Remarkable advances in science are often due to bold innovations by highly creative minds. Yet history of science is replete with instances of new ideas meeting with stiff resistance. This is illustrated in the context of mathematics by eminent Indian mathematician Prof. S.Ramanan in the recent convocation address at the Chennai Mathematical Institute, which is reprinted below. Prof. Ramanan also bemoans the compartmentalised training in and approach to mathematics, especially in India. The fragmental attitude is an obstacle not only to creativity, but also to improved job opportunities.

S Ramasubramanian

The Spirit of Creativity and Innovation in Mathematics

S Ramanan

Introduction

It gives me great pleasure to greet all of you, young graduating students, on this occasion. Graduation is indeed something special. In England, Europe, United States and elsewhere, it is considered a turning point in one's life. Even at the so-called High School graduation, they make a big song and dance, I mean, literally! High School Graduates take part in Proms. and even parents and other relatives of graduating students wear T-shirts proclaiming their wards' achievements.

This is the time when one finishes formal learning of a certain kind. For those of you who go on for further academic studies, learning from a teacher *Gurumukha* – is replaced by learning mostly by oneself. For those who take up other vocations, formal learning more or less comes to an end. I consider Chennai Mathematical Institute a unique institution in our country, where students are trained to think for themselves even at the undergraduate level. Creation of such an institution was my dream and I am delighted that Seshadri, using his formidable reputation and goodwill, has brought it to fruition. I think that by all parameters, the CMI experiment has been a success.

Text of Convocation Address at the Chennai Mathematical Institute delivered on 1st August 2006.

Padma Bhushan Rama Rao is an outstanding researcher and scientific administrator and I consider it a privilege to be delivering this address in his presence.

Intellectual Conservatism

I am a great admirer of India's first Prime Minister, Pandit Jawaharlal Nehru. His vision was profound and many-sided. In my opinion, many institutions of lasting intellectual value in our country owe their existence to his enthusiastic support. The Lalit Kala Academy, the Sahitya Academy, Indian Statistical Institute, the Indian Institutes of Technology are some of the Institutions spawned during the early years of Independence. The Tata Institute of Fundamental Research, where I was first a graduate student and later a professor for over forty years, was one of the beneficiaries of his progressive thinking.

In his *Discovery of India*, Nehru bemoans the stagnation that intellectual life in India underwent for several centuries in the last millennium. This cannot be entirely blamed on foreign conquerors. In this era, scholarship did not mean bold and new thought, but interpretation. Pandits vied with each other in quoting this or the other poet or philosopher. The general belief was that all one needed to know had all been said and that one had only to understand what had already been said. Research did not mean discovery of new truth but deeper understanding of the truth established by the ancients. I am not unaware that there were a few exceptions to this. The blossoming of classical music, indigenously in the south and under Islamic influence in the north, provides, for example, a noteworthy exception.

Having decried quotations, let me make one myself. It is a verse at the beginning of Kalidasa's play *Sâkuntalam* and is a favourite of mine.

> Pur ânamityevas âdhu sarvam
> Na châpi k âvyam navamityavadyam
> Santa: pareekshya anyatarat bhajante
> Moodha: parapratyaya neya buddhi:

Something is not good just because it is ancient, nor is a piece of work to be thrown out as new-fangled. Sensible people analyze the pros and cons and come to conclusions. Only fools are led by the opinions of others.

I would like intelligent people like you to reflect even on this and come to your own conclusions! Personally I am willing to take it as my credo and have it on my T-shirt. I feel that suspicion of things new, and reluctance to adopt new techniques and modes of life, continues in the daily life of India consciously or otherwise, even today. Happily however this is beginning to change, thanks to the visible prosperity that new technology has brought.

This conservatism is present even in creative art and is not confined only to Indians. However, in the West, daring new innovators, alongside die-hard conservatives, are not in short supply. Let me illustrate this with some examples from mathematical progress in the last two centuries.

George Cantor was a mathematician of extraordinary originality. He was attacked fiercely by the general community of mathematicians for his innovation. Apparently there were murmurs even from the Church. Cantor tried to cope with the problem by claiming that his theory was the result of Divine Intervention, and that he was only a messenger! Thankfully, his efforts in this direction were successful and the church acknowledged that his theory did not contradict official doctrine! There were secular objections from established mathematicians as well. Fortunately, for every acerbic comment from a Poincaré that future generations would regard Cantor's theory as a disease from which it has recovered, there was a counter from a Hilbert which said 'From the paradise created for us by Cantor, no one can drive us out'.

Let me give another, more recent, example. In the 40's, a group of French mathematicians realized, to their horror, that there was no definitive version of even fundamental mathematical theorems, with assumptions and conclusions unambiguously stated and proved. They embarked on the project of writing up the whole of mathematics *ab initio*. They pretended to be a mathematician called Bourbaki. Thanks to their efforts, by the time I became a graduate student half a century ago, it was very comforting to have at one place, precise definitions and statements. It may seem surprising today that there was at all any controversy about the desirability of having the basic results so proved. In view of the precise style which mathematical writing has acquired since then, you may wonder if there is, or was, any other way of expressing mathematics. If you go through Hardy's *Pure Mathematics* – not a bad book by the way – and read through two full pages of an essay on the 'notion of a function', you will appreciate what a liberation it was to realize that a function is simply a map of one set to another! But the fact is that many mathematicians, among whom were some distinguished ones of achievement, carried on a tirade against Bourbaki and Bourbaki-ism.

In Paris, the State Universities are known by numbers. Université 5 and Université 6 were situated in a huge complex of interconnected towers, located in the center of Paris called Jussieu. The administrative offices of the two universities were located in different towers that were connected across by a walkway corridor of about ten meters' length but there was considerable antipathy between the Bourbakists of cinquiéme and the others of sixiéme. When I mis-addressed a letter as 5 to a well-known mathematician who was a professor at 6, the letter came back, saying 'addressee not known'! This was explained later to me in terms of the hostility between the two wings! Today, much of this puerile back biting has, happily, ended.

For some time, my research interests centered around Abelian varieties, named after Abel, one of the great innovators from Norway, of the nineteenth century. I was trying to understand what is known as Riemann's theta transformation formula, in my own way. Carl Ludwig Siegel, a famous German mathematician, was instrumental in applying it to number theory deeply. Siegel was unfortunately ultra-conservative and carried on a campaign against abstraction. He was even credited with the statement 'the degeneration of modern mathematics started with Riemann'! I made some progress in formalizing Siegel's identities and found that the 'correct' modern set-up for this, was to define a functor from the category of quadratic spaces over the integers into the category of polarized Abelian varieties. Being an avid lover of modern mathematics, I derived great, and slightly perverse, satisfaction from calling it the *Siegel functor*.

Grothendieck was a great mathematical philosopher and was awarded the Fields Medal in 1966. He was one of the most creative mathematicians of the last half-century. He came up with fundamentally different ways of looking at things by what is sometimes described as 'pure thought'. He was not afraid of thinking anew as and when he learnt. Consequently, he often asked elementary, and sometimes even stupid, questions. I want to interject an advice to students here. 'Do not be afraid to ask a question for fear that it may be considered stupid'. Most lecturers would be happier with stupid questions than with none at all!

André Weil, another great mathematician of the century, had formulated his famous conjectures relating the number of solution of a set of Diophantine equations, with the topology of the locus defined by the same set of equations over complex numbers. These came to be known as 'Weil conjectures' and influenced the direction algebraic geometry took, for several decades. In retrospect, it is clear that the most appropriate framework in which this question has to be looked at is the language of schemes, invented by Grothendieck. But André Weil, instead of hailing this progress and quickly embracing the new framework, acted sullen and negative. Ultimately the conjectures were settled in the affirmative, based chiefly on Grothendieck's ideas, supplemented by Michael Artin, the coup de grace being given by another great mathematician and Fields medallist, Pierre Deligne. It is difficult to understand why mathematicians like Weil, who were *avant garde* in their youth, became so conservative later. Although I am sure that ageing has something to do with it, that perhaps does not explain this phenomenon completely.

Fermat's famous problem is also concerned with the solution of Diophantine equations. He considered the explicit equation

$$x^n + y^n = z^n$$

and claimed that if $n \geq 3$, the only integral solutions are the obvious ones, namely those with one of the coordinates zero. Gerd Faltings, another Fields medallist, proved that it can have only finitely many solutions (up to multiplication of all the three coordinates by the same integer). Note that when $n = 2$, the above equation has infinitely many solutions. In fact, the identity

$$\left(\frac{a^2 - b^2}{2}\right)^2 + (ab)^2 = \left(\frac{a^2 + b^2}{2}\right)^2$$

where a, b are integers, both even or both odd, provides *all* the solutions. Pythagoras' theorem tells us that these correspond to the lengths of the three sides of a right-angled triangle. Some of those, like (x=3, y=4, z=5) and (x=5, y=12, z=13), corresponding to $(a, b) = (3, 1)$, and $(a, b) = (5, 1)$ respectively, are familiar to high school students. Faltings makes a sharp attack on the conservatives, saying "*It is strange that people who have no compunction in using analytical methods in the study of numbers, taking after nineteenth century mathematical thought, should be so vocal in deriding new ideas which have come up in the last decades*".

He was referring again to the new ideas of Grothendieck. Perhaps most of you know that Fermat's conjecture itself was solved recently by Andrew Wiles.

In the early days of Tata Institute of Fundamental Research, there were representatives of both the conservative and the progressive streams. Two facts, however, were influential in the breaking of the shackles of the past. Prof Chandrasekharan, whose personal mathematical work cannot be termed as 'modern', recognized nevertheless the desirability of exposing young students to new ideas. He managed to get many of the top mathematicians of the world to visit India and lecture to the students at Tata Institute. The second is the lucky circumstance that the Institute got extremely gifted students like Narasimhan, and Seshadri, and their early success served as a spur to others who followed.

Specialization vs Versatility

Another aspect which I wish to touch upon, is the dichotomy between specialization and versatility. It is on the one hand, necessary to focus on particular topics which one wishes to work on. On the other, it has also to be ensured that specialization does not become very narrow and one's skills do not become superfluous with the efflux of time.

I am sure that in the paleolithic age, some humans were better than others in making fire by friction. These skilful people were surely in great demand at that time and their work was highly valued. As technology progressed, these skills were less and less useful and of no value by the time they became fossils! Even a few decades ago, there were typists who prided themselves in typing fast and without any mistakes. The advent of word processors made these skills less valuable. During my graduate student days, many of my fellow researchers from experimental disciplines spent hours computing with slide rules at a point in time and a little later, with punching cards for the computer. These skills are now as useful as archery in present day warfare.

I am laboring therefore the obvious when I say that technological progress makes many skills superfluous in the market place. Less obvious is the phenomenon of obsolescence in the intellectual domain. Many experts in mathematical analysis had trained themselves in the effective use of summability theories and development of new tools in this area. The rise of functional analysis and the development of the theory of distributions, made many of the tricks of regularization superfluous. I will paraphrase this by saying that as soon as the community of mathematicians considers that the broad contours of a theory are well understood, people look to other areas for interesting problems.

In the fifties and early sixties, algebraic topology was in the center of mathematical interest, along with algebraic geometry. Soon, it was generally felt that higher dimensional topology had been understood. The aim of algebraic topology is to understand the topology of a space by means of numerical invariants. One of the outstanding achievements of the above period, thanks to the deep work of John Milnor, Stephan Smale and others, was the near accomplishment of this task.

A beacon light for such a project was the so-called Poincaré conjecture which seeks to solve this question for higher dimensional spheres. Poincaré himself only stated this as a conjecture for the three-dimensional sphere. It was ironical that the obvious generalization of this problem, namely, characterization of spheres of any dimension was solved for all dimensions except the originally proposed three!

Subsequently therefore, the general interest turned to topology in low dimensions. With Perelmann's solution of the original Poincaré conjecture, characterizing the three dimensional sphere – and it is generally expected that he would receive the Fields Medal in Madrid later this month – the curtain has fallen on this specific problem. It remains to be seen whether the methods employed in the solution of the above problem spawn new territories or if this rings the closure on the general interest in low-dimensional topology.

Therefore one ought to beware of too narrow a focus and try to have a broad perspective of theories beyond those one is interested in for the bread and butter of the day. As one grows older, it becomes difficult to have the nimbleness needed to understand new things. (Conversely, attempting to understand new things is one of the ways one keeps young)! This may be because having acquired a lot of expertise in one area, one may feel that time is better spent using what one has learnt over decades, than acquiring piddly little details of a new theory. Therefore, the broader the base one has built in youth, the better the agility with which one can deal with this problem through life.

There are two aspects to mathematical research: Learning mathematics, and learning to do mathematics. Although they are closely inter-related, they are not the same. For doing worthwhile research in an area of mathematics, it is, generally speaking, necessary to be an expert in that field. Occasionally, it may happen that one just strays into a subject and sees that some question the experts in that area are struggling over, can be solved by a clever trick one has thought of in one's own. Some collaborative efforts are indeed of this kind. But even to make such felicitous interaction possible, one has to keep one's eyes and ears open. What I said above amounts in practice to the following. Have a sharp focus on your topic of research at the moment, but also try to have a broad understanding of other areas. This, like thrift, may save something for the future.

Moreover, one does not have to be so utilitarian in one's approach to things intellectual. Humans do not live by bread alone. While one has to focus on a small area – indeed often on a specific problem – at a professional level, one has, for one's own enjoyment, to try and have a wider knowledge of other branches of mathematics. All the details in a painting are not equally sharp and detailed. So in knowledge, there will be areas which one knows deeply, those in which one has expertise, others which one knows well and some more on the horizon, with which one has only a nodding acquaintance. After all, an enriched life is marked by sensitivity to culture in broad terms, and to help students build it in themselves is one of the main aims of education.

Pure Mathematics as a Career

In our days, pure mathematics graduates had hardly any avenues of employment. Administrative service and academic positions were perhaps the only possibilities. Here, permit me a personal digression. I remember that it was not an easy decision for a lower middle class person like me to go into mathematics.

When I was a student of the second form, that is, the seventh grade, we had a Tamil non-detailed text book, entitled '*Moonru Indiap periyorgal*' or 'Three great men of India'. It must have been well-written, since I still remember it after so many decades. The three great men in question are Jagadish Chandra Bose, P C Ray and Srinivasa Ramanujan. The book had reproduced a letter which Ramanujan wrote to Hardy, in which he claimed that the series

$$1 + 2 + 3 + \cdots$$

gives the value –1/12, adding,

after this claim you may think that I am fit to be in a lunatic asylum, but I can justify what I say

This intrigued me, and I went about asking seniors at school, and also older relatives. They expressed incredulity, and essentially said that the author, likely a non-mathematician, did not know what he was talking about. Somehow this explanation did not satisfy me and I said to myself that some day I was going to understand what this meant. I am not suggesting that this was the only reason why I went into mathematics! We had good teachers at high school and undergraduate levels. In particular, the ability of one of them, Prof Raghava Sastry, effectively to convey his enthusiasm for projective geometry, played a part in my becoming an algebraic geometer ultimately.

From the economic point of view, things have changed for the better for mathematics graduates. Rightly or wrongly, industry managers think that someone with a mathematical background is better capable than others, of analyzing a problem into components, and synthesizing the solutions. Mathematics graduates and PhDs in the United States, do not have much problem getting into lucrative Wall Street jobs, for example!

In fact, if you are of the type which romanticizes penury and wish to take to mathematics as a career for that reason, you may be disappointed! You can look around and see that people who have had reasonable success as mathematicians, are not badly off indeed! In addition, by and large, they do not have to pander to a boss, their time is their own, with plenty of foreign travel thrown in with consequent enrichment of outlook, social and cultural, besides professional.

Conclusion

I wish you all the best in life, and hope that, whenever you recall your stay at CMI, it will be with pleasure. I hope that, later in life, many of you will think of giving something back to quality education in India in general, and to CMI in particular.