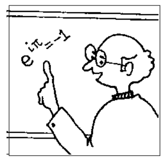

In this section of *Resonance,* we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

A Note on Sum-Product Numbers

Soubhik Chakraborty
Lecturer in Statistics
Marwari College
TM Bhagalpur University
Bhagalpur 812 007, India.
Email:soubhik@yahoo.co.in

This note defines a sum-product number and shows why there are only three of them.

Introduction

A positive integer is called as a *sum-product number* if and only if the sum of its digits multiplied by the product of its digits equals the number itself. Example:144 = (1 + 4 + 4) (1.4.4).

Claim: There are only three sum-product numbers, namely, 1, 135 and 144.

Proof: We are giving only a sketch of the proof due to D Wilson. Assume n as an r-digited sum-product number with s and p as the sum and product of its digits respectively. Then we must have

$$10^{r-1} \leq n;\ s \leq\ 9r;\ p \leq\ 9^r.$$

Also, we must have $n = s\,p$ by the assumption that n is a sum-product number. Combining these results, we get

$$10^{r-1} \leq\ n = sp \leq\ (9r)(9^r).$$

But the inequality $10^{r-1} \leq (9r)(9^r)$ holds provided only $r \leq 84$ so that

Keywords
Sum-product number, divisibility.

n has at most 84 digits. (Hint: it pays to rewrite the inequality as $(10/9)^r \leq 90r$).

This gives $s \leq 9r \leq (9)(84)=756$; $p \leq n < 10^{85}$.

Now p being a product of digits must be of the form $2^a3^b5^c7^d$. However from our knowledge of divisibility by 10, p cannot be divisible by 10 as otherwise 10 would also divide $n(=sp)$ in which case n would end in 0 whereby the product of its digits $p = 0$ which is ruled out. Thus p can be either of the form $2^a3^b7^c$ or $3^a5^b7^c$. This certainly reduced the search space for sum-product numbers and allowed Wilson to verify the needful (for details, see [1]).

Suggested Reading

[1] **Eric W Weisstein, *Sum-Product Number*, From Mathworld (http://www.mathworld.wolfram.com/Sum-Product Number.html)**

Errata

***Resonance*, Vol.10, No.12, pp.67–70, 2007.**

Classroom:
Root Test and Ratio Test in the Context of (C, k) Summability of Series by P N Natarajan

$k \geq 0$ should read as $k > 0$ in the following lines :

Page 67: line 5 from top;
Page 68: line 5 from bottom;
Page 69: line 2 & line 3 from top;
Page 70: line 8 from top & line 6 from bottom

$k > -1$ should read as $0 > k > -1$ in the following lines:

Page 69: line 7 from bottom;
Page 70: line 3 from bottom. .