

Think It Over



This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

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On a Result of Ramanujan

The following claim is made in Ramanujan's *Notebook* (page 23, entry 11):

$$\text{If } x = y + \left(\frac{z}{y} - y^3\right)^{1/3}, \text{ then } 3y = x + \left(\frac{9z}{x} - x^3\right)^{1/3}.$$

We shall show that the '3' and '9' in this relation cannot be replaced by any other pair of numbers. We first write the claim in rationalized form by clearing the roots:

$$\text{If } (x - y)^3 = \frac{z}{y} - y^3, \text{ then } (3y - x)^3 = \frac{9z}{x} - x^3. \quad (1)$$

Suppose that we replace the '3' and '9' in equation (1) by literals a and b ; we get:

$$\text{If } (x - y)^3 = \frac{z}{y} - y^3, \text{ then } (ay - x)^3 = \frac{bz}{x} - x^3. \quad (2)$$

The 'If' condition in (2) is equivalent to $x^3 - 3x^2y + 3xy^2 = \frac{z}{y}$, and therefore to

$$x^2 - 3xy + 3y^2 = \frac{z}{xy}.$$

Keywords

TIO on Ramanujan's result.



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The 'Then' in (2) is equivalent to

$$a^3y^3 - 3a^2y^2x + 3axyx^2 = \frac{bz}{x}, \text{ and therefore to}$$

$$3ax^2 - 3a^2xy + a^3y^2 = \frac{bz}{xy}.$$

The two conditions in (2) are therefore equivalent iff

$$\frac{1}{3a} = \frac{3}{3a^2} = \frac{3}{a^3} = \frac{1}{b}.$$

These lead to $a = 3$ and $b = 9$, as claimed.

We can deduce an analogous result for quadratics, by arguing in the same way. The result is:

$$\text{If } (x + y)^2 = \frac{z}{y} + y^2, \text{ then } (4y + x)^2 = \frac{8z}{x} + x^2.$$

This may be stated using radicals as:

$$\text{If } x = -y + \left(\frac{z}{y} + y^2\right)^{1/2}, \text{ then } 4y = -x + \left(\frac{8z}{x} + x^2\right)^{1/23}.$$

We leave the proof to the reader.

A Problem in Graph Theory

A conference of ' n ' leaders is proposed to be held (where ' n ' is an odd number). Suppose that the organisers decide to conduct it in such a way that the leaders will have their dinner together at a circular table and each leader will have different neighbours on each day. Then

- 1) For how many days will the leaders have to meet to have all combinations and arrangements?
- 2) How can the leaders be arranged in their position each day?

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