Our Readers Write ...

In July, 2006 issue of *Resonance*, Dr Jasbir S Chahal has written a very nice article about some basic ideas in geometry. In this connection I would like to point out that the famous Indian mathematician Brahmagupta discovered a method for generating primitive Pythagorean triplets in the form $(a^2 - b^2,\ 2ab, a^2 + b^2)$ although the conditions imposed on a and b by him are different than those described in the theorem of the article by Dr Chahal. Also, there are some other rules for obtaining Pythagorean triplets. Those rules are described below including that of Brahmagupta.

(1) **Brahmagupta's Rule**

If a and b are two natural numbers and $a > b$, then $a^2 - b^2$, 2ab, $a^2 + b^2$ will be a Pythagorean triplet. Moreover, if $a^2 - b^2$ is add, then the triplet will be a primitive one.

(2) **Rule of Pythagoras**

Pythagoras discovered a rule for generating a Pythagorean triplet from an odd number. The rule states that the square of the odd number has to be divided into two parts such that one part is 1 larger than the other. Then, the odd number itself and these two parts will be a Pythagorean triplet. This means that for any natural number n, $(2n + 1)$, $(2n^2 + 2n)$, $(2n^2 + 2n)$ will be a Pythagorean triplet. This rule was used by Proclus in about 460 BC.

(3) **Euclid's Rule**

If, (i) a, b are two even numbers such that they have no common factor greater than 2 or (ii) a, b are two odd numbers and in either case ab is a square number then $\sqrt{ab}$, $\frac{a-b}{2}$, $\frac{a+b}{2}$ is a Pythagorean triplet. For instance, if $a=25$ and $b=1$, then we get (5,12,13) as Pythagorean triplet.

(4) **Master's Rule**

If a and b are either two odd or two even numbers such that $a > b$ and $\frac{a^2 + b^2}{2b}$ is a natural number, then $\left(a, \frac{a^2 - b^2}{2b}, \frac{a^2 + b^2}{2b}\right)$ is a Pythagorean triplet. For example, if a=9 and b=3 then we get (9,12,15) as a triplet.

(5) **Dixon's Rule**

If a, b ($a > b$) are prime to each other and one of them is odd while the other is even and $2ab$ is a square number then we get $\left(a + \sqrt{2ab}, b + \sqrt{2ab}, a + b + \sqrt{2ab}\right)$ as a Pythagorean triplet. For instance, if $a = 9$ and $b = 2$ then the Pythagorean triplet is (15,8,17).

Finally, it should be mentioned that all other rules except that due to Brahmagupta have limitations. Although Dixon's rule is more general than that of Pythagoras, Euclid and Maseres, but Brahmagupta's rule is most general.

Utpal Mukhopadhyay

I have read the article "Inverting matrices constructed from roots of unity" in the August 2006 issue of *Resonance*. The theorem, and the analogue for cyclic groups was indeed interesting. My research and teaching are in the area of system theory and I could immediately identify the "Discrete Fourier Transform" we talk about in digital signal processing. From this perspective, I would like to add that the symmetric Vandermonde matrix M constructed from roots of unity has another interesting property that the pair of columns j and j+1 are orthogonal, and hence we might readily expect that the inverse of M is M itself, of course with some suitable scale factor. Noticing that the magnitude of each column vector is $\sqrt{n}$, we may fix the scale factor and thus the inverse of M is 1/n times M.

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