A Paradox in Radiation Heat Transfer

J Srinivasan
Centre for Atmospheric and Oceanic Sciences
and Mechanical Engineering
Indian Institute of Science
Bangalore 560 012, India.
Email:jayes@caos.iisc.ernet.in

Heat transfer between two surfaces usually increases when the temperature difference between the two surfaces increases. Here we highlight an unusual situation in radiation heat transfer wherein the heat transfer decreases when the temperature difference increases.

Introduction

In 1701, Newton proposed that heat transfer from a surface to the surrounding fluid is proportional to the temperature difference between the surface and the fluid. This statement is called 'Newton's cooling law'. The rate of heat transfer per unit area is given by

$$Q = h(T_1 - T_2), \tag{1}$$

where h is called the convective heat transfer coefficient and T_1 and T_2 are the temperatures of the solid surface and the fluid respectively.

Newton's cooling law has been used widely for the design of heat exchangers, power plants, rockets, refrigerators and automobiles. Can a similar law be used for radiation heat transfer? The radiation heat transfer between two infinite *black* surfaces can be written based on the Stefan–Boltzmann law as (see, for example [1])

$$Q_{\text{rad}} = \sigma(T_1^4 - T_2^4),$$

where σ is the Strefan–Boltzmann constant and T_1 and T_2 are the absolute temperatures of the two surfaces. This equation can be rewritten as

$$Q_{\text{rad}} = \sigma(T_1 + T_2)(T_1^2 + T_2^2)(T_1 - T_2). \tag{2}$$

Based on equations (1) and (2) we see that heat transfer increases as the temperature difference increases in both convection and radiation.

Keywords
Radiation, emissivity, heat transfer.

How about surfaces that are not blackbodies? A gray surface is defined as a surface whose emission is a uniform fraction of the emission by a blackbody at the same temperature. We define the total emissivity (ε) of a gray surface as the ratio of radiation emitted per unit area by the gray surface to that emitted by a blackbody at the same temperature. We can estimate the radiation transfer between two infinite parallel gray plates as follows (*Figure* 1):

The radiant energy emitted by surface 1 is $\varepsilon_1 \sigma T_1^4$, where ε_1 is the total emissivity of the gray surface 1. The radiation absorbed by surface 2 is $\varepsilon_2 \varepsilon_1 \sigma T_1^4$, where ε_2 is the emissivity of surface 2. (Note that based on Kirchoff's law for gray surfaces we have assumed that the total absorptivity α_2 is equal to the total emissivity ε_2). If ρ_2 is the reflectivity of the gray surface, then $\rho_2 = (1 - \varepsilon_2)$.

The radiation reflected by surface 2 is then $(1-\varepsilon_2)\varepsilon_1 \sigma T_1^4$.

The radiation reflected by surface 2 is reflected back to it by surface 1 and hence the absorption by surface 2 of the radiation emitted by surface 1 (after *two* reflections) is $(1 - \varepsilon_2)(1 - \varepsilon_1)\varepsilon_2 \varepsilon_1 \sigma T_1^4$.

If all the rays after zero, two, four, six and more reflections are considered, the total radiation flux absorbed by surface 2 is given by the infinite series

$$Q_{2,\text{abs}} = \varepsilon_2 \varepsilon_1 \sigma T_1^4 \{1+(1-\varepsilon_1)(1-\varepsilon_2)+(1-\varepsilon_1)^2(1-\varepsilon_2)^2 \ldots\}.$$

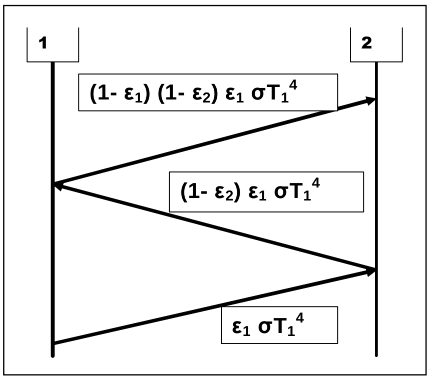

Figure 1. Ray tracing between parallel plates.

This can be simplified to

$$Q_{2,\text{abs}} = \varepsilon_2\varepsilon_1\sigma T_1^4\{1-(1-\varepsilon_1)(1-\varepsilon_2)\}^{-1}.$$

Similary, we can calculate the radiant flux emitted by surface 2 that is absorbed by surface 1 as

$$Q_{1,\text{abs}} = \varepsilon_2\varepsilon_1\sigma T_2^4\{1-(1-\varepsilon_1)(1-\varepsilon_2)\}^{-1}.$$

Hence the net radiation heat transfer between the two surfaces is

$$Q_{\text{net}} = Q_{2,\text{abs}} - Q_{1,\text{abs}}$$

$$Q_{\text{net}} = \varepsilon_2\varepsilon_1\sigma(T_1^4 - T_2^4)(\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2)^{-1}. \qquad (3)$$

This can be rewritten as

$$Q_{\text{net}} = \sigma[\varepsilon_2\varepsilon_1(\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2)^{-1}(T_1+T_2)(T_1^2+T_2^2)](T_1-T_2).$$

If we assume that the term within the square bracket is approximately constant, then we can conclude that radiation heat transfer between gray surfaces *increases* when the temperature difference between the surfaces *increases*. This conclusion is, however, not valid for all radiation heat transfer problems. We discuss, in the next section, an interesting case wherein the radiation heat transfer between two surfaces *decreases* when the temperature difference between the two surfaces *increases*!

Construction of a Dewar Flask

A Dewar flask is a cylindrical container used to store cryogenic fluids. It has several features designed to reduce conduction, convection and radiation heat transfer. Conduction and convection heat transfer are almost eliminated by evacuating the air gap between the inner and outer surfaces of the flask. This is similar to the thermos flask used at home to keep coffee hot. The vacuum ensures that there is almost no medium to transfer heat by conduction or convection. Radiation heat transfer does not require a medium and hence radiation heat

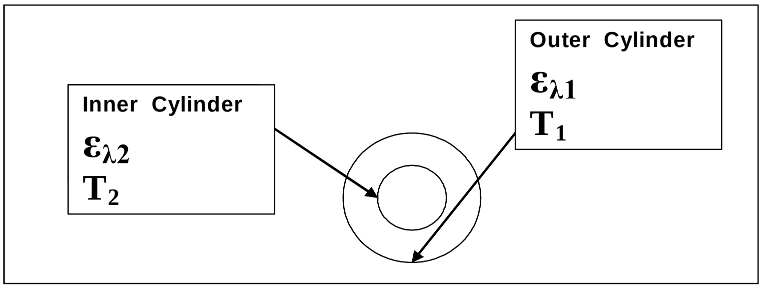

Figure 2. Radiation between two cylinders.

transfer has to be reduced by other means. The outer surface of the inner cylinder and the inner surface of the outer cylinder are silvered (see *Figure* 2). This ensures that the radiation emitted by the outer cylinder (at ambient temperature) is mostly reflected rather than absorbed by the inner cylinder. Hence the heat leakage to the inner cylinder by radiation is reduced substantially. To understand the role played by the reflectivity of the surfaces we undertake a detailed analysis of radiation heat transfer between two cylinders in the next section.

Heat Transfer in Dewar Flask

Dewar flasks are used widely to store cryogenic liquids like nitrogen, hydrogen and helium. The boiling point of hydrogen is 20 K when the pressure is 1 bar while the boiling point of nitrogen at 1 bar is 76 K. If the ambient temperature is 300 K, then the temperature difference between the outside and inside walls of the flask will be 280 K when liquid hydrogen is stored in it but it will be 223 K when liquid nitrogen is stored in the flask. Hence one should expect that the heat leakage into the Dewar flask with liquid hydrogen will be more than that in the flask with liquid nitrogen. Laboratory observations show, however, that the heat leakage into the flask with liquid nitrogen is about *60% more* than the heat leakage into the Dewar flask with liquid hydrogen in it. Why should the heat leakage be *higher* when the temperature difference is *lower*? To understand this unusual phenomenon we need to understand the complexities inherent in radiation heat transfer between non-gray surfaces. The spectral emissivity of a non-gray surface is a function of wavelength and this has a profound influence on radiation heat transfer.

Radiation Heat Transfer between Two Cylinders

If the gap between the two cylinders is small compared to the radius of the cylinder, we can approximate the radiation heat transfer between the two cylinders by the heat transfer between two infinite parallel plates. We can calculate the spectral radiation heat transfer between non-black infinite parallel plates by ray tracing. We define the spectral emissivity (ε_λ) as the ratio of radiation emitted per unit area by a real surface (between λ and dλ) to that emitted by a blackbody (between λ and dλ) at the same temperature. Here λ is the wavelength of the radiation that is emitted. By following the procedure outlined for radiation transfer between two gray surfaces, the net radiation heat transfer between the two surfaces in the range λ and dλ is obtained as

$$Q_{\mathrm{net},\lambda} = \varepsilon_{\lambda 2}\varepsilon_{\lambda 1}(e_{\lambda 1} - e_{\lambda 2})(\varepsilon_{\lambda 1} + \varepsilon_{\lambda 2} - \varepsilon_{\lambda 1}\varepsilon_{\lambda 2})^{-1},$$

where $e_{\lambda 1}$ and $e_{\lambda 2}$ are the spectral blackbody emissions by surface 1 (outer cylinder) and surface 2 (inner cylinder) respectively. Here e_λ, the spectral emissive power of a blackbody, is given by

$$e_\lambda = C_1\{\lambda^5[\exp(C_2/\lambda T) - 1]\}^{-1},$$

where C_1 and C_2 are contants.

For silvered surfaces we can show that the spectral emissivity varies as: (see [1])

$$\varepsilon_{\lambda 1} = B(T_1/\lambda)^{0.5},$$

where B is an empirical constant that is related to the electrical resistance of silver.

Since the emissivity of silvered surfaces is very low (around 0.01) we can neglect the product term in the denominator in the above equation and simplify the expression to

$$Q_{\text{net},\lambda} = \varepsilon_{\lambda 2}\varepsilon_{\lambda 1}(e_{\lambda 1} - e_{\lambda 2})(\varepsilon_{\lambda 1} + \varepsilon_{\lambda 2})^{-1}.$$

Using the above expression for spectral heat transfer we can obtain the following expression for the net radiation heat transfer by integrating over all wavelengths:

$$Q_{\text{net}} = \int_0^\infty Q_{\text{net},\lambda} d\lambda$$

$$Q_{\text{net}} = \sigma B^*(T_1^5 T_2^{0.5} - T_2^5 T_1^{0.5})(T_2^{0.5} + T_1^{0.5})^{-1}, \qquad (4)$$

where $B^* = 4.26 \times 10^{-5}$ K^{-1} for a silvered surface.

Note that equation (4) for radiation between non-gray surfaces is very different from equation (3) that is valid for gray surfaces. We will now show that (4) predicts that radiation heat transfer between the two surfaces *increases* when the temperature difference *decreases*.

For a flask containing hydrogen, $T_2 = 20$ K, while for nitrogen it is 77 K. In both cases T_1 can be assumed to be 300 K (ambient temperature). Hence the ratio of radiation heat transfer to the flask containing liquid nitrogen to that containing liquid hydrogen can be written as (after assuming that $T_1 >> T_2$)

$$Q_{\text{net,N2}}/Q_{\text{net,H2}} = (77)^{0.5}(20)^{-0.5}[(20)^{0.5} + (300)^{0.5}]$$
$$[(77)^{0.5} + (300)^{0.5}]^{-1} = 1.63.$$

We have been able to show that the radiation heat transfer to the Dewar containing liquid nitrogen should be about 63% higher than the heat transfer to the Dewar containing liquid hydrogen. This is consistent with laboratory observations.

We can estimate the heat transfer for other values of T_2 between 4 K and 300 K. These results are shown in

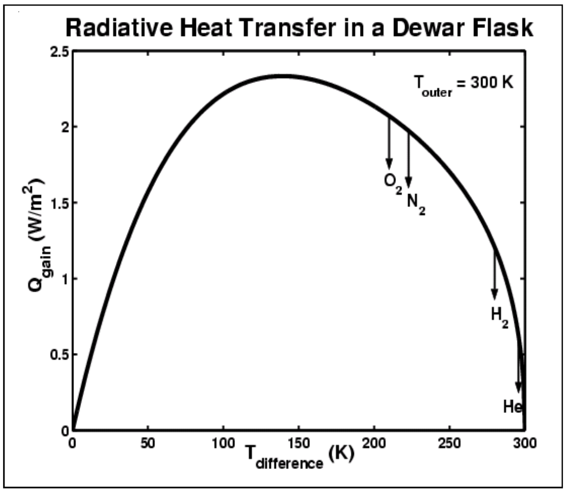

Figure 3. Heat leakage into Dewar Flask as a function of temperature difference.

Figure 3. We find that heat loss is small when the temperature difference between the inner and outer cylinder is very small or very large. The heat transfer between two surfaces should go to zero when the temperature difference between the two surfaces goes to zero. However, we find that the heat transfer goes to zero even when the temperature difference is large. This result does not conform to what we expect when heat transfer occurs by conduction or convection. Also, this trend will not be seen when radiation heat transfer occurs between two black or gray surfaces.

This anomalous behaviour occurs primarily because the spectral emissivity of a silvered surface at 20 K is less than that of a surface at 77 K. Hence the radiation absorbed by the inner cylinder in a Dewar flask containing liquid hydrogen will be less than that absorbed in a Dewar flask containing liquid nitrogen. The change in spectral emissivity between 77 K and 20 K is more than the change in temperature difference (i.e, 280 to 223) and hence we obtain the apparent paradox. Note that to obtain the result that agrees with observations, we have to take into account the dependence of spectral emissivity of a silvered surface on both the temperature and the wavelength. This paradox occurs because emissivity is a strong function of temperature and wavelength.

Suggested Reading

[1] R Siegel and J R Howell, *Thermal Radiation Heat Transfer*, Taylor and Francis,4th Edition, London, 2002.