Controlling Uncertain Dynamical Systems

Basic Ideas of Adaptive Control

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This article is meant to be a simple introduction to the science of adaptive control. Adaptive control is commonly used in airplanes and chemical plants, besides other engineering systems.

1. Some Basic Notions

Dynamical systems are ubiquitous in nature. Classic examples of dynamical systems are an oscillating simple pendulum or a spring-mass-damper system. Mathematically, these systems can be modeled in the following form:

$$m\ddot{x} + c\dot{x} + kx = f(x, \dot{x}, t), \tag{1}$$

where x represents the angular displacement of the pendulum, or the translational displacement of the mass in the spring-mass-damper system, m is the mass/inertia, c is the damping coefficient, and k is the stiffness coefficient; f in this model is an external force/torque acting on the system. Many other practical systems can also be modeled in this form. For example, the dynamic response of an airplane after its nose is slightly displaced by a gust of wind can be modeled in precisely this manner; this motion is called the *short period mode* of oscillation of the airplane. The sloshing motion of the free surface of a liquid in a partially-filled container is often modeled as described above; these models are popularly called *pendulum models* for slosh.

For convenience, we choose $m = 1$ in the following discussion – there is no loss of generality due to this choice as we could have equally well divided every term in (1) by m, and defined c/m, k/m, and f/m as the new damping coefficient, new stiffness coefficient, and new external force/torque (acceleration, actually) respectively.

The unforced ($f = 0$) dynamics of the system in (1) are well known. Given a perturbation from its equilibrium state ($x = 0, \dot{x} = 0$), the system response depends on its eigenvalues. The eigenvalues λ_1, λ_2 of this system are easily obtained to be

$$\lambda_{1,2} = -\frac{c}{2}\left(1 \pm \sqrt{1 - \frac{4k}{c^2}}\right). \qquad (2)$$

The system response is therefore damped or undamped, oscillatory or monotonic, depending on the sign of c and the sign of $(1 - 4k/c^2)$. Often, the system response is unsatisfactory – perhaps the damping is too low or even negative, or the frequency is undesirably close to resonance with a harmonic of another component. Sometimes, a damped, oscillatory response is desired, but the system behaves in a monotonic manner. In principle, these shortcomings can be overcome by appropriately choosing the c and k parameters, but many a times it is physically impossible to design a system with desired values of c, k due to geometric, structural, or other design limitations. One must then take recourse to automatic feedback control.

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A feedback controller, in its simplest form, consists of three components:

1. A sensor that measures (senses) the system states (in this case, x and $\dot{x}$);
2. A control law that operates on these measurements to arrive at a value of force/torque, $f(x, \dot{x})$, that needs to be applied to the system; and
3. An actuator that applies the required force/torque on the system.

To keep matters simple, we assume that the system states can indeed be measured by use of suitable sensors, and that the sensors measure these states accurately, introducing no errors in the measurement. We also assume that the required force/torque, as calculated by the control law, can actually be realized by the actuator with no error. In practice, of course, a good model will consider measurement noise, actuator dynamics, actuator limits such as saturation and rate limits, controller time delays, and so on, but these complications may easily be avoided in an introductory discussion.

A typical control law for this problem is linear, not an explicit function of time, and of the following form:

$$f(x, \dot{x}) = c_f \dot{x} + k_f x, \tag{3}$$

where c_f, k_f are constant parameters called *controller gain parameters*, or just *gains*, for short. A block diagram of the dynamical system with the feedback controller is sketched in *Figure* 1. It is seen that the controller uses the available values of $x, \dot{x}$ at any instant and computes f as per the mathematical law in (3), which it *feeds back* to the dynamical system.

The equation for the dynamics of the controlled (closed-loop) system is written by merging (1) and (3), as follows

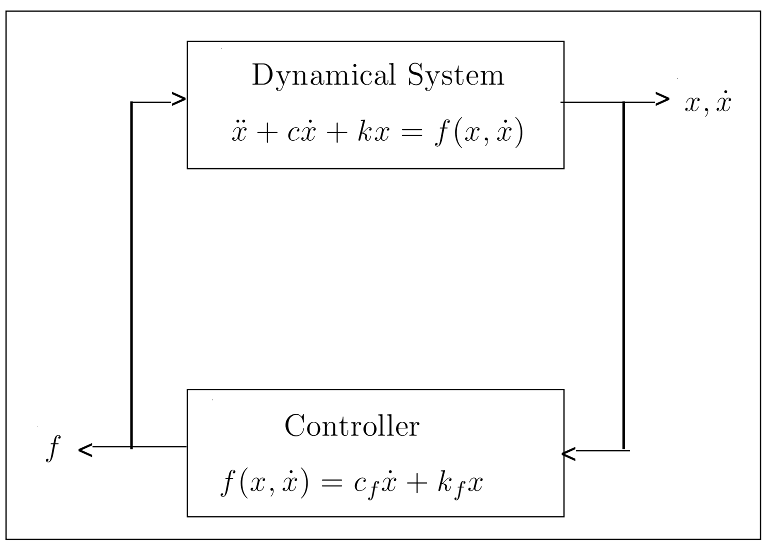

Figure 1. Block diagram of dynamical system with feedback controller.

(remember that m has been set to 1):

$$\ddot{x} + (c - c_f)\dot{x} + (k - k_f)x = 0, \qquad (4)$$

where $c - c_f = c_c$ and $k - k_f = k_c$; c_c, k_c being the desired values of the damping and stiffness coefficients for the dynamical system under study. Knowing c_c, k_c, and the values of damping and stiffness coefficients for the uncontrolled (open-loop) dynamical system c, k, it is a simple matter to choose the correct values of controller gains c_f, k_f, in order to obtain the desired closed-loop dynamics.

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Controllers of this kind are very popular and find wide application in a whole range of engineering systems. However, engineering systems – even relatively simpler ones – are reasonably complex. They cannot always be definitively described by a single dynamical equation such as (1) with fixed damping and stiffness coefficients, c and k. For example, in case of airplanes, the coefficients c, k vary with speed, altitude (height) of flight, amount of fuel consumed or bombs dropped, etc. For many systems, it is just not possible to estimate c, k accurately in the first place, and there is always a window of uncertainty about their values. With aging, wear and tear, misalignments, and such factors creeping in with constant use, the values of c, k may slowly drift from their original values with time. It is a challenge to design controllers for systems with varying, uncertain, or unknown values of coefficients.

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Fixed-gain controllers (controllers designed with constant values of c_f, k_f, assuming some nominal value of system coefficients c, k) will not perform satisfactorily when the values of c, k change from the assumed nominal values. One solution to this problem is to have somewhat 'intelligent' controllers that can anticipate or estimate the changes in system coefficients c, k and alter their gains c_f, k_f suitably such that the closed-loop system dynamics is always as desired (as specified by

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c_c, k_c). Such controllers that can 'adapt' themselves to changes in the system while in operation are called *adaptive controllers*. Specifically, adaptive controllers use control algorithms that allow the controller to remain effective in varying process conditions (system coefficients). This article discusses two of the most popular adaptive control schemes: gain scheduling, and model reference adaptive control.

2. Gain Scheduling

Gain scheduling is widely used in the control industry and is very effective when the system coefficients are known *a priori* to vary as a function of a slowly-varying parameter, say α. The dynamical system can then be represented as

$$\ddot{x} + c(\alpha)\dot{x} + k(\alpha)x = f(x, \dot{x}, t). \qquad (5)$$

In the case of an aircraft, for instance, α could be its speed or flight altitude. The challenge is, no matter what value of α the system is operating at, for the operator (e.g., the pilot, in the case of an airplane) to always see the closed-loop system with a fixed value of damping and stiffness, c_c, k_c.

2.1 *Case 1*

For some systems, it might be known that at times $t_1, t_2, \ldots, t_n$, the slowly-varying parameter α takes on values $\alpha_1, \alpha_2, \ldots, \alpha_n$, respectively. Then, the dynamical system of (5) can be taken as a collection of n dynamical systems with constant parameters c, k; one each for each of the time instants, $t_1, t_2, \ldots, t_n$. One can then design a collection of fixed-gain controllers,

$$f(x, \dot{x}; t_i) = c_f(t_i)\dot{x} + k_f(t_i)x \qquad (6)$$

as discussed previously, each with gains $c_f(t_i), k_f(t_i)$ changing as a function of time, such that the closed-loop, controlled dynamical system has the same coefficients,

c_c, k_c, at all times, $t_1, t_2, \ldots, t_n$. At intermediate instants of time, the controller gains are not pre-calculated, but are simply obtained in real time by *linear interpolation* between the gains at the nearest two time instants. This may result in an error in the sense that the coefficients of the closed-loop dynamical system at intermediate time instants may not exactly be equal to the desired values, c_c, k_c. However, this error can be limited by making the interval between successive time instants small enough by choosing a suitably large number of times $t_1, t_2, \ldots, t_n$, so that the error is held within reasonable bounds.

At intermediate instants of time, the controller gains are not pre-calculated, but are simply obtained in real time by *linear interpolation* between the gains at the nearest two time instants.

2.2 *Case 2*

For many systems, however, the variation of α with time cannot be predicted in advance, i.e., α cannot *a priori* be indexed to definite instants of time, $t_1, t_2, \ldots, t_n$. Here, α is used as the parameter rather than time t. The range of all possible values of α is first determined, say $[\alpha_{min}, \alpha_{max}]$. Various discrete values of α within this range are then chosen as design points for the controller. Let these values be labeled $\alpha_1, \alpha_2, \ldots, \alpha_l$. These design points need not be equispaced in α, but could be more crowded in regions where the system is expected to operate more often. The dynamical system, equation (5), can be considered to be a collection of l dynamical systems, each with constant c, k, as before, and fixed-gain controllers,

$$f(x, \dot{x}; \alpha_i) = c_f(\alpha_i)\dot{x} + k_f(\alpha_i)x \qquad (7)$$

with gains $c_f(\alpha_i), k_f(\alpha_i)$ designed for each of them such that the closed-loop system at each design point, $\alpha_1, \alpha_2, \ldots, \alpha_l$, is identical (with coefficients c_c, k_c). Once again, linear interpolation can be used to obtain controller gains at intermediate α values, with the same comments on accuracy as before.

A block diagram of the dynamical system with the feed-

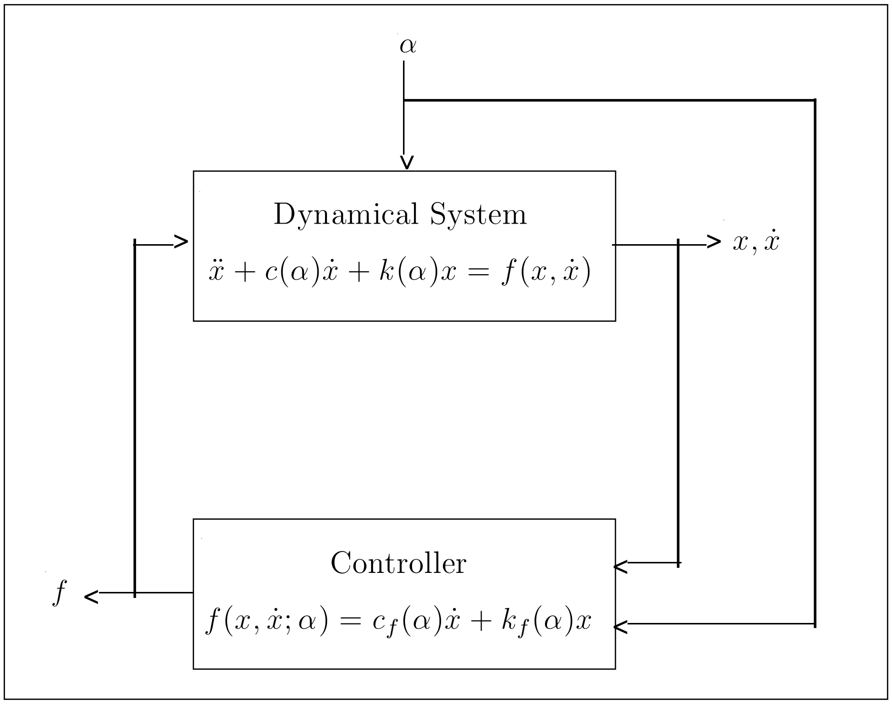


Figure 2. Block diagram of dynamical system with gain scheduled feedback controller, incorporating feedback of slowly-varying parameter α.

back controller for this case is sketched in *Figure* 2. The main difference, as compared to the previous case, is that the dynamical system is a function of the slowly-varying parameter α, and the value of this parameter also needs to be sensed and fed into the controller. The controller will use this value to evaluate the gain parameters c_f, k_f from the set of pre-calculated gains for different fixed values of α. The controller gains are said to be 'scheduled' as a function of the parameter α, and this procedure is called *gain scheduling*. Surprisingly, this fairly simple method of control is essentially what is used in practice for most airplanes flying today.

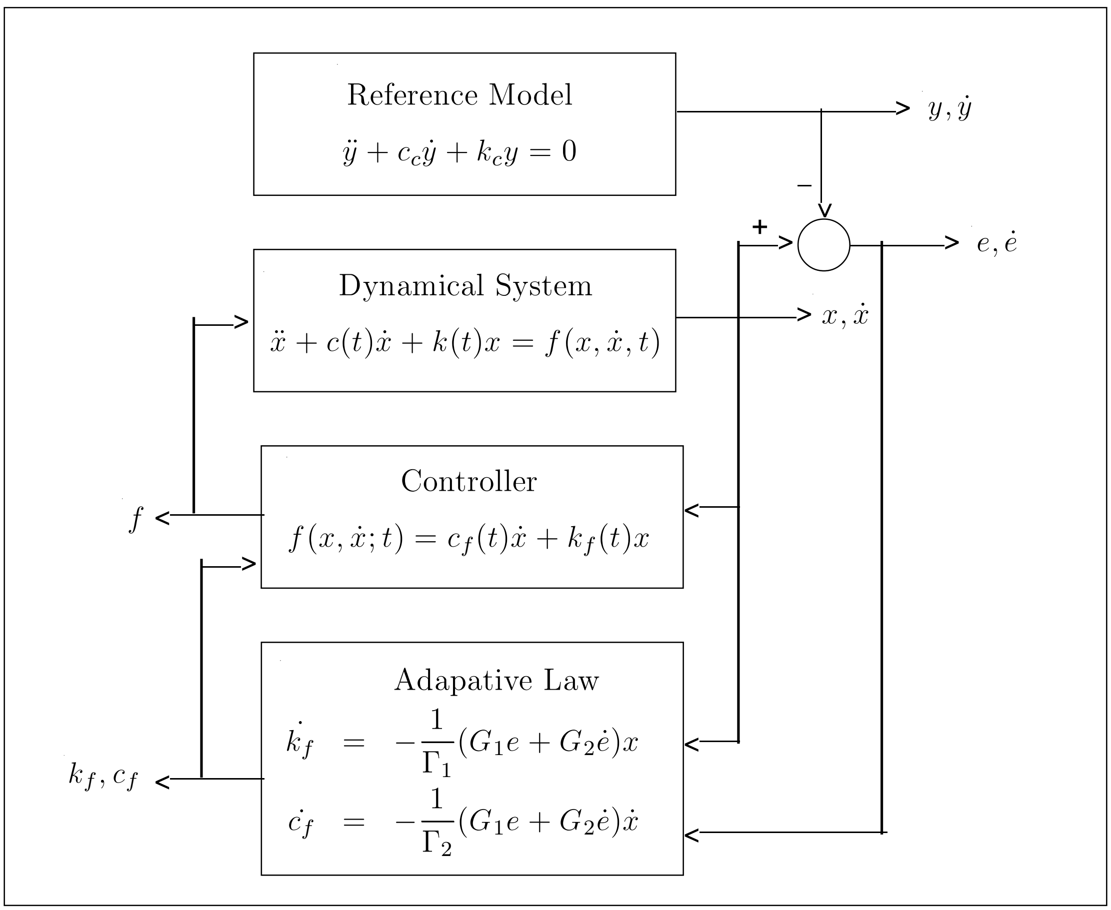
However, for typical engineering systems, e.g., aircraft, chemical plants, the process of designing a gain scheduling controller is both laborious and expensive. Also, gain scheduled controllers are not really 'intelligent' in the sense that they do not have a mechanism to adjust the gain parameters c_f, k_f in real time if the system coefficients happen to be different from the assumed values

c, k in the design process. In the next section, we discuss another method called the *model reference adaptive scheme*, which is capable of 'tuning' the controller gain parameters in real time to account for any uncertainty or change in system coefficients c, k. By doing so, it aims to have the controlled system always respond with the desired values of damping and stiffness, c_c, k_c, no matter what the system coefficients c, k may be.

3. Model Reference Adaptive Control

A block diagram of the model reference adaptive control scheme is shown in *Figure* 3. It consists of a dynamical system, modeled by a linear differential equation as before,

Figure 3. Block diagram of model reference adaptive control scheme.



$$\ddot{x} + c(t)\dot{x} + k(t)x = f(x, \dot{x}, t) \qquad (8)$$

except that the coefficients c, k are not assumed to be known. The system states $x, \dot{x}$ are fed back to a controller which appears to be of the same form as in (3),

$$f(x, \dot{x}; t) = c_f(t)\dot{x} + k_f(t)x \qquad (9)$$

but whose gain parameters c_f, k_f are generally unknown, because without knowing c, k, it is not possible to calculate the required values of c_f, k_f that will give a closed-loop (controlled) system with damping and stiffness, c_c, k_c, respectively. In fact, starting from arbitrary values, the controller gains c_f, k_f have to be 'tuned' as a function of time, as indicated in equation (9). For this purpose, the closed-loop system (dynamical system plus controller),

$$\ddot{x} + (c - c_f)\dot{x} + (k - k_f)x = 0 \qquad (10)$$

is run in parallel with a reference model,

$$\ddot{y} + c_c\dot{y} + k_c y = 0, \qquad (11)$$

whose coefficients c_c, k_c are the desired damping and stiffness of the closed-loop system. Ideally, $c_f(t), k_f(t)$ should evolve to particular values c_f^*, k_f^*, such that

$$\begin{aligned} c - c_f^* &= c_c \ , \\ k - k_f^* &= k_c \ . \end{aligned} \qquad (12)$$

These are called the *model matching conditions*. When these conditions are satisfied, the closed-loop dynamical system is identical to the reference model; hence, its dynamics matches with that of the reference model with the desired damping and stiffness, c_c, k_c respectively. In practice, the error between the states $x, \dot{x}$ of the dynamical system, and the states $y, \dot{y}$ of the reference model, is considered:

$$\begin{aligned} e &= x - y \\ \dot{e} &= \dot{x} - \dot{y}. \end{aligned} \qquad (13)$$

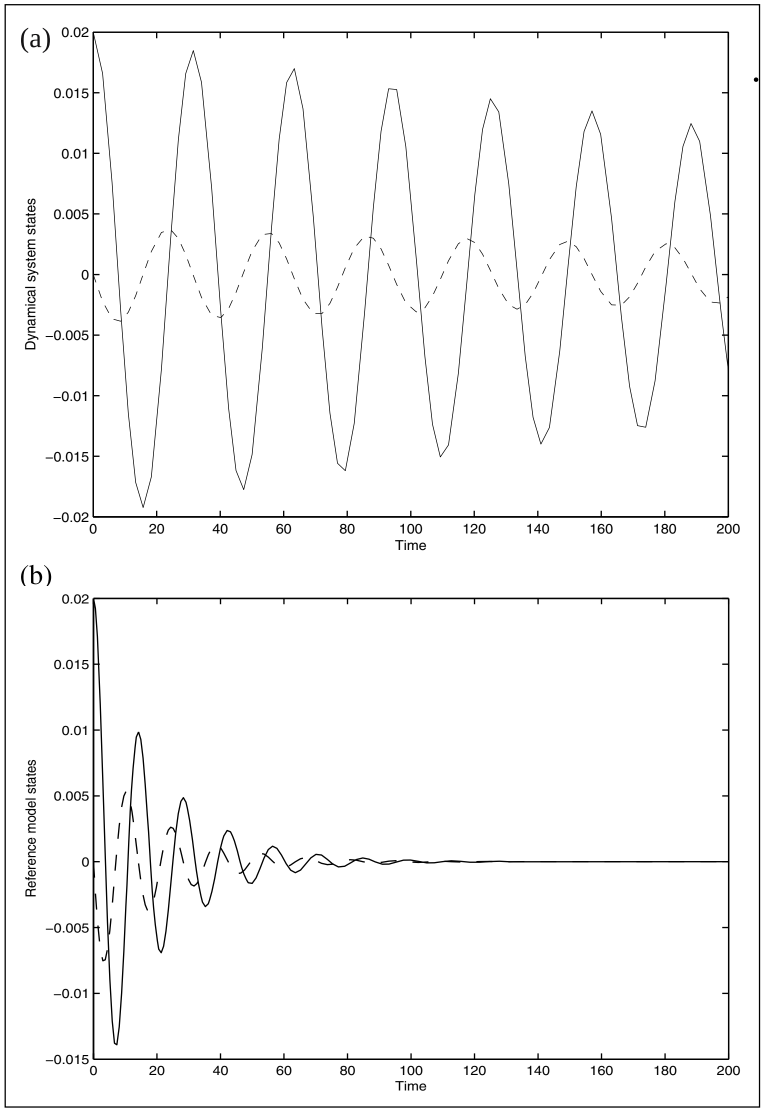
The error vector $(e, \dot{e})$, and the state vector $(x, \dot{x})$, are together used to run an adaptive law:

$$\dot{k_f} = -\frac{1}{\Gamma_1}(G_1 e + G_2 \dot{e})x \ ,$$
$$\dot{c_f} = -\frac{1}{\Gamma_2}(G_1 e + G_2 \dot{e})\dot{x} \ , \qquad (14)$$

which evolves the controller gain parameters as a function of time. The constants G_1, G_2 are chosen to ensure that the error vector $(e, \dot{e})$, tends to zero with time. This implies that the state vector $(x, \dot{x})$, *tracks* the reference model states $(y, \dot{y})$, which damp out to zero with time; hence, the dynamical system states also tend to zero with time. That is, the dynamical system is controlled, and made to behave in a manner similar to the reference model, whose dynamics has been selected to have the desired behavior. The parameters Γ_1, Γ_2 in (14) are 'weighting' constants that decide how much faster the adaptation law must run as compared to the dynamical system-reference model dynamics. These are chosen based on experience or certain rules of thumb.

The working of the model reference adaptive scheme is now illustrated with an example. *Figure* 4a shows an underdamped linear dynamical system whose states die down very slowly with time. Using an adaptive control scheme as sketched in *Figure* 3, it is possible to alter the dynamics of the system, e.g., by increasing its damping and changing its stiffness (resulting in increased frequency of oscillation), as illustrated in *Figure* 4b. This is achieved by running a reference model with the desired damping and stiffness, as shown in *Figure* 5a, and using an adaptive law of the form in (14), which drives to zero the error between the controlled dynamical system states and the reference model states, as shown in *Figure* 5b. The controller gain parameters are initially totally unknown, and are taken to be zero, and the adaptive law then evolves the controller gain parameters with time, as shown in *Figure* 6. These simulations were

Figure 4. Typical time histories of dynamical system states, x (full line) and x (dashed line), (a) in open-loop (uncontrolled) showing poor damping (c = 0.0025, k = 0.04), and (b) in closed-loop (controlled) with desired damping and higher frequency (c_c = 0.1, k_c = 0.2).



carried out using the `ode45` integration routine in MATLAB/SIMULINK.

4. Applications

The use of adaptive control has seen a resurgence in recent years, and it is being considered a promising technology for the future. A partial list of industrial applications of adaptive control, compiled by Björn Wittenmark, includes: Distillation column, chemical reactor

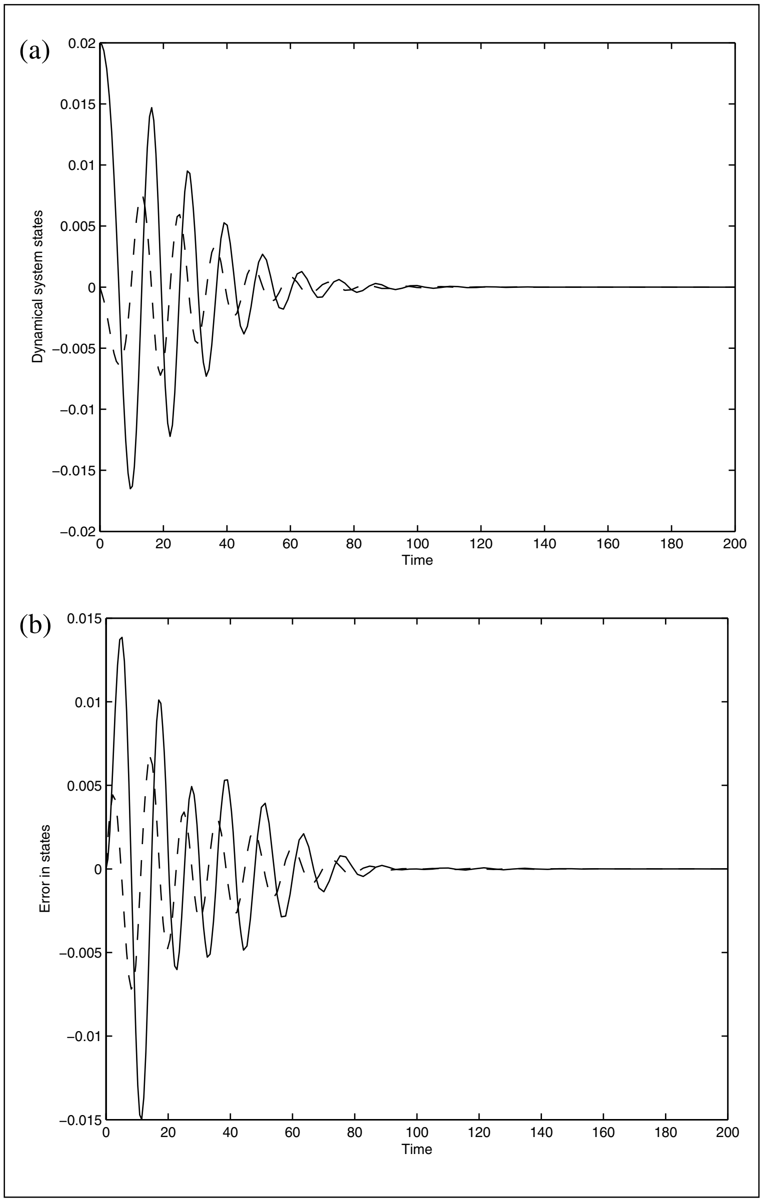


Figure 5. Typical time histories of (a) reference model (c_c = 0.1, k_c = 0.2) states, y (full line) and y (dashed line), and (b) error between closed-loop dynamical system and reference model states, e (full line) and e (dashed line).

control, pulp digester, rolling mill, engine control, ship autopilot, disk drives, solar plant, thermal power plant, heat exchanger, and materials testing machine.

Adaptive control in one form or another is employed on most modern airplanes, especially military aircraft. The F-111 is an early example of a fighter airplane that

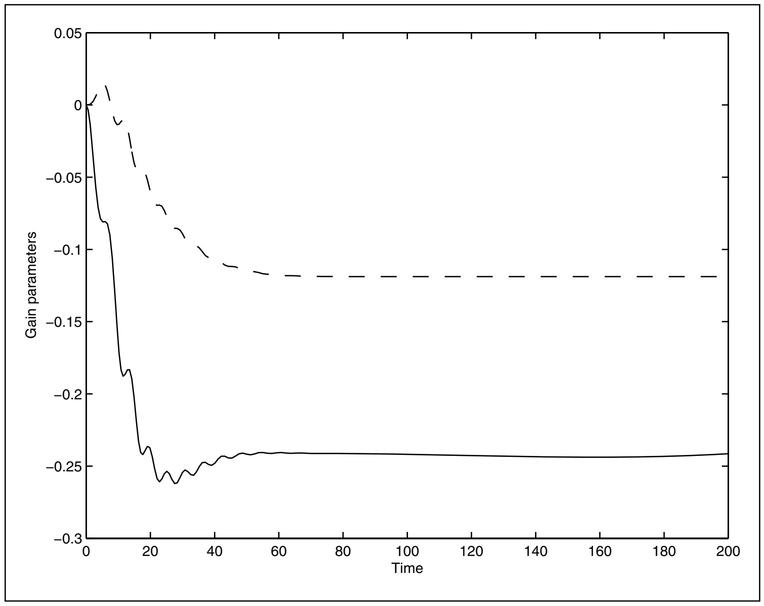


Figure 6. Typical time history showing controller gain parameter adaptation by the adaptive law, k_f (full line) and c_f (dashed line).

used an adaptive controller. The use of active control using neural networks for aircraft autopilots is being actively pursued by guidance, navigation and control engineers in the aerospace community[1]. On the ground, the benefits of adaptive control have been realized for safe operation of locomotive engines[2]. Control of combustion acoustics in rockets and manipulation of robots are some other exciting areas where adaptive control is being fruitfully employed. The books listed below give more information about the theory and applications of adaptive control techniques for those wishing to dig deeper into this subject.

[1] Refer to work by Anthony Calise and his group at Georgia Tech.
[2] Brian Anderson at the Australian National University has been a pioneer in this field..

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Suggested Reading

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[2] K S Narendra and A M Annaswamy, *Stable Adaptive Systems*, Dover Publications, 2005.