 255 K or −18 °C. In fact this is close to the temperature that satellites have measured outside the Earth's atmosphere.

We can also apply Stefan's law to work out the temperature of the Earth.

Black-Body Spectrum

We have so far discussed the total radiation emitted by the black body. It is important to know how the radiant energy is distributed amongst the different wavelengths in its spectrum. Lummer and Pringsheim addressed this problem. As a result of their laborious and painstaking work, the general nature of the black-body spectrum became clear. *Figure* 1 shows some typical curves that represent the spectrum at different temperatures T of the black body. Each curve gives $u = u(\lambda)$, the energy per unit volume (energy density), at different wavelengths λ, in unit wavelength interval ($d\lambda$ = unity). Notice three important features:

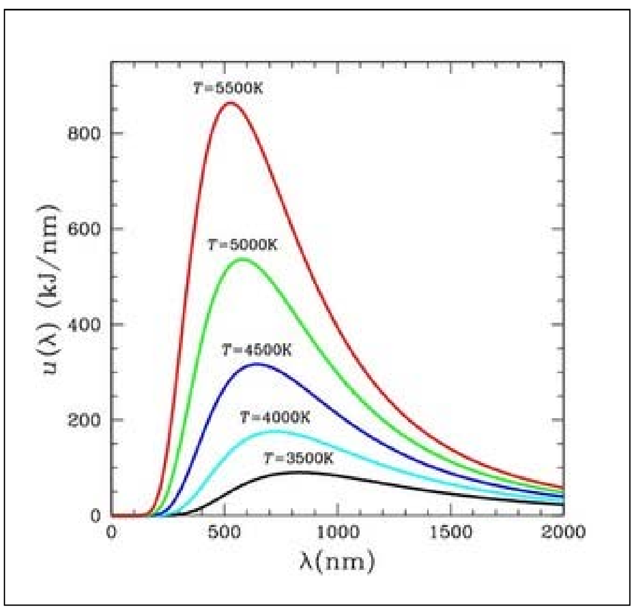


Figure 1. Black-body spectrum.

Boltzmann took a bold step in applying thermodynamics to radiation.

1. The spectrum is continuous. It has all the wavelengths λ, from 0 to ∞.

2. It has a peak at a particular wavelength.

3. The peak height increases and shifts to shorter wavelengths as the temperature of the black body increases.

Note: The visible part of the spectrum extends from 400 nm to 700 nm.

Thermodynamic Description of Radiation

Stefan–Boltzmann Law

It is great to know that the black-body radiation obeys Stefan's law. But to get this law from the science of heat or thermal physics is greater still. This became the next big challenge for physicists. In this game, Ludwig Boltzmann, a student of Stefan, broke new ground in 1884. Till then physicists had applied thermodynamics only to material objects. Boltzmann took a bold step in applying thermodynamics to radiation. The pressure P of the electromagnetic radiation in terms of its total energy density U can be calculated. It turns out that

$$P = \left(\frac{1}{3}\right) U.$$

So, Boltzmann conceived a Carnot 'Ether Engine' with radiation as the working substance and driven purely by pressure of radiation. This engine was assumed to have perfectly reflecting walls. The source and receiver were taken to be black bodies. During the isothermal expansion, the heat (energy) absorbed is $\mathrm{d}Q = (U\mathrm{d}V + P\mathrm{d}V)$. Work done in the Carnot cycle is $\mathrm{d}W = \mathrm{d}P\mathrm{d}V$. Thus the efficiency of this engine is

Boltzmann conceived a Carnot 'Ether Engine' with radiation as the working substance and driven purely by pressure of radiation.

$$\eta = \frac{\mathrm{d}W}{\mathrm{d}Q} = \frac{\mathrm{d}U}{4U}.$$

For a Carnot engine this should be equal to $\mathrm{d}T/T$. It is an easy matter then to show that the total energy density U of the thermal radiation depends on absolute temperature T according to the relation:

$$U = bT^4,$$

where b is a constant. From this, Boltzmann easily arrived at the Stefan's law of radiation with $\sigma = (bc/4)$, c being the velocity of light. That is how in later literature, Stefan's law came to be known as the Stefan–Boltzmann Law.

Wien was the first one to account for some of the features of the black body spectrum. He, like Boltzmann, applied thermodynamics to radiation.

Wien's Law

The features of the black-body spectrum remained a mystery till Wien appeared on the scene. He was the first one to account for some of the features of the black-body spectrum. In 1893, he, like Boltzmann, applied thermodynamics to radiation. He considered the problem of radiation confined inside a chamber of volume V, whose walls are under adiabatic expansion. Because the walls of the cavity are moving, relatively, the wavelengths of the waves reaching the surface get Doppler shifted in wavelength. Calculations led Wien to the surprising result that in this process (V/λ^3) remains a constant. From this, he easily calculated the energy $u(\lambda, T)$ emitted per unit volume per sec in unit wavelength interval. He found that:

$$u(\lambda, T) = \frac{f(\lambda T)}{\lambda^5},$$

where the nature of the function 'f' remained undetermined [1]. Incidentally integration of u over λ gives U, the total energy density of a black body. In the literature this relation is often referred to as Wien's law. Wien showed from this relation that the peak value u_p of u and the wavelength λ_p at which this peak occurs, are related to the temperature T of the black body through the relations:

$$T\lambda_p = \mathrm{A},$$

It is not generally appreciated that water vapour is a greenhouse gas. Water molecules absorb electromagnetic radiations of wavelength more than 10 microns.

$$u_p = \mathrm{B}T^5,$$

where A and B are constants. These extraordinary relations bring out two important aspects of the black-body spectrum, one pertaining to the position of its peak and another to the height of its peak. Though Wien did this fine piece of work in 1893, he published it only in 1896. In 1900 Lummer and Pringsheim from their experiments on cavity radiation showed that both these relations of Wien agreed well with their results. They found the constant A to be 2.898×10^{-3} mK. The first relation is known in the literature as Wien's Displacement Law. It may be mentioned here that Wien got the 1911 Nobel Prize in Physics for the discovery of this law.

Two implications of Wien's displacement law are worth mentioning here. Our first example refers to a tungsten filament lamp. It emits radiation at about 2000 K. Thus it emits mostly at a wavelength of 1.5 microns i.e. heat and not light. It is a very inefficient lamp. Our second example has to do with the temperature of the Earth. As already mentioned above, the temperature of the Earth due to solar heating alone is about 255 K or -18 °C. Such a black body emits peak radiation at a wavelength of about 12 microns. We know that the Earth is surrounded by a blanket of water vapour. It is not generally appreciated that water vapour is a greenhouse gas. Water molecules absorb electromagnetic radiations of wavelength more than 10 microns. Water molecules in the atmosphere, after absorbing these long wavelength radiations, radiate them back to the Earth thus raising its temperature. A new radiation equilibrium is reached and the Earth's temperature rises to about 300 K i.e 27 °C.

Microscopic Description of Radiation

Wien's Formula

Wien was also the first to suggest a microscopic view of the black-body radiation. Inspired by the kinetic theory of gases, he cooked up through 'loose' arguments an exponential form for the function 'f' and proposed in the same paper of 1896:

$$f(\lambda T) = \text{ C exp } \left(\frac{-\text{D}}{\lambda T}\right),$$

thus getting for u the following form:

$$u(\lambda, T) = \frac{\text{C exp } \left[\left(\frac{-\text{D}}{\lambda T}\right)\right]}{\lambda^5},$$

where C and D are constants. This is often referred to as 'Wien's formula'. Lummer and Pringsheim showed that it agrees very well with experimental results near the low wavelength end of the black-body spectrum. But they found that it deviates, giving lesser values, at the higher wavelengths with large deviations near the long wavelength tail of the spectrum.

Lord Rayleigh and James Jeans looked at the black-body radiation from the point of view of statistical mechanics.

Rayleigh–Jeans Law

Lord Rayleigh and James Jeans looked at the black-body radiation from the point of view of statistical mechanics. They were probably the first to develop a self-consistent classical microscopic picture of black-body radiation. They considered the black-body radiation confined in a cavity. Since radiations are nothing but electromagnetic waves, these waves will have to 'fit' into the cavity space. Hence, any such wave will have, like a vibrating string, an integral number of half wavelengths over the length it covers inside the cavity. Each such permitted wave is called a 'mode'. Rayleigh and Jeans worked out the number Z of the possible electromagnetic modes per unit volume that can exist inside the cavity, at a wavelength λ for unit wavelength interval. They got after taking into consideration the polarization of the electromagnetic waves:

$$Z(\lambda) = \frac{8\pi}{\lambda^4}.$$

The famous Rayleigh–Jeans Law completely fails near the low wavelength end.

Next they invoked the equipartition theorem of Maxwell's classical statistics. According to this, the thermal energy of a system is equally distributed amongst the different degrees of freedom of the system. Each degree of freedom accommodates an energy of $(kT/2)$, k being the Boltzmann constant equal to 1.38×10^{-16} erg deg^{-1} ($\times 10^{-23}$J/K). Rayleigh and Jeans applied this theorem to radiation inside the cavity. They recognized that the energy of radiation is distributed amongst its different modes inside the cavity. Now each mode is like a harmonic oscillator and hence has two degrees of freedom. As a consequence each mode inside the cavity has an energy of $\varepsilon = kT$. And there are Z modes per unit volume inside the cavity. Therefore the total radiation energy per unit volume at λ in unit wavelength interval is:

$$u(\lambda, T) = \varepsilon Z = \frac{(8\pi k)T}{\lambda^4}.$$

This is the famous Rayleigh–Jeans Law. Rayleigh got this result in 1900 with an undetermined constant in place of the numerical coefficient (8π). Later in 1905 Jeans worked out this constant. For all its beauty, this formula does not agree well with the experimental black-body spectrum excepting at high wavelengths near the tail of the spectrum. Further it completely fails near the low wavelength end where it diverges to an infinite value for $u\ (\lambda, T)$. This is often referred to as the Ultraviolet Catastrophe. It is interesting to note that we can get Rayleigh–Jeans law from Wien's law if we take the function $f(\lambda T) = 8\pi kT\lambda$.

Planck's Formula

So, we have two forms for $u(\lambda, T)$: the one due to Wien that works well at the low wavelength end and the other one due to Rayleigh and Jeans that works well at the high wavelength tail of the black-body spectrum. Both the formulae fail in the intermediate region of the spectrum. Having got formulae at the two wavelength limits, it seems a reasonable step to get a full formula by interpolating between these two. This important step was taken by Max Planck around 1900. Planck suggested an interpolation formula by taking for Wien's function 'f' the form

$$f(\lambda T) = \frac{(8\pi \ k\beta)}{\left\{\exp\left(\frac{\beta}{\lambda T}\right) - 1\right\}},$$

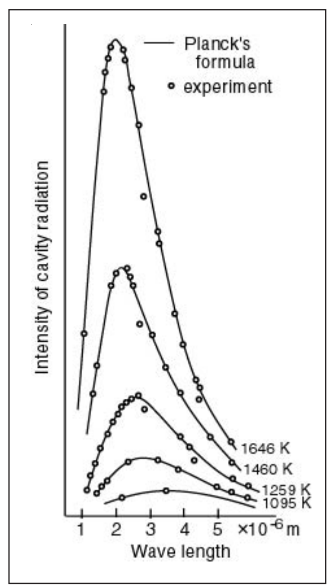
with k as the Boltzmann constant and β as an adjustable parameter. Hence for the black-body spectrum he got

$$u(\lambda, T) = \frac{(8\pi \ k\beta)}{\left[\left\{\exp\left(\frac{\beta}{\lambda T}\right) - 1\right\} \lambda^5\right]}.$$

Planck presented the discovery of Planck's formula at the German Physical Society meeting that took place on October 9, 1900.

This is the famous Planck formula. We can quickly verify that this goes over to the Rayleigh–Jeans law at high wavelengths and to the Wien's formula at low wavelengths with the constants C = $8\pi k\beta$ and D = β. In view of what has been said of the Wien and Rayleigh–Jeans formulae, we conclude that the Planck formula agrees with experimental results at both the short and long wavelength limits. How is it in the rest of the spectrum? It is of historical interest in this context to know that on Sunday evening of October 7, 1900, Planck sent his formula on a postcard, to his friend Heinrich Rubens which he received the following morning. A couple of days later, Rubens informed Planck that the formula worked perfectly. A typical data shown in the *Figure* 2 illustrates this fact.

Figure 2. Experimental black body spectrum and Planck's formula.



Planck presented the discovery of this formula at the German Physical Society meeting that took place on October 9, 1900. Later he communicated the same to the Berlin Academy of Sciences.

Planck's Theory

The extraordinary success of his formula would by itself have given Planck a permanent place in the annals of physics. The greatness of Planck lies in trying to make

Planck quantized the permitted energies of an oscillator.

sense out of his formula. Its very success must have set him thinking deeply about the underlying physics. Till then only the Rayleigh–Jeans formula had been derived on 'sound' physics. Since the classical description did not lead to the right 'formula' Planck had to think of a non-classical approach.

He knew that the radiation that was inside the cavity was being continuously emitted and absorbed by the 'material' of the cavity. At that time, the physicists thought that the atoms of a material which absorbed or emitted radiation as harmonic oscillators. Hence, oscillators were in equilibrium with radiation exchanging energy with it. He knew that radiations are electromagnetic waves. Hence the cavity was filled with these waves. These waves were absorbed and emitted by the oscillators that behaved like classical 'pendulums'. Enormous experimental evidence had backed the wave nature of electromagnetic waves. Planck could not ignore these and thus retained the wave nature of radiation as such. But the oscillator mechanism of absorption and emission of electromagnetic waves was more a theoretical model. He therefore, ventured to amend the laws governing its behaviour. He must have tried many alternatives and failed, to arrive at his own formula for the black-body spectrum. Then finally, in desperation as he has confessed, he suggested that a harmonic oscillator cannot have any energy but only in integral multiples of a quantum of energy $\varepsilon_0 = h\nu$, where ν is the natural frequency of the oscillator and h a constant to be determined. In other words, Planck quantized the permitted energies of an oscillator. Thus, they were strictly non-classical in nature with energies $\varepsilon = nh\nu$, n being an integer. These oscillators are in thermal equilibrium at any temperature. Therefore, Planck invoked the Boltzmann distribution to describe them. Accordingly, the number N of oscillators of energy ε is given by:

$$N = N_0 \exp\left(\frac{-\varepsilon}{kT}\right) = N_0 \exp\left(\frac{-nh\nu}{kT}\right)$$

with N_0 as the total number of oscillators of zero energy. Then the average oscillator energy ε of the system of oscillators becomes

$$\begin{aligned} \varepsilon &= \frac{\text{Total energy}}{\text{Total number of oscillators}} \\ &= \frac{(\sum N\epsilon)}{\sum N} \\ &= \frac{\sum\left\{\exp\left(\frac{-nh\nu}{kT}\right)\right\}\{nh\nu\}}{\sum\left\{\exp\left(\frac{-nh\nu}{kT}\right)\right\}} \end{aligned}$$

(summation is over n)

$$= \frac{(h\nu)}{\left\{\exp\left(\frac{h\nu}{kT}\right) - 1\right\}}.$$

Now each permitted mode that is inside the cavity is absorbed or emitted by an oscillator that is in the material of the cavity. Therefore Planck identified each oscillator of natural frequency ν with a cavity mode of the same frequency. Hence in terms of the wavelength $\lambda(= c/\nu)$ of the mode, we get the average energy of a mode to be given by

$$\varepsilon = \frac{\left(\frac{hc}{\lambda}\right)}{\left\{\exp\left(\frac{hc}{\lambda kT}\right) - 1\right\}}.$$

Planck repeated the procedure of Rayleigh and Jeans. There are Z modes per unit volume per unit wavelength interval. Further, each mode has an average energy ε. Hence, the total energy of radiation per unit volume at λ per unit wavelength interval is

$$u_(\lambda, T) = \varepsilon Z(\lambda),$$

$$u(\lambda, T) = \frac{(8\pi hc)}{\left[\left\{\exp\left(\frac{hc}{k\lambda T}\right) - 1\right\}\lambda^5\right]}.$$

This is nothing but the Planck formula with $\beta = (hc/k)$. Thus, his quantized oscillator model led Planck to the formula that he had earlier discovered. $u(\lambda,\mathrm{T})$ has a peak at a wavelength λ_p given by:

$$T\lambda_p = \frac{hc}{4.9651k},$$

and the total energy density U is obtained by integrating $u(\lambda, T)$ over λ from 0 to ∞,

$$U = \left(\frac{8\pi^5 k^4}{15c^3h^3}\right) T^4.$$

Hence,

$$b = \left(\frac{8\pi^5 k^4}{15c^3h^3}\right).$$

Planck used the value of $T\lambda_p = 0.294$ cm deg as obtained by Lummer and Pringsheim in 1900. Further he used the data on the total radiation emitted by a black body as measured by Kurlbaum in 1898. This gave him $b = 7.061 \times 10^{-15}$ erg cm^{-3} deg^{-4}. From these values Planck got

$$k = 1.346 \times 10^{-16} \text{ erg deg } (= 1.346 \times 10^{-23}\text{J/K}),$$

$$h = 6.55 \times 10^{-27}\text{erg sec } (= 6.55 \times 10^{-34}\text{J sec}).$$

The close agreement between the computed and known values of the Boltzmann constant k is a testimony to the success of his formula. This probably emboldened him to report his findings in the German Physical Society meeting that took place on December 14, 1900. Here the quantization of oscillator energy was presented for the first time. That is why many look upon this as the date of birth of Quantum Theory. Planck communicated his findings to the Berlin Academy of Sciences in 1900 and later published his work in *Annalen der Physik* in 1901. Planck had to wait for more than a decade for the scientific community to accept his revolutionary

Planck had to wait for more than a decade for the scientific community to accept his revolutionary ideas on quantization.

ideas on quantization and hence his theory of radiation. In fact Einstein was a critic of Planck's work. He used Wien's formula in his study of black-body radiation that led him to propound the quantum nature of light in his famous 1905 paper. Gradually physicists started appreciating Planck's theory. These admirers were indeed happy when Planck got the 1918 Nobel Prize in Physics for this great work.

Admirers were indeed happy when Planck got the 1918 Nobel Prize in Physics for this great work.

Planck and the Zero Point Energy

It has already been stated that Planck's formula goes over to the Rayleigh–Jeans formula in the long wavelength or equivalently high temperature limit. Therefore, it was natural for Planck to expect the average oscillator energy of his quantized oscillator to go over, at high temperatures, to the classical value of (kT). It has been shown above that the average energy of a Planckian oscillator is

$$\varepsilon = \frac{\left(\frac{hc}{\lambda}\right)}{\left\{\exp\left(\frac{hc}{\lambda kT}\right) - 1\right\}}.$$

It is very easy to show by expanding the exponential term and retaining only the first power $(hc/\lambda kT)$, that this expression for ε reduces to kT in the high temperature limit. This is what Planck did in his 1901 paper on the quantum theory of black-body radiation. Later in 1912, for reasons that are not clear, he reassessed his earlier work. He found that when he went to the next higher power in $(hc/\lambda kT)$ and calculated ε, he was in for a surprise. He got

$$\varepsilon = kT - \frac{\left(\frac{hc}{\lambda}\right)}{2} = kT - \frac{(h\nu)}{2}.$$

It did not go over to the classical value of kT. To force the classical limit on his model, he proposed in his 1912 paper that his quantized oscillator has its energy levels

The first experimental evidence for the existence of the Zero Point Energy came in 1924 from a work of the famous American chemist Robert Mulliken.

shifted upward by $(\frac{1}{2}h\nu)$. That is, the oscillator energies are

$$\varepsilon = \left(n + \frac{1}{2}\right) h\nu,$$

instead of $\varepsilon = nh\nu$. In other words, even in the ground state $(n = 0)$, it has a finite energy of $\frac{1}{2}h\nu$. Thus, Planck's oscillator was non-classical in two different ways. It had not only discrete energies but would not be at rest even in the lowest energy state. In other words, it would have a finite energy even at absolute zero of temperature. In later literature this was referred to as the Zero Point Energy of a quantum oscillator. As a consequence of this, the black-body radiation spectrum is described by

$$u(\lambda, T) = \frac{(8\pi hc)\left[1/\left\{\exp\left(\frac{hc}{k\lambda T}\right) - 1\right\} + \frac{1}{2}\right]}{\lambda^5}.$$

Planck himself called this the Second Law of Radiation.

It is not out of context to point out here that probably the first experimental evidence for the existence of the Zero Point Energy came in 1924 from a work of the famous American chemist Robert Mulliken. He was studying the spectra of Boron Monoxides (BO), with two different Boron isotopes, in one case with mass 10, in another mass 11. (As a consequence, these molecules will have slightly different reduced masses). By comparing the vibrational and the rotational spectra of these molecules, he discovered that the observed spectra had a better fit with the theory, if the energy levels were assumed to start at $(h\nu)/2$ rather than 0.

Quantized Oscillator

Quantization of the energy of an oscillator is central to Planck's theory of black-body radiation. There is an impression that the justification for this comes only from formal quantum mechanics where the Schrödinger Wave Equation is solved for an oscillator. The method employed to solve the Schrödinger equation is mathematically quite complex. It is not generally appreciated that the essential result i.e., $\varepsilon = nh\nu$ can be arrived at in a much simpler way by exploiting the Matter Wave Concept as enunciated by de Broglie. In this description a particle of mass m moving with a velocity v behaves like a wave of wavelength

$$\Lambda = \frac{h}{(mv)}.$$

As the particle travels a distance 'ds' in its path, the number of wavelengths it covers is (ds/Λ). In a periodic motion as in a motion on a circle or in a harmonic oscillator, the particle comes back to the same state once in a cycle. In the wave description this means that in that cycle it must have traversed an integral number of wavelengths. In other words, over a cycle

$$\int \left(\frac{\mathrm{d}s}{\Lambda}\right) = \int \left(\frac{mv}{h}\right) \mathrm{d}s = n \text{ (an integer)}.$$

When the particle is moving on a circle with a uniform velocity v, the integration is easy and leads to the result $mvr = nh/2\pi$, the well-known Bohr quantum condition which is at the heart of his theory of atoms. We describe a harmonic oscillator in terms of its displacement X given by

$$X = X_0 \sin\,(2\pi\nu t).$$

The amplitude of its oscillations is X_0 and its natural frequency is $\nu = (\alpha/m)^{1/2}$ with α as the spring constant. Now the energy of a harmonic oscillator is

$$\varepsilon = \frac{mv^2}{2} + \frac{\alpha X^2}{2}.$$

As already said over one period we must have an integral number of wavelengths. i.e.

$$\int \left(\frac{mv}{h}\right) \mathrm{d}X = n.$$

Integrating over a cycle, we get the required result,

$$\varepsilon = nh\nu.$$

Epilogue

Planck's unorthodox solution to the problem of the black-body spectrum heralded the era of Modern Physics. The study of black-body radiation by later investigators became extremely fruitful and important. We list some of them here:

1. Einstein analysed the low wavelength end of the black-body spectrum in the Wien approximation and suggested that 'Light' behaved like particles with each particle, later christened as Photons, having an energy of $h\nu$. This led to his Nobel Prize winning work on the photoelectric effect.

2. Einstein studied carefully the thermal equilibrium between radiation and atoms in a cavity and concluded that radiation not only behaved like particles of energy $h\nu$ but each particle also had a momentum of $h\nu/c$. Further, he showed that the atoms absorbed or emitted radiation of energy $h\nu = |(E_2 - E_1)|$ in a transition of an atom from state of energy E_1 to another of energy E_2 thus justifying Bohr's assumption concerning emission and absorption of radiation. His later study of the same system showed that there can be both spontaneous as well as radiation-induced transition of an atom from a higher energy state to a lower energy state. This is at the heart of Laser action.

3. Indian physicist Bose showed that Einstein's photon picture of radiation would lead to Planck's radiation formula if the photons are assumed to be not only indistinguishable but also to obey a statistical distribution totally different from the well-known Maxwell–Boltzmann distribution. This led to the development of Quantum Statistics.

Suggested Reading

[1] J K Roberts and A R Miller, *Heat and Thermodynamics*, Blackie and Sons Ltd., 1966.

[2] J Stefan, *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften, Mathematische Naturwissenschaftliche Classe Abteilung*, Vol.279, p.391, 1879.

[3] L Boltzmann, *Annalen der Physik und Chemi.*, Vol.22, p.31, 291, 1884.

[4] W Wien, *Ann. Physik.*, Vol.58, p.622, 1896.

[5] J W S Rayleigh, *Phil. Mag.*, Vol.49, p.539, 1900.

[6] J H Jeans, *Phil. Mag.*, Vol.10, p.91, 1905.

[7] M Planck, *Berlin Academy of Sciences*, Vol.X, p.19, 1900.

[8] M Planck, *Berlin Academy of Sciences*, Vol.XII, p.14, 1900.

[9] M Planck, *Annalen der Physik.*, Vol.l4, p.553, 1901.

[10] M Planck, *Annalen der Physik.*, Vol.37, p.642, 1912.

4. The cosmic microwave background radiation discovered by Wilson and Penzias has a black-body spectrum with a peak radiation around a wavelength of about 1 mm and corresponds to a temperature of about 3 K. This strongly supported the Big Bang Theory according to which this radiation is a remnant of the Big Bang. They got the 1978 Nobel Prize in Physics, for this work.

Address for Correspondence
G S Ranganath
422, 10th Cross, 8th Main
Padmanabhanagar
Bangalaore 560 070, India.
Email:
gs.ranganath@gmail.com

Max Planck in Memoriam

A man to whom it has been given to bless the world with a great creative idea has no need for the praise of posterity. His very achievement has already conferred a higher boon upon him.

Yet it is good – indeed, it is indispensable – that representatives of all who strive for truth and knowledge should be gathered here today from the four corners of the globe. They are here to bear witness that even in these times of ours, when political passion and brute force hang like swords over the anguished and fearful heads of men, the standard of our ideal search for truth is being held aloft undimmed. This ideal, a bond forever uniting scientists of all times and in all places, was embodied with rare completeness in Max Planck.

Even the Greeks had already conceived the atomistic nature of matter and the concept was raised to a high degree of probability by the scientists of the nineteenth century. But it was Planck's law of radiation that yielded the first exact determination – independent of other assumptions – of the absolute magnitudes of atoms. More than that, he showed convincingly that in addition to the atomistic structure of matter there is a kind of atomistic structure to energy, governed by the universal constant h, which was introduced by Planck.

This discovery became the basis of all twentieth-century research in physics and has almost entirely conditioned its development ever since. Without this discovery it would not have been possible to establish a workable theory of molecules and atoms and the energy processes that govern their transformations. Moreover, it has shattered the whole framework of classical mechanics and electrodynamics and set science a fresh task: that of finding a new conceptual basis for all physics. Despite remarkable partial gains, the problem is still far from a satisfactory solution.

In paying homage to this man the American National Academy of Sciences expresses its hope that free research, for the sake of pure knowledge, may remain unhampered and unimpaired.

– Albert Einstein

Statement read at the Memorial Services for Max Planck, April, 1948.