Resonance reprints with permission the text of a colloquium given by David Speiser at CERN in 1978 titled *'Newton's Principia'*. Following a brief biography of Newton, Speiser describes the structures and contents of the three parts of this great work; an indication of what were known before Newton to others, and of Newton's own unique contributions; and the ways in which his work was carried forward mainly in continental Europe. The complexity of Newton's personality is referred to. And finally in Speiser's view, the only other figure Newton can be compared with is the poet Dante Alighieri.

N Mukunda

Newton's *Principia*

D Speiser

Newton's life and work are briefly summarized and it is reasoned that we are in a better position today than a century ago for evaluating his accomplishment. The *Principia* is then presented and discussed in some detail. Stress is laid especially on the difference between the first two books, where Newton put together into one coherent "general mechanics" his own and his predecessors discoveries, and the third, where the theorems are applied for explaining the solar system, the motions of planets and comets, and the tides. There follows an explanation of how Newton's ideas were propagated, even though the *Principia,* unlike the *Opticks,* was understood by only a few scientists. Through the work of D. Bernoulli and L. Euler, especially, Newton's mechanics was transformed and expanded into an endeavour of endless application. It is shown that the theory of relativity, although marking a limit to the validity of Newton's mechanics, has made clear how much better than most of his critics Newton understood the problems behind his work. Lastly Newton's enigmatic personality and his almost superhuman work are compared with the personality and the work of Dante.

Few books were less understood than Newton's *Principia* [1] but few books had a bigger effect. How were Newton's ideas propagated?

At the beginning of this century, the theory of relativity and quantum theory established restrictions on the validity of Newton's mechanics: what is its significance for today's physics and how do we now judge Newton's accomplishment?

* * *

No scientist ever knew such fame or was so honoured as Newton. Copernicus' book appeared only during the last few days before its author's death [2]. Kepler, the "Imperial Mathematician", knew fame, but until his death he remained a poor man persecuted by worries, and he died exhausted by the many material struggles which he had to go through during his entire life. Galileo, already spoiled by glory as a young man, ended his days as a man severely restricted in his freedom. Newton, however, held several high public offices after his scientifically active period, and he remained President of the all-powerful Royal Society for 25 years. No stranger of rank who came to London would forget to pay him reverence. At his funeral, four Dukes and Marquises were his pallbearers, and his tomb stands at the most conspicuous place in Westminster Abbey.

His name, as no one else's, became synonymous with science itself. Euler refers to him often as "Summus Neutonus". In one of his poems, the German poet Schiller calls all scientists briefly "Newtons". And when the philosopher Kant, whose ideas were deeply influenced by Newton, wants to state that a mathematical biology is impossible he says simply, "There will never be a Newton of the grass-blade". Only two scientists after Newton seem to have gained such legendary fame: Darwin in the last century and Einstein in our own, the latter precisely because he overthrew Newton's mechanics.

We must now ask: In what respect are we particularly prepared *today* for understanding Newton the scientist, and for appreciating his work? As an answer, one may point out the following:

Firstly, while Newton's personality remained for a long time hidden behind a veil of venerability (to the point that, for example, David Brewster's biography sometimes reads rather like a hagiography), we know it better today than ever before [3]. The American Louis Trenchard More's impressive book, the article of John Maynard Keynes, the well-known economist who had saved a great part of Newton's manu-

scripts, an analysis by Frank Manuel and others, have shown us the great scientist as a man. We now see not only the teacher highly venerated by his disciples, but also a secretive and diffident personality filled with passions, whose behaviour towards his colleagues was often motivated by rancour and who was not above unworthy means.

Secondly, the history of science has taught us a lot during recent decades. Alexandre Koyre, Bernard Cohen, Derek Whiteside, A Rupert and Mary Boas Hall, John Herivel, and many others have made Newton's work much more accessible [4]. And with respect to the later development of mechanics, C A Truesdell has removed many prejudices that had become entrenched in the course of time [5]. He corrected the opinion, formulated among others by the physicist E. Mach, that "After Newton, mechanics neither advanced nor went back one single step" [6]. Today, we know that this is erroneous. Much of what we today call Newton's mechanics was already known before him, and much was only found after him. And since in what follows I shall often talk of the accomplishments of Newton's successors, I hasten to add that nothing is further from my intention than to diminish Newton's importance. I shall do this rather in order to show the vastness of science itself, with respect to which even such a grandiose work as Newton's may appear small.

But above all, for more than 200 years Newton's mechanics was thought to be unchangeable. Yet in this century, its limitations were discovered. The special theory of relativity, the general theory of relativity and finally quantum mechanics, gave limits to the applicability of classical mechanics, and today, more than 50 years thereafter, at the beginning of the last quarter of the century, we are perhaps for the first time at a sufficient distance to correctly appreciate the significance of Newton's discoveries.

And finally, one must also be aware of the fact that, perhaps for the first time, physics and chemistry are no longer at the forefront of scientific progress, but that molecular biology has taken over this role during the past 20 years.

Therefore, I shall not be speaking now as a historian of Newton, which I am not, but merely as a physicist trying to understand his book.

* * *

A few words must suffice as a sketch of Newton's life.

Isaac Newton was born on Christmas day, 1642 (old calendar) in Woolsthorpe. He was

the posthumous son of an independent farmer, brought up first by his mother and, after her second marriage, by his grandmother. Because of his extraordinary gifts and also because people did not think that he would be strong enough for the life of a farmer, he was sent to Cambridge, presumably in order to study theology. And theology indeed remained during his whole life the subject of his principal interest, to which he devoted the greater part of his time and energy. But once at Cambridge his extraordinary gift for mathematics was soon discovered, and he followed there among others the courses of the famous Isaac Barrow, whose successor he later became.

At the end of his undergraduate years (as we would say today) during the great plague of 1665–66 he was forced to return to Woolsthorpe, and there he made the three great discoveries which made him immortal, or to be precise, he took the first steps towards them: his calculus of fluxions, the theory of light and colours, and the explanation of the attraction of the moon. But what he discovered he kept to himself. In truth, in none of the three cases do we know exactly how much Newton had already discovered at this date. With respect to gravitation, one thing is certain: he must have grasped intuitively something decisive, but he probably thought that he had not yet a systematic and solid enough foundation for his theory. It was at this time too that there occurred the famous incident of the apple in his mother's garden, which he later recounted to his friend Stukeley.

However, back at Cambridge, he became involved again with other subjects. Only in 1679 and at the beginning of the 1680's did he begin to work once again on the problems of mechanics, which he had left untouched for almost 15 years. It appears that it was an exchange of letters with his rival Robert Hooke that made him return to the still unsolved problem of gravity. And *then,* in a few years, he composed the *Principia,* or more fully the *Philosophiae Naturalis Principia Mathematica* [1]. It was printed in 1687, thanks mainly to the efforts of his disciple Edmund Halley, with the imprimatur, given on 5 July 1686, of the president of the Royal Society, the famous Samuel Pepys, whose often indiscreet diaries are still today a delight to many readers.

But a few years after this publication, Newton's life changed drastically. In the autumn of 1693 he suffered a breakdown, the significance of which has perhaps been sometimes exaggerated. However that may be, according to Lord Keynes, his friends, especially Charles Montague, Lord Halifax, decided that nothing less than a complete change in his life was needed. Thanks to Halifax's influence, in 1696 he was appointed first

"Warden" and then "Master of the Mint", in which capacity he was to supervise the coining of the new English money that was introduced by decision of Halifax and the Whig government. His predecessors had considered the office mostly as a sinecure, but not he: Newton, it seems, was an extremely able administrator; he had in fact been a very good one already at Cambridge. Keynes calls him "one of the greatest and most efficient of our civil servants", and he performed his task splendidly; indeed, his chemical studies had prepared him well. False money was common at this time, and coins were often devalued by cutting or clipping little bits, so that the honest people lived in a state of permanent inflation. Dr Fierz has told me that we owe to Newton the invention of the "milled edge" for protecting money against such assaults, an invention which each of you carry in his pocket. Thanks to Newton's energy, the output of new money was increased by a factor of eight. He also defended money in a more active way, namely by organizing his own police and a net of informers. According to Manuel, at least 19 coiners of false money were hanged as a result of Newton's efforts, for counterfeiting was considered high treason [7]. However we may feel about this today, England certainly profited greatly from his activities.

In London, he published in 1704 his second great work, the *Opticks* [8], which is based on work done in the 1660's and on earlier publications, and towards the end of his life, he brought out two more books, one on the *Prophet Daniel and the Apocalypse of St. John* and one on the *Chronology of the Ancient Kingdoms.*

During these last years he was a figure of universal respect, and became almost the centre of a cult. He was now the well-established protector and often generous friend of his many disciples: Edmund Halley, Roger Cotes, Henry Pemberton, and for a time, the unfortunate Nicolas Fatio de Duilliers from Geneva.

But at the same time he was the all-powerful president of the Royal Society, punishing and persecuting with all available means anyone who dared to oppose him or who he felt was attacking him. These dark, even sombre, traits appear in the last portraits by the painter John Vanderbank. Thus Arthur Koestler, in one of his books, calls him "monster and saint" [9]. There are light and dark features in his personality, both in almost superhuman measure. But it is tragic that *both* should have had a paralysing, almost stifling effect on the continued development of science in England.

Twice he was to reprint the *Principia* in revised form: in 1713 with Roger Cotes and in 1726 with Henry Pemberton.

DEFINITION I

The quantity of matter is the measure of the same, arising from its density and bulk conjointly.[2]

DEFINITION II[3]

The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.

DEFINITION III

The vis insita, *or innate force of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forwards in a right line.*

DEFINITION IV

An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of uniform motion in a right line.

DEFINITION V

A centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre.

Of this sort is gravity, by which bodies tend to the centre of the earth; magnetism, by which iron tends to the loadstone; and that force, whatever it is, by which the planets are continually drawn aside from the rectilinear motions, which otherwise they would pursue, and made to revolve in curvilinear orbits.

The quantity of any centripetal force may be considered as of three kinds: absolute, accelerative, and motive.

DEFINITION VI

The absolute quantity of a centripetal force is the measure of the same, proportional to the efficacy of the cause that propagates it from the centre, through the spaces round about.

DEFINITION VII

The accelerative quantity of a centripetal force is the measure of the same, proportional to the velocity which it generates in a given time.

DEFINITION VIII

The motive quantity of a centripetal force is the measure of the same, proportional to the motion which it generates in a given time.

[From Ref. 13]

During the last 6 years of his life he was in poor health but remained active. He died in his 85th year, on 20 March 1727, after a short illness.

* * *

I must now give you an idea of the content of the *Principia* and of Newton's'purpose, his goal, and what he accomplished.

At the beginning we find a short introduction in which Newton states his goals, and an ode by his friend Edmund Halley, who edited the *Principia* against Newton's resistance and at his own financial risk.

The work then begins with a series of definitions, by which some fundamental notions like "quantity of motion", "inner" or "inertial force", central force", etc., are explained and where it is stated how these quantities are to be measured (see opposite). There follows in a scholium his famous doctrine of absolute space and absolute time, about which more will be said later.

Newton then formulates the three important "axioms" or "laws of motion" (see p. 6 and 7), which still today all girls and boys learn in high school. These axioms are, in his words but a bit shortened:

1)   in the absence of an external force,every body remains at rest or in the state of uniform motion,

2)   the change of the quantity of motion is proportional to the external force,

3)   the reaction is always equal and opposite to the action, or the actions of two bodies are always equal and in opposite directions.

These laws are followed by a number of corollaries, among others the law of the parallelogram of forces.

And *then only* follow the three main parts of the work: first, two books *On the Motion of Bodies, and* then the third book *On the system of the World,* which crowns the whole work. In order to follow Newton's reasoning and to penetrate into his philosophy of nature and of science, one must clearly grasp the significance of this arrangement. The first two books, which comprise about 4/5 of the work, contain what we would today call

"General Mechanics". There, starting from theorems of geometry and from his own new axioms of mechanics, he derives and proves a great number of theorems. However, with a few exceptions they are not applied to natural phenomena. This he does only in the third part, the "Celestial Mechanics".

Since dynamics describes motions, i.e. changes, its formulation required a new mathematical language. Newton had discovered this language himself: his *calculus of fluxions,* which, however, he had never published before in a systematic form. Therefore, the first book opens with a chapter containing 11 mathematical lemmas, which in the sequel are used constantly. But for this very reason – and it seems that this was Newton's intention – this first chapter presents for the reader an almost insurmountable obstacle. Even today, knowing what Newton is driving at and being familiar with the theorems he proves, we have great difficulty following his reasoning; and this holds true for the whole work.

The content of the following 13 chapters is organized into 50 theorems, 48 problems, some lemmas and a few general scholia. Chapters 2 to 7 contain the investigation of the orbits of a body that moves under the influence of a central force; first the description of the curve itself, then the way in which this curve is traversed in time.

The presentation – and in order to understand Newton's purpose this must be kept in mind – is always conceived in view of two opposite goals. One is practical, directed towards applications, the other one systematic, aiming at a rigorous deduction. On the one hand, it is clear that the needs of practical astronomy suggested many of the problems he treats, such as the 14th problem of Section V: "Construct the conic section that passes through 5 given points"; this is obviously a problem of astronomy, namely, the determination of the orbit of a comet. But – and this is important – this motivation is never stated explicitly. Discussion of the planetary system is postponed to the third book, since *the systematic presentation must not be interrupted or obscured.*

What at once catches the eye of the present day reader in these first chapters is the direction which Newton's investigation takes. Unlike us, he does not start from a differential equation. In fact, in the entire work, there is no differential equation at all, not even the one which today we call "Newton's equation". Rather he proceeds in the opposite direction and solves the inverse problem: given a curve, for instance an ellipse, he shows that if the centre of attraction is at the focus, the centripetal force must decrease with the square of the distance. Thus he finds that all conic sections are

solutions of his universal law of gravitation. But only Johann Bernoulli, armed with all his analytic equipment, succeeded in proving that the conic sections are the only solutions to the problem [10]. And it would be wrong to see only a completion of Newton's results in Bernoulli's theorem. Rather we must say that Johann Bernoulli succeeded in attacking the problem from a new point of view, and this, as we all know, is in science one of the most difficult accomplishments.

There follow the sections which today we would call the "advanced" ones. In these, he treats general forces and the motion of the apsides, and Section X contains a theory of the pendulum, based on the work of Huygens. I cannot talk today about Newton's predecessors; however, I should like to take this opportunity to state that in my opinion the importance of Huygens' work, especially its influence on Newton, is greatly underestimated in the literature. Surely, the appearance of the *Horologium Oscillatorium* was one of the decisive events before the composition of the *Principia*. While in these first ten sections Newton considers (as we would say today) the motion of a single body in an external field, beginning in Section XI, he considers systems of *several* bodies, simple bodies first and then extended ones. In particular, his theorem 26 and the following 22 corollaries are the mathematical basis for his attack on the famous three-body problem. His lunar theory in the third book is based on these theorems and it was then developed further in the second edition. The same holds for his theory of the tides. Sections XII and XIII treat the mutual attraction of extended bodies. It seems that the solutions of these problems kept Newton occupied for a long time and that these difficulties were one of the main reasons of the delay of the *Principia*.

The last section of the first book contains the mechanical foundation of his corpuscular optics. There he discusses the passage of a particle through parallel sheets and he derives Snell's law. This is the *first example* of how the new mechanics transcends itself and becomes the basis of another domain of physics. For, of all motives which Newton has left to the physicists, this probably proved itself to be the most successful: to conceive a phenomenon of nature as the propagation of a system through space and time.

This short summary may show to what extent the first book is organized systematically, and how much of its spirit thereby has remained modern. Indeed, the sequence of its presentation is essentially the same as we still find today in our textbooks. The enormous difference from a modern book is due mostly to Newton's mathematics, and partly also, but to a minor degree, to differences in the point of view from which he

considers the problems.

However, the foregoing dry enumeration could not do justice to the enormous riches that we find in this book. In all sections, wherever we look, we find new theorems, and quite often they are theorems which still today present great difficulty to mathematicians and physicists in spite of the powerful tools which the mathematicians have provided us with during the past 300 years.

The second book does not exhibit the same unity in its structure. It culminates in the refutation of the theory of vortices, by which Descartes had tried to explain the solar system. But even though this book was conceived primarily in view of this goal, it nevertheless contains much more.

In the first four sections Newton treats the motion of bodies in a viscous fluid in which friction is proportional to the speed or to the square of the speed. This "Ansatz" is one of Newton's most important discoveries, but strangely enough, as Truesdell has noted, it was overlooked by his successors for more than a century.

Section V is devoted to hydrostatics, and Section VI to the motion of a pendulum subject to friction, a problem that had been recognized by Huygens as fundamental for the construction of clocks.

But the most important sections with respect to both size and content, are Sections VII and VIII. Section VII contains his hydrodynamics and Section VIII his acoustics, in particular his investigations on the speed of sound. His accomplishment in this domain was summed up by Truesdell in the following words: "Newton had the genius to propose the problem, Euler to solve it" [11].

Section IX, the last, presents the refutation of Descartes theory and Newton's triumph.

The text of the second book testifies – and this is worth mentioning – that Newton did not work only as a mathematician and theoretical physicist, but that also in the domain of mechanics he performed original experiments [12].

The structure of the first two books is dominated by his intention to systematize the whole doctrine of mechanics by trying to derive it from a few axioms stated at the beginning of the books, and thus to realize mechanics as one system. Newton in this respect is the successor of Euclid, and his immediate predecessor is Huygens. It is true, however, that with respect to this endeavour he did not completely succeed in the

second book.

Finally the third book, the most admired of all. It contains the application of his theoretical mechanics to the solar system, that is, to the motion of planets, moons (at the time of Newton ten were known), comets and, last but not least, the sun itself. For, according to his third law of motion, the sun too moves around the solar system's centre of inertia. Newton made significant and fundamental contributions to the understanding of all these phenomena.

Having shown in the first book that motion on an ellipse can be understood as a consequence of the force law $1/r^2$, he now concludes in the third book that the inverse-square law of gravity is valid in the solar system without exception. This conclusion is based *both* on Kepler's laws, which rest on observation, and on the theorem from the first book.

It is worth while following his line of reasoning in detail. In the first edition, the third book begins with nine hypotheses, of which the first three are general rules of correct procedure in science. The last five are statements of observed facts, for example that the five planets Mercury, Venus, Mars, Jupiter and Saturn move according to Kepler's laws around the sun and that the same laws are also valid for their moons. But with respect to the motion of the earth itself, which was in dispute, he leaves the question open initially. Thus he first demonstrates the validity of the law of gravity for planets and moons, and only after having established that Kepler's laws hold for the planets not with respect to their motion relative to the earth but relative to the *sun,* does he decide in favour of the Copernican doctrine.

One sees that he perceives the connection between theory and experiment in a way which even Galileo and Kepler could not yet grasp.

But he does not leave off there. After having deduced Kepler's laws from the principles of his mechanics he goes on far beyond anything that had been achieved before him, and this expecially in two directions. We owe to Newton the first quantitative description of the orbit of a comet: it is either a parabola or a hyperbola, and he establishes the following theorem: "the proportion between the instantaneous speed of a comet on a parabola and the speed of a planet on a circle is proportional to the square root of the proportion of the diameter of a planetary orbit to the instantaneous distance of the comet from the sun" [13]:

$$\frac{v_{com}}{v_{pl}} = \sqrt{\frac{2r_{pl}}{r_{com}}} \quad or \quad r_{com}\,v_{com}^2 = 2r_{pl}\,v_{pl}^2$$

which may be contrasted with

Kepler's third law for circular orbits: $r_1 v_1^2 = r_2 v_2^2$

This law, the analogue to Kepler's third one, is an extension of our astronomical knowledge far beyond what we owe to Kepler. The comets, which were still an enigma for Descartes and for Jacob Bernoulli, thereby became accessible to observational and computational astronomy [14]. Thus was broken what has sometimes been called "the last bulwark of superstition".

The Italian Algarotti says mockingly, in a book to which we will return: "[Les comètes] sont les ennemies les plus déclarées que les tourbillons [de Descartes] aient dans le ciel et it semble en général qu'elles n'ont été faites que pour déconcerter les systèmes" [15].

The attentive reader of the *Principia* will notice that the observations of Flamsteed, the Astronomer Royal, have in part disappeared in the later editions. This was a counter-blow developed by Newton in one of these battles in which he fought his enemies with all means, allowed ones and sometimes not allowed ones. And he never hesitated to attack an enemy publicly and, if possible, to destroy him: Newton's luminous genius was surrounded by dark traits of character.

The other major achievement is the foundation of a lunar theory. Everyone knows that Newton's mechanics permits one easily to solve the two-body problem, and since the sun is so much heavier than the planets these solutions are excellent approximations to reality.

But in order to have an exact result one must solve a many-body problem; for example, in order to predict the moon's orbit accurately it must be computed under the influence of the earth *and* the sun. Still today we do not have an exact solution and must use approximations. Newton took the first enormously ingenious steps in this direction, especially in the second edition of the *Principia*, but to follow these steps is enormously difficult.

The comparison of the first edition with the second one is revealing. Besides an

expansion of the lunar theory we find other changes. Thus the "hypotheses" at the beginning disappear, or more precisely they change their name. Between 1686 and 1713, Newton again pondered intensively and systematically the philosophical principles of science, and this must have led him to a revision of his concepts.

He adds at the end of the second book the famous "Scholium Generale" which brings the totality of what he had achieved in science into contact with theology. There he also explains his "inductive method" and now states proudly the often quoted sentence, *"Hypotheses non fingo"* ("I do not invent hypotheses").

In fact, however, the hypotheses have *not* completely disappeared. We still find two of them in the second edition. First, one of the lemmas, the third one, now became a hypothesis. Probably this means no more than that Newton could not prove it. But of the original nine hypotheses, one, the fourth, still remains. It says: "the centre of the world is at rest". As I said, in the first edition he had left open the question of whether the earth or, as we would say today, the centre of inertia of the solar system, is at rest. But it is clear that the statement: "the centre of the world is at rest" is neither a "rule for proceeding correctly" nor a "phenomenon", but a true hypothesis. But it appears strange, and I do not know whether people have sufficiently emphasized*) the fact that this hypothesis is not only superfluous, it directly contradicts Newton's mechanics. For, according to Newton, physical processes evolve in a homogeneous space and homogeneous time. His mechanics postulates what we call today the principle of relativity. This is the meaning of his first and his third axioms. And in such a space, on which (as we say today) a group acts, there is no centre: the centre is everywhere and nowhere. It is surprising to see that even Newton felt here a great difficulty and that even he was not clearly aware of the consequences of his own ideas. This is another example of Wolfgang Pauli's observation that "a fundamental scientific discovery, sometimes even against the resistance of its creator, gives birth to further fruitful developments following its own autonomous course" [16]. Only much later did physicists grasp this point clearly.

Let us now compare the first two books with the third one: *the truth* of the theory springs from its principles: this is pursued and explained in the two books *De Motu Corporum;* but *its validity* is proved by its agreement with the observations: this is the purpose of the

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* After this lecture was delivered I found in a paper by I.B. Cohen that H. Weyl indeed insisted on this contradiction. See Ref. 30.

third book, *De Mundi Systemate*. It is precisely in the accomplishment of having recognized and achieved this double goal that we must see what is sometimes called a bit vaguely the "Newtonian synthesis".

It is from this point of view that one should judge Newton's achievement. His first law goes back to late antiquity, and Galileo's successors, as well as Descartes, Wallis and especially Huygens, had grasped it clearly. The famous second law was understood by Galileo and after him by Beeckmann. The law of gravity goes back to Kepler, and others, for example Boulliaud, Borelli and Hooke, stated it before or independently of Newton. If one should put one's finger on one special point, my choice would be his third axiom: action = reaction (for once I agree with Mach [17]). The law in this form, and indeed the whole idea, belongs to Newton. Perhaps it was suggested to him by Kepler's discovery that the tides are caused by the moon. This law, even today difficult to grasp, made possible the principle of relativity, which is fundamental for all of physics, including classical physics. And, after Newton, this axiom became a heuristic principle of great value. But to ask this question (what is Newton's most important single discovery?) in this way means that because of the trees, one does not see the forest! Newton's specific achievement is to have *put together* what his predecessors found, into *one coherent system* [18]. The motions of the planets and comets could now be computed starting from only four fundamental principles, and beyond this, as we shall see, his system is capable of an extension to the whole of physics.

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We may pause for a moment and reflect on the sobering fact that the book which, more than any other one, supported the claim of the enlightenment, that all questions could be solved with the help of rational thought, was understood only by a mere handful of scientists!

How then did Newton's discoveries propagate? It is revealing to know how big the edition of such an important work was. Dr Whiteside (Cambridge), the editor of Newton's mathematical manuscripts, informs me that in Newton's time 250 copies constitued a large edition, and that 1000 copies meant a big enterprise. His estimate is that the first edition printed in London numbered something between 330 and 350 copies [19]. For the second edition printed in Cambridge we are fortunate to possess exact data: 750 copies were printed, of which 711 were distributed. The others were used

by Newton for corrections and he sent a few to his friends. The third edition, printed again in London, comprised 1200 copies. These numbers testify to the great success of the book. But, in spite of it, one must ask how many readers really understood it? Together with Dr. Fellmann in Basle, who knows Newton's mathematics and who rediscovered in Geneva the copy of the *Principia* owned by Leibniz [20], we tried to enumerate the possible candidates. Our lists were not identical, but we agreed that during Newton's life-time at best six "savants" understood the book and, up to the present, probably not more than one dozen or, if one is generous, two; moreover, Dr Fellmann and I agree that both of us are *not* on this list.

A fourth edition, still in Latin, with a commentary, was edited in Rome after Newton's death by the Rev. Fathers Jacquier and Le Seur [21]. This edition is noteworthy in several respects. First for the fact that it was prepared in Rome under the very eyes of the Santo Officio. Jacquier and Le Seur had to declare that, with respect to certain assertions, they did "only wear a mask", that is that they did not express their own opinion; which was not much more than a formality. Afterwards this book was printed in Geneva of all places, so that one might say that this edition was an oecumenical collaboration "avant la lettre".

But what interests us here mostly is that the two Reverend Fathers added to Newton's book three papers which had been submitted in 1740 to the French Academy of Sciences [22]. For its annual prize, the Academy had asked for an investigation on the tides, a subject about which the theory of Descartes and the more recent one of Newton were in competition. The prize was divided and attributed equally to Colin MacLaurin, one of Newton's disciples, Daniel Bernoulli, and Leonhard Euler, all of whose papers took Newton's *Principia* as their starting point. Thus, one may see in the attribution of these prizes a sort of official recognition of Newton's physics in France and on the continent in general.

At one point in his paper, Daniel Bernoulli writes, in the year 1740, "I see that this corresponds to what Newton says in the *Principia,* where he compares the length of the earth's axis with the radius of the equator. As far as his reasoning goes, he is probably the only one to understand it, for this great man sees even through a veil what another can hardly see with a microscope" [23] In other words: one of the best mathematicians and physicists of his time could see, when he read the *Principia,* that Newton was right in his contentions, but he could not always follow his proofs.

The difficulty met by his contemporaries is also illustrated by the following: we have books written by Newton's disciples Henry Pemberton (1728) and Colin MacLaurin (1748) where, as they themselves say, they "explain the philosophy of Sir Isaac Newton" [24]. Both books contain few formulae or none at all, for those were understood only by a handful of mathematicians who shared the secret of the "New Math". And this still in the middle of the 18th century, at the time when on the continent Voltaire was popularizing Newton's physics with the help of his friend, Mme du Châtelet, who herself like Voltaire quite probably did not understand it either.

Count Algarotti, one of the foreign luminaries that Frederic the Great attracted to his Court, whom I have already quoted, wrote in his youth, in 1732, two volumes entitled *Il Newtonianismo per le Dame,* a book that was translated into French [15]. But the major part of the work is devoted to Newton's *Opticks.* Only in the sixth and last part does the author approach the *Principia.* The *Principia* was praised, but the *Opticks* was read.

How then was the new mechanics propagated? If we want to investigate how a new science is disseminated and how it penetrates into research itself, we must ask, Where did it stimulate new creative work? Thanks to the authority of its creator, Newton's doctrine became popular in the British Universities comparatively early, but we find little creative response there. The first such came from the continent, first in a critical, even polemic way, which I shall mention only briefly.

Science became intertwined with a sterile polemic between Newton and Leibniz and their supporters concerning the priority of the discovery of calculus, or more precisely, the independence of Leibniz's discovery. This quarrel has often been told more or less exactly; perhaps even now we do not know all the relevant facts, and therefore I shall not enter into this episode.

An essential and important step was however taken by Jacob Herrmann, a disciple of Jacob Bernoulli who, in 1716, published the first systematic presentation of Newton's mechanics after the *Principia* itself. It was the first book of all on mechanics after Newton, and had the title *Phoronomia, sive de viribus et motibus corporum solidorum et fluidorum* [25]. In this book we find, for the first time, Newton's laws in differential form [26]. In spite of its historical importance, the *Phoronomia* has unfortunately been neglected by historians, until now, but we can hope that, thanks to C A Truesdell, this situation will change during the coming years.

But one may justly say that the first important creative response to the *Principia* came from Daniel Bernoulli. Daniel Bernoulli had learned mathematics and physics from his father Johann and his elder brother Nicolaus. In physics he was first guided by the ideas of Descartes, then by those of Huygens and Leibniz, whom the father had venerated and defended in his priority dispute with Newton. But one day, we do not know exactly when and how, he began to read and carefully study the *Principia.* His father owned a copy of the second edition, which is still today at the University Library in Basle. By studying Daniel's papers we can see how he penetrates, step by step, into Newton's ideas; how, with the help of the mathematics of Leibniz, of his uncle, and of his father, he begins to reformulate them; and how in his hands these different currents begin to merge. So far, these currents had been in competition on the ideal plane, and on the plane of daily life they had simply quarrelled. Now, a new synthesis was to emerge.

But each progress has its price, and in this case the price was a family tragedy. We know how old Johann behaved towards his son. In 1738, when Daniel was finally able to publish in Strasbourg his *magnum opus,* the *Hydrodynamica* [27] the father published at the same time his much shorter *Hydraulica,* backdating it by several years and implying thereby that his son's discoveries had followed his own [28]. This has often been attributed to jealousy, which certainly played an important part, but a more profound motive was the great bitterness of the father at seeing the son take over the enemy's cause and lead it to triumph. For, as we saw, it was Daniel's paper on the tides, just then in preparation and later to become his most printed work, which sealed on the continent the victory of "Newtonianism" (as some would say today).

At about the same time, or even a bit earlier, the final and irreversible step was taken. In 1736, the young Leonhard Euler (not yet 30 years old) published in two volumes his *Mechanica sive Motus Scientia Analytice Exposita* [29]. This synthesis of Newton's and (we must insist) Huygens' physics with the mathematics of Descartes, Leibniz and the Bernoullis, is a *second* turning point. While Newton's book remained forever closed to almost everybody, we find here, for the first time, a work that provides an easy access to mechanics; as easy as this science permits. For it was in these books that the language in which we understand and present mechanics until this very day was coined and fixed. In these books Newton's mechanics was developed into a method which *will and must* attack all phenomena of the physical world: it has become a program for the whole of the natural sciences. The book's success was enormous. The *Mechanica* does not equal with

respect either to beauty or to originality some of Euler's later work, but it made accessible for the first time the new science. Laplace later called Euler "notre mâitre à tous", wherein he was thinking not only of the mathematician but also of the author of the *Mechanica.* Euler's accomplishment is the more astonishing as his book was only the third systematic presentation of mechanics, and yet its appearance established mechanics in the academies and universities as a domain of knowledge. In fact, it made possible the modern way of studying. A few years later in his *Anleitung zur Naturlehre,* which unfortunately appeared only after his death, Euler again discussed in detail the fundamentals of mechanics [30]. The mathematician and physicist Hermann Weyl says of this work [31]: "Euler exposes in wonderful clarity the sum of the whole knowledge in the natural sciences of his age".

There Euler discussed especially inertia and the principle of relativity. Later these questions were neglected, even forgotten. Had these problems been understood, the quarrel about Einstein's theory of relativity would never have become so silly. For, as everyone who receives anti-relativist articles from time to time can testify: the opponents of the theory of relativity not only do not understand Einstein's mechanics, they also do not understand Newton's, whom they try to defend.

Euler's *Mechanica* and his later works, especially those in hydrodynamics, have given to physics its modern character. They have also opened the way to mathematical technology, a connection so characteristic of modern science.

During the 18th century, Newton's computations were extended especially by Clairaut and by Euler, the true founders of analytical lunar theory [32]. For quite a time mathematicians and astronomers were not sure whether or not Newton's law of gravity would suffice to explain all observations made in the planetary system or whether additional terms would be needed. The situation was not unlike that in quantum electrodynamics today. Only after having performed long and complicated computations did Clairaut, Euler and their successors become convinced that no discrepancy between observation and theory existed. After enormous efforts of many mathematicians and astronomers, by the middle of the last century only a few small effects remained still unexplained. And of all those, eventually *one only* remained: the perihelion of the planet Mercury advances too fast by a minimal amount of 43 seconds of arc per century, which is a tenth of a second per revolution. This discrepancy, as you know, was only explained by Einstein in 1914 [33]. But then, Einstein showed that the

discrepancy is due not only to the law of gravity but to a limitation in Newton's mechanics itself.

It is also worth recalling that just a few years ago, Kenneth Nordvedt in Montana showed how a precise measurement of the moon's orbit permits one to confirm the Newtonian principle of equivalence to the order of the gravitational interactions themselves [34]. Two years ago those measurements were made independently by I. I. Shapiro and by R. H. Dicke *et al.*, with the help of laser techniques which permit a determination of the earth–moon distance to an accuracy of 10 cm, and they were able to confirm the principle.

Beyond astronomy, Euler's hydrodynamics and the theory of elasticity, due mainly to Euler and Cauchy but based in part on ideas of Newton and Jacob Bernoulli, have helped extend the triumph of Newton's mechanics to all domains of physics. Also through Euler it found its way to Faraday and through him definitively into magnetism and electricity.

If, during this century, we have been forced to replace Newton's mechanics, which we call "classical", on the one hand by Einstein's mechanics, on the other by quantum mechanics as the foundation of physics, precisely this replacement permits us to appreciate more justly what Newton accomplished with his own theory. During the entire 19th century and still at the beginning of the 20th, Newton's formulation of the principles of mechanics underwent continual criticism from Mach among others. The first and third of his axioms were often degraded to the rank of mere definitions, and some (d'Alembert already) went even so far as also to see nothing more than a definition in his second axiom: "The change in the quantity of motion is proportional to the external force". Today it has become clear to us that Newton was right. The second axiom was modified by special relativity, *in order to take account of experiments;* thus, *it cannot be* a definition. The first axiom was modified by the general theory of relativity *in order to take account of experiments,* thus *it cannot be* a definition. All of the three are true axioms of classical mechanics.

The list however, as Truesdell and others have shown, is not complete [35]. Euler recognized in 1775 as a fourth independent axiom for the mechanics of continua that the change of angular momentum is equal to the moment of the force. The independence of this axiom too remained forgotten for a long time, and according to Truesdell, Boltzmann was the first to recognize again its true role [36]. The same may be said

concerning Newton's doctrine of absolute space and absolute time [37]. That he saw in them attributes of God may astonish us today. But what is important is that, like Euler after him, he saw here a fundamental problem. Namely, in which system must physical quantities be measured? His doctrine was derided; only Einstein again recognized the importance of the question. Therefore, even though today Newton's answer has been slightly modified, the glory remains his and we can see today more clearly than ever that he understood the character of his own answers better and more exactly than did most of his successors. In this respect, Newton triumphed over his critics.

Permit me now to conclude with a few words about Newton's personality. In no less than three domains, mathematics, optics and mechanics, this extraordinary man achieved accomplishments of unique importance and left us incomparable works. And in stating this, one must be conscious of the fact that these occupations were only a small part of his activities. In addition, he was a great chemist and alchemist. For example, his famous law in thermodynamics grew out of these occupations. But in fact, as we know today through the enormous mass of papers which he left, his principal interests were theology and history, to which he devoted probably the greater part of his time. And here too, as in the exact sciences, Newton did not see himself as a scientist among others, but as a prophet, to whom it was given to reveal the great secrets of nature and of history.

With whom can we compare this infinitely gifted man, of such a complex character, generous towards his pupils, but also timid, defiant, sensitive, easily offended, rancorous, and filled with strong passions, especially an insatiable desire for power? Galileo, Kepler, Euler, Faraday, Riemann, Maxwell and Einstein do not show traits of similar passions to the same degree, and none was filled with such a sense of a prophetic mission. Kepler, it is true, felt such a mission, but his ideas lay always open and unconcealed, and he was not given to being the judge of others. Besides his splendid discoveries, Newton, certainly more than any other great scientist, has left science also with an uncomfortable legacy. You all know science and research can become the victim of ambitions and of power, and certainly many places fell foul of entrenched power. I think it is fair to say that, in this respect, Newton's grasp for power was accepted with the acclamation of the majority of his fellows and successors, and thus he has more than any other man infused European science with this often deadly danger. We should not overlook this constant menace, because it is our work and with it our dignity that are at stake.

If we want to find a kindred spirit in Europe's history, I think we have to look for it outside science. What other man was so imbued with a quasi-religious mission to reveal to the world the secrets of the universe and of its history, and to be judge of all previous periods? I think that we may find Newton's closest spiritual relative in the Italian *poet and scientist Dante.* Each saw himself as a prophet sent to reveal what had been secret [38]. Each accomplished this mission in a work whose composition must have required an almost unimaginable degree of intellectual concentration and physical strain. Of both we can see the boundless dedication to their work and feel the enormous hidden passions, and of both we can admire their sense of responsibility and dedication to the political community in which they lived.

Both works are not easily accessible and present to the reader great difficulties. Like Dante's *Divine Commedia,* the *Principia* too has a majestic beauty which fascinates and captivates the reader. The impression one receives for the *Principia* can be compared to an immense dark cavern, full of fascinating byways revealed in occasional flashes of light. And at the far end burns a brilliant fire whose dazzling reflections are seen from time to time. I hope to have given you a little reflection of this fire and of the fascination it radiates. But as with Dante's work, the full impression, and you know this much better than I do, is reserved for the reader.

Acknowledgements

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* I am indebted to David Finkelstein who directed my attention to these books.

12) See, for example, Ref. 1 p. 271 (Reg. 7), p. 316 (Corol. 5), p. 330 (Prob. IV), and especially p. 354.

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18) Whatever one may think of Newton's behaviour towards Hooke, it is from this point of view that one must understand the following sentence in his famous letter to Halley, 20 June 1686: "Now is not this very fine? Mathematicians that find out, settle and do all the business must content themselves with being nothing but dry calculators and drudges and another that does nothing but pretend and grasp at all things must carry away all the inventions as well of those that were to follow him as of those that went before."

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I keep the subject of my inquiry constantly before me, and wait till the first dawning opens gradually, by little and little, into a full and clear light.

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I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

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If I have ever made any valuable discoveries, it has been owing more to patient attention, than to any other talent.

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Truth is ever to be found in the simplicity, and not in the multiplicity and confusion of things.

— *Newton*