Bernhard Riemann

The line of succession of outstanding mathematicians, who occupied the Chair of Mathematics at the University of Göttingen from early 1800, is a formidable list – Gauss, Dirichlet, Riemann, Clebsch, Klein, Hilbert, Courant, Minkowski, to name a few. Bernhard Riemann was born the son of a Lutheran pastor on 17 September 1826 in Breslenz, Germany, a small town close to the River Elbe. Till the age of 14 he was taught by his father and later went to the high school in Hanover and the Johanneum in Luneburg. At an early age, it was evident that he had mathematical talent, but following the wish of his father, he started to study theology and philosophy at the University of Göttingen in the summer of 1846. He also attended lectures in mathematics and could eventually convince his father to permit him to devote his entire attention to the study of "poorly paid" mathematics. The level of mathematics teaching at Göttingen then was still elementary. Gauss lectured only to the beginners and reserved his mathematical expertise for a small group of chosen students, to which Riemann did not belong at that time. So Riemann went to Berlin in the summer of 1847, where Jacobi, Dirichlet, Eisenstein and Steiner had built around them a group of students, with whom they shared their mathematical findings at lectures. Dirichlet in particular, who was acknowledged as the leading mathematician after Gauss and who was at the height of his mathematical career, had a profound influence on Riemann. After four semesters at Berlin, Riemann returned to Göttingen, where the situation had improved. On an invitation from Gauss, Wilhelm Weber had returned to Göttingen, where he held outstanding lectures in physics accompanied with experiments, which was unusual at that time. This awoke in Riemann a liking for that side of mathematics which was related to problems of physics.

In 1851 Riemann obtained his doctorate degree with Gauss as his guide. His thesis was a path-breaking work on analytic functions, in which he placed the idea of an analytic function, which till then was vague and ambivalent, on a firm basis and established its connection to conformal mapping. In his habilitation thesis, Riemann worked on the representation of periodic functions using Fourier series, where he introduced the idea of integration, now well known as Riemann Integration. In his now-famous habilitation lecture, where he spoke on 'On the hypotheses on which geometry rests', he

Based on an article in German '*Göttingen – a Centre of Mathematics*' – a compilation by the Mathematisches Institut, Universität Göttingen, 1992.

developed ideas, which only Gauss could appreciate at that time and which now form the basis for various fields of mathematics and the general relativity theory of Einstein. In 1857 Riemann was appointed associate professor and after the untimely death of Dirichlet, the successor of Gauss, he occupied the Chair of Mathematics at Göttingen University in 1859.

In the short span of time till his death in 1866, Riemann did pioneering work in the theory of analytic functions, in analytic number theory and in mathematical physics. Among the mire of difficulties surrounding the treatment of many-valued analytic functions, he found a new path in that he specified a suitable surface for its domain of definition, which is now called a Riemann surface. This idea explained the work on algebraic curves done over the previous fifty years, it unified the earlier work and opened the door to new discoveries. In the only work he did in the area of number theory, Riemann found the key to the problem of the distribution of primes, in that he associated it with a special analytic function, now known as the Riemann Zeta function. One of the problems associated with it is the famous Riemann Hypothesis, the proof of which would have far reaching consequences for our understanding of primes among other things. Till today it has neither been proved nor has a counter-example been found. (In fact, it is one of the problems identified by the Clay Mathematical Institute as a Millenium Problem for the solution of which a prize is offered). Riemann's contribution in the field of mathematical physics gave impetus to the study of gravity, sound waves, electricity and magnetism.

Riemann's all-encompassing ideas took a long time to be understood, but they have influenced the growth of mathematics in a slow but steady manner and determine the direction of research even today.

In 1862 Riemann was already suffering from ill health, the consequence of years of meager resources and of denying himself proper nourishment. He had tuberculosis, for which at that time there was no cure. He spent a lot of time in Italy, where the climate was mild, but that could only delay the inevitable and he died on 20 July 1866 in Selasca at the early age of 39 years.

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