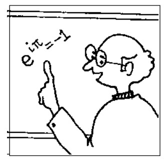
Classroom



In this section of *Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

Experimental Measurement of Viscosity and Flow Velocity

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An experiment involving the measurement of viscosity using Poiseuille's equation was performed in a workshop at Presidency College, Calcutta. The results obtained are presented along with a brief theoretical discussion.

Experimental Procedure

Two exactly similar large glass vessels with graded scale of volume in cc on their outer surface were taken. Water, chosen as the most suitable liquid, was poured in one of the two vessels while the other was kept empty. The vessel with water was kept on a small box of vertical height 'h_1', while the empty vessel was kept simply on the table. Then a U-tube, sharp at its bends and made of glass and also with suitably chosen cross-section so as to be noncapillary with respect to water was taken. It was kept overturned to connect both the vessel (*Figure* 1). Water was made to flow from vessel with water to the empty vessel through the U-tube by means of siphon action. As soon as the flow starts, the digital stop-watch was switched on. The volume of liquid 'S' was being continuously measured directly from the graded scale on the

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Poiseuille's equation, measurement of viscosity, laminar flow, Reynold's number.

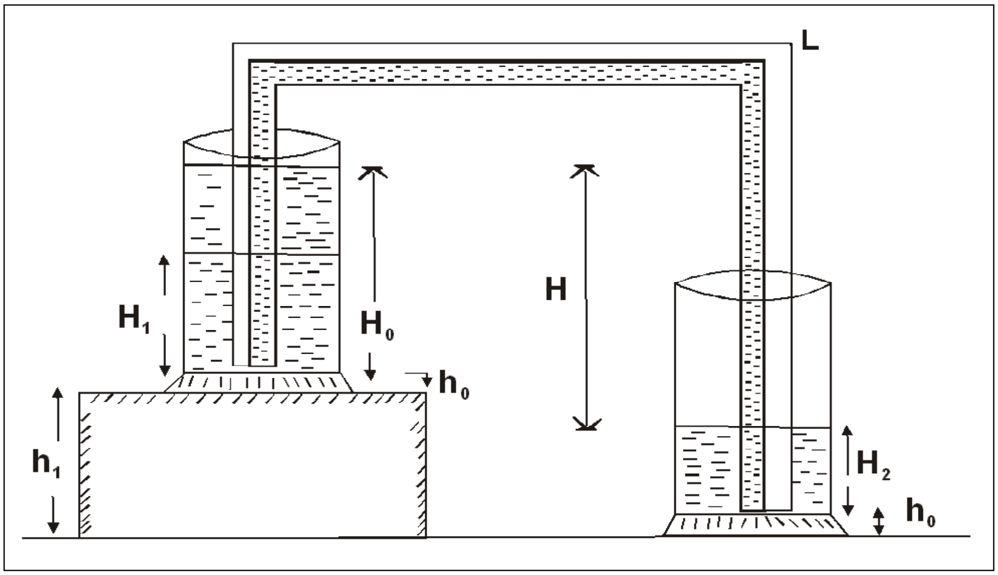

Figure 1. Schematic diagram of the experimental set-up along with labeling of different level-height.

wall of the glass-vessel with respect to time taken using a digital stop watch. From 'S' values the corresponding H_1 values were found by dividing each 'S' value with the cross-section area (πR^2) of vessel.

Data as Recorded in the Specific Form of the Experiment

It can be seen from *Figure* 1 that

$$H = (h_1 + h_0 + H_1) - (h_0 + H_2) = h_1 + H_1 - H_2$$

But, for incompressible fluid (i.e. liquid)

$$H_1 A + H_2 A = H_0 A (\text{vol. of liquid conserved})$$

$$\text{or } H_1 + H_2 = H_0 \text{ or } H_2 = H_0 - H_1$$

Hence, $H = h_1 + H_1 - H_2 = h_1 + H_1 - H_0 + H_1 = 2H_1 + h_1 - H_0$

For our experiment, $h_1 = 0.17\text{m}$, $H_0 = 0.2\text{m}$

$\rho = \rho_w \approx 10^3$ kg/m^3 (approx.), $g = 9.8\text{m/sec}^2$,

$L = 0.945\text{m}$, $R = 0.034\text{m}$

t (time in secs.)	S (vol in m^3) (Graded scale reading)	H_1(m) (=s/35) (cross sectional area =0.0035 sq. m)	H (m) ($2H_1 - 0.3$)	1n H
0	700×10^{-6}	0.200	0.370	–0.99
18	650×10^{-6}	0.186	0.342	–1.07
31.56	600×10^{-6}	0.171	0.312	–1.16
47.75	550×10^{-6}	0.157	0.284	–1.25
62	500×10^{-6}	0.143	0.256	–1.36
81.5	450×10^{-6}	0.129	0.228	–1.43
98	400×10^{-6}	0.114	0.198	–1.61
121	350×10^{-6}	0.100	0.170	–1.77
145	300×10^{-6}	0.086	0.142	–1.95
173	250×10^{-6}	0.071	0.112	–2.18
210	200×10^{-6}	0.057	0.084	–2.47
259	150×10^{-6}	0.043	0.056	–2.88

Table 1. Data as recorded in the experiment.

Considering the tube to be of right circular cylindrical shape the radius 'a' of the tube is found out to be

$$a = \left(\frac{V}{\pi L}\right)^{1/2}$$

$$= \left(\frac{\text{volume of liquid contained within the tube}}{\pi \text{ X total length of the tube}}\right)^{1/2}$$

$$= \left(\frac{5.5 \times 10^{-6} m^3}{\pi \times 0.945 m}\right) = 1.36 \times 10^{-3} m$$

So, $H = 2H_1 + 0.17 - 0.20 = 2H_1 - -0.03$

Now, 'H' was observed and recorded as a function of time (*Table* 1). (*Figure* 2) and an empirical equation corresponding to that graph using least square method of curve fitting was found out to be the following:

$$lnH = -7.0 \times 10^{-3} t - 0.96$$

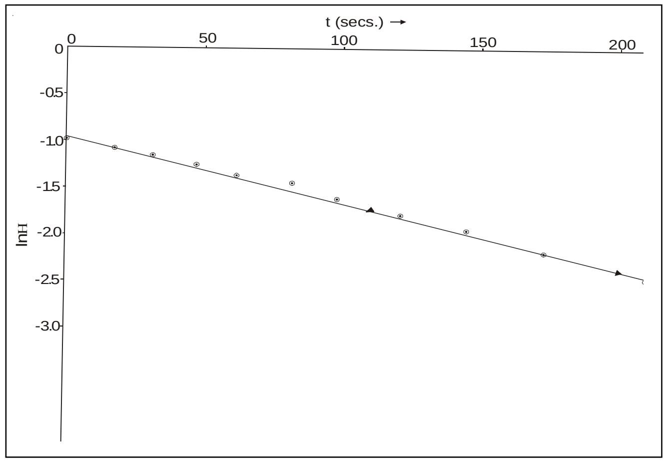

Figure 2. Graphical plot of lnH versus time, as obtained from experiment.

The slope is directly found out from that equation as follows:

$$\text{Slope} = \left(\frac{d(lnH)}{dt}\right) = 7.0 \times 10^{-3} \text{ per sec.}$$

Table 2 gives us the average percentage deviation in the calculation of the slope and that is found out to be -1.03%. This negligibly small deviation confirms the validity of considering the above value of the slope.

Table 2. Percentage deviation of values of observed data with respect to least square fit data (Least square – Fit equation (empirical) ln H = –7 × 10^{-3} t –0.96).

t (time in secs.)	ln H (theo.)	1n H (obsd.)	Percentage deviation	Average percentage deviation
0	– 0.96	– 0.99	3.0%	
18	– 1.09	– 1.07	– 1.9%	
31.56	– 1.18	– 1.16	– 1.7%	
47.75	– 1.30	– 1.25	– 4.0%	
62	– 1.39	– 1.36	– 2.2%	
81.5	– 1.50	– 1.43	– 4.1%	– 1.03%
98	– 1.65	– 1.61	– 2.5%	
121	– 1.81	– 1.77	– 2.2%	
145	– 1.98	– 1.95	– 1.5%	
173	– 2.17	– 2.18	0.4%	
210	– 2.43	– 2.47	1.6%	
259	– 2.78	– 2.88	3.5%	

The coefficient of viscosity is [1] given by

$$\eta = \frac{a^4 \rho g}{4R^2 L} \times \frac{1}{\left(\frac{d(lnH)}{dt}\right)} \qquad (1)$$

in case of a streamline laminar siphon flow of water.

Here,

$$lnH = lnH_0 + \left(\frac{-a^4 \rho g}{4\eta L R^2}\right) t$$

While deriving this equation, losses in the bend and exit losses have been neglected.

Interpretation of Symbols Used in This Srticle are Given Below

a - the radius of the tube, ρ - the density of liquid

g - acceleration due to gravity, η - coefficient of viscosity

L - the total length of the tube,

R - the radius of cylindrical vessels (both the vessels have same geometry),

t - time

H - the height difference of the exposed liquid levels in two vessels at any time 't', $H_0 = H(t = 0)$

Putting all the numerical values of variable related to (1), we get,

$$\eta = \frac{(1.36 \times 10^{-3})^4 \times 10^3 \times 9.8}{4 \times (0.034)^2 \times 0.945} \times \frac{1}{7 \times 10^{-3}}$$

$$= \frac{7.7 \times 10^{-6}}{7.0 \times 10^{-3}} = 0.0011 Ns/m^2$$

Limit of error, in the calculation of η is found out from *Table* 3 to be equal to + 6.4%.

t (secs.)	1n H (obsd.)	Slope m (SI unit)	Average value of m	Value of η Ns/m^2	Least square fit value of m	Value of η (Ns/m^2)
0	– 0.99	–				
18	– 1.07	-6.1×10^{-3}				
31.56	– 1.16	-6.3×10^{-3}				
47.75	– 1.25	-6.1×10^{-3}				
62	– 1.36	-6.4×10^{-3}				
81.5	– 1.43	-5.8×10^{-3}	-6.6×10^{-3}	1.17×10^{-3}	-7×10^{-3}	1.1×10^{-3}
98	– 1.61	-6.6×10^{-3}				
121	– 1.77	-6.7×10^{-3}				
145	– 1.95	-6.8×10^{-3}				
173	– 2.18	-7.0×10^{-3}				
210	– 2.47	-7.2×10^{-3}				
259	– 2.88	-7.4×10^{-3}				

Table 3. Calculation of limit of error in the calculation of η semi-empirical equation, ln H= mt – 0.96. Limit of error in the calculation of η is found to be approximately + 6.4%.

Velocity of Emergence of Liquid Through Tube as a Function of Time

The time rate of volume of liquid discharged is given by

$$\frac{dV}{dt} = \frac{\pi a^4}{8\eta L}(\rho g H) = -\frac{-\pi R^2}{2}\left\{\frac{dH}{dt}\right\} \tag{2}$$

For liquid being incompressible

$$\frac{\pi R^2}{2}\left\{\frac{dH}{dt}\right\} = \left|\frac{dV}{dt}\right| = \pi a^2 v,$$

where v is the mean velocity of liquid through tube at time 't'.

From equation (1) we can write

$$H = H_0 \exp\left(-\left\{\frac{a^4 \rho g}{4\pi\eta R^2 L}\right\} t\right) \tag{3}$$

Combining equations (2) and (3), one can write

$$\pi a^2 v = \frac{\pi R^2}{2}\left\{\frac{dH}{dt}\right\}$$

$$= \frac{a^4}{8\eta L}\left(\rho g H_0 \exp\left\{-\frac{a^4\rho g}{4\pi\eta R^2 L}\right\}t\right)$$

$$\text{or} \quad v = \left(\frac{a^2\rho g H_0}{8\pi\eta L}\right)\exp\left(\left\{-\frac{a^4\rho g}{4\pi\eta L R^2}\right\}t\right). \qquad (4)$$

The Reynold's Number (Re) is given by

$$R_2 = \frac{v\rho a}{\eta}$$

It is generally said that if $R_e < 2000$ the flow is laminar and if $R_e > 3000$, the flow is turbulent.

Here,

$$\begin{aligned} R_{e\ \max} &= \frac{v_{\max}\rho a}{\eta} = \frac{v_0\rho a}{\eta} \\ &= \frac{\rho^2 a^3 g H_0}{4\eta^2 L} = \frac{(1.36\times10^{-3})^3\times(10^3)^2\times9.8\times0}{4\times(.0011)^2\times0.945)} \\ &= 1994\text{(approx.)} \end{aligned}$$

Within the specific limit of error in the calculation of η from *Table* 3 the value ($R_{e\ \max}$) is found to lie between 1793 and 1994.

Hence $R_{e\ \max} < 2000$ and the flow, we worked with, is laminar.

Moreover, critical velocity $v_c = R_{e\ \max}\times\frac{\eta}{\rho a}$ $= 2000\times\frac{0.0011}{10^3\times1.36\times10^{-3}} = 1.48$ m/sec(Approx)

Therefore $v_{\max} = v = 1994 = \left(\frac{0.0011}{10^3\times1.36\times10^{-3}}\right) =$ 1.468 m/sec (Approx.)

Therefore $v_{\max} < v_c$.

Hence the flow remained laminar in our experiment.