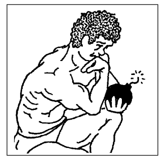
Think It Over



This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

Solution to 'A Problem in Graph Theory'

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A conference of '*n*' leaders is proposed to be held (where '*n*' is an odd number). Suppose that the organisers decide to conduct it in such a way that the leaders will have their dinner together at a circular table and each leader will have different neighbours on each day. Then

1) For how many days will the leaders have to meet to have all combinations and arrangements?
2) How can the leaders be arranged in their position each day?

Think-it-Over question appeared in *Resonance*, Vol.12, No.1, p.81, 2007.

One way of solving the problem is by using Hamilton cycle. A Hamilton cycle in a graph G is a cycle which contains every vertex of G.

Here the leaders are represented by vertices and two vertices are joined by an edge if the leaders represented by those vertices are neighbours in a particular arrangement. We are searching for Hamilton cycles in the complete graph on n vertices such that each edge in the complete graph appears in one and only one Hamilton cycle.

Since there are $\frac{n(n-1)}{2}$ edges in the complete graph on n vertices, each Hamilton cycle has n edges and each edge appears exactly in one Hamilton cycle, the number of possible Hamilton cycles is equal to $\frac{n(n-1)}{2} \div n = \frac{n-1}{2}$.

Keywords
Hamilton cycle.

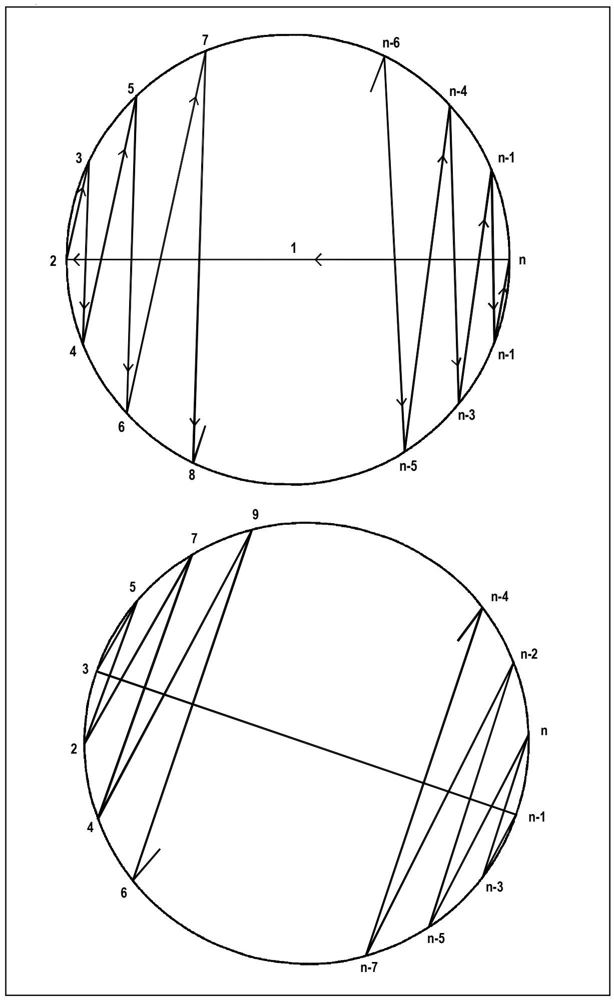

***Figure 1 (top). The arrangement is* 1–2–3–4–5–6–*...*–(*n*–3)–(*n*–2)–(*n*–1)–*n*–1.**
***Figure 2 (bottom). The arrangement is* 1–3–5–2–7–4–...–(*n*–2)–(*n*–5)–*n*–(*n*–3)–(*n*–1)–1.**

That is, the leaders will have to meet on $\frac{n-1}{2}$ days at least.

That the number $\frac{n-1}{2}$ suffices is demonstrated as follows:

Consider the initial arrangement shown in (*Figure* 1). Fix the circumference of the disc and rotate the cycle through an angle $\frac{2\pi}{n-1}$. Then we obtain the second arrangement (*Figure* 2). This can be repeated by rotating the cycle through the same angle. Each time we obtain a new arrangement, so that the total number of arrangements is equal to $\frac{n-1}{2}$ which is also the required number of days.