Editorial

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What is it that convinces a brilliant mathematician to reject the most prestigious prize in Mathematics – the Fields Medal? For the first time in the history of the Fields Medal, a recipient has refused to accept the award. This editorial is devoted to Grigori Perelman and his quiet rise to fame.

While the award of the Fields Medal is a carefully guarded secret, mathematicians below the age of 40 eagerly await the phone call that tells them (and only them!) a few weeks in advance about their getting the award at the International Congress of Mathematicians (ICM) held every four years. This year at the twenty fifth ICM, it was an open secret that Perelman would be one of the Fields Medalists. Instead of the usual phone call, Sir John Ball, president of the ICM, made a trip to Saint Petersburg to speak to Perelman in person. The meeting took place at a conference centre in a stately mansion overlooking the Neva River. Perelman told Ball that he had no intention of accepting the medal. "I refuse", he said simply. Ball, in an interview at ICM, said that Perelman had spoken to him of some personal experiences with the mathematical community that "had caused him to remain at a distance." The chairperson of the Congress, Manuel de Leon said "The reason Perelman gave me is that he feels isolated from the mathematics community and therefore has no wish to appear as one of its leaders."

What is it that alienates a brilliant young mind? Has the mathematics community been supportive or has it been cruelly indifferent? It is worthwhile to take a look back at Perelman's research career. He worked on several problems at the same time. One of his themes of research was the Poincaré conjecture, formulated by Henri Poincaré over a hundred years ago. In basic terms, what Poincaré proposed was that all simply connected, three dimensional manifolds were spheres. To the astonishment of most mathematicians, it turned out that manifolds of the fourth, fifth and higher dimension were more tractable than those of the third dimension. Stephen Smale announced a proof of the Poincaré conjecture in high dimensions ($n \geq 5$) in 1960 and won the Fields Medal. So did Michael Freedman twenty years later for his work on the fourth dimension. In the 1970s William Thurston proposed that every three dimensional manifold could be broken down into one or more of eight types of

component – Geometrization Conjecture, for which he won a Fields Medal. In 1982, Richard Hamilton identified a particular evolution equation, which he called the Ricci flow, as the key to resolving the Poincaré and Thurston Geometrization Conjecture. Hamilton's idea was to use the Ricci flow to homogenize the geometry of 3-manifolds. Over more than twenty years, Hamilton and other geometric analysts made progress in understanding Ricci flow, but could not resolve the problem of "singularities", which are regions where the geometry, instead of getting homogenized, suddenly exhibits uncontrolled changes.

Perelman listened to Hamilton's lectures, while on a two year fellowship (1993–95) at Berkeley. Perelman realized that a paper he wrote earlier on Alexandrov spaces might help Hamilton to prove Thurston's conjecture and told Hamilton about it, but Hamilton showed little interest. In 1996, Perelman wrote to Hamilton a long letter outlining how one could overcome the obstacles, with the hope of collaborating with him. Hamilton did not reply and Perelman decided to work alone.

In an unorthodox way, on November 11, 2002, Perelman posted a thirty nine page paper with ground breaking results on the Ricci flow and its singularities on arXiv.org, a web site used to post preprint articles. By casually posting a proof on the internet of one of the most famous problems in mathematics, Perelman was taking a great risk. If the proof had errors, there would be no way of preventing another mathematician from fixing it and claiming to have solved the conjecture. Perelman was not particularly concerned. He was then invited to a lecture tour of the US in April 2003. To Perelman's disappointment, Hamilton did not attend his lectures at Stony Brook. On the invitation of John Morgan, he then went to Columbia University, where Hamilton was. At his lecture there, Hamilton came late and asked no questions.

By mid July 2003, Perelman had posted the final two installments of his proof on the internet. Three sets of mathematicians began the task of painstakingly retracing his steps. Tian and Morgan wrote a book of 473 pages to explain the 55 pages of Perelman.

It is an encouraging and comforting thought that there are still Perelmans in the field of science. The Fields Medal holds no importance for him. To him – "It was completely irrelevant. ***Everybody understood that if the proof was correct, then no other recognition is needed.***"

You may read more about the issue in the article entitled 'Manifold Destiny – a Legendary Problem and the Battle Over it' in *The New Yorker* 28th August 2006 and *Daily News*, ICM, Madrid, August 2006.