C S Smith's Development of a Viewpoint

Complex Ideas and their Demonstration in the 2D Soap Froth

Denis Weaire

Cyril Smith's distinguished career in industrial metallurgy was wonderfully transformed into his masterful engagement with the history of science and technology. These two phases were bridged by a piece of research that particularly pleased him, as epitomising his general ideas of form and organisation. It was based on his conception of the 2D soap froth as a prototype for metallic grain growth (and much else). He rightly sensed that it would be an enduring source of understanding of some of the complex effects that he so admired in materials.

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Following a Philomorph

Invited to address the American Philosophical Society in Philadelphia as a member of a Royal Society delegation, I told of the grounding of my recent research in that of Cyril Stanley Smith, and described myself as a *philomorph.* I am not a member of the club, but I certainly adhere to it in spirit.

I have other things in common with him, including a British education, and a sojourn in the US (including Cambridge and Chicago, at the Institute that he founded). He was a fine example of the cross-fertilisation of those two English-speaking cultures, an Alistair Cook of science.

My talk at Philadelphia [1] may be consulted for some details of the path along which he pointed my way, which now leads all the way to the Beijing Olympics [2]. Here I shall concentrate on some of his general ideas, and the way in which his two-dimensional soap froth became a cornerstone of the physics of foams.

Keywords
Bubble, soap froth, grain growth, philomorph.

Key Ideas

Smith appreciated the freedom that two great universities gave him in later life. It allowed him to let his curiosity roam freely and playfully, and to publish impressions and suggestions that were only half-formed. Much of his writing is tentative, outlining a mere sketch of a design yet to be fulfilled, or as he put it himself "experiments in the development of a viewpoint" [3]. Try writing that as a grant proposal...

In part he was driven by a reaction against the narrow reductionism of his day. In his own discipline it took the form of a concentration on the perfect (or near-perfect) single crystal. Such rarified solid state physics gave us the silicon age, so its positive achievements can hardly be discounted. But Smith saw that there was much more in heaven and earth than dreamed of in that philosophy. It was as if a great microscope had been focussed on just one corner of a mighty tapestry, of infinite variety and complexity.

I wonder if he ever recognised a soul-mate in the poet Hopkins (*Glory be to God for dappled things...*) with whom he shared a sense of wonder at complexity and individuality in nature. They came to the same middle ground from opposite sides of the arts/science (and mystical/rationalist) divide.

Like Hopkins, Smith was a man out of tune with current fashion. His work was a wine that – like much good poetry – needed to be kept for a generation. Whole subjects have now developed around topics that he grasped at. The theories of *chaos* and *fractals* and indeed *complexity* are some of them. This has been brought about not just by a change of heart among scientists but by the power of the computer, about which – in its more primitive form – Smith was somewhat sceptical. He did not see, as did his colleague John von Neumann, that the computer would release us from the narrow confines of that part of mathematics that is tractable in practice (roughly speaking, linear theory).

A Search for Structure

The collection of essays *A Search for Structure* contains representative excerpts from his later work. In those parts that deal with his essential ideas he repeatedly returns to certain sweeping themes, as keys to understanding complex systems:

- hierarchical organisation, in which each level has its own laws but interacts with those above and below
- change as a process of local nucleation and growth;
- hidden order (usually of a statistical character) within disorder;
- the history dependence of structure and properties.

He saw these ideas as relating to more than the materials science for which he advocated their wider recognition.

Funicity

The science of the twentieth century took too seriously the equilibrium thermodynamics of the previous one. This attributed to every system a unique state dependent on its conditions (such as temperature). In such a state it lives in ignorance of its previous existence. But many real physical systems are, like ourselves, largely products of their experience. In the case of metals, they may have been heated, stretched, and beaten in the forge of the smith. (Has anyone noted this appropriate name for our subject, a man who forged ideas?)

So in reality, he insisted:

A complex structure is the result of and to a large extent the record of its past.

He coined a word for this property of dependence on history. It was to be *funicity* (hence *funeous* and *afuneous*), after a character in a short story of Borges. It deserves to be used. Let us not forget to do so.

Whereas this was in the main a message to science of something that is commonplace in the arts, that which dealt with nucleation was rather the reverse. This has long been a well recognised and difficult aspect of the science of materials. Many changes of state, particularly crystallisation, require an initial nucleus, a starting point for growth of the new order. Its occurrence may be a rare and chance event, and inadequate nuclei will fail to grow.

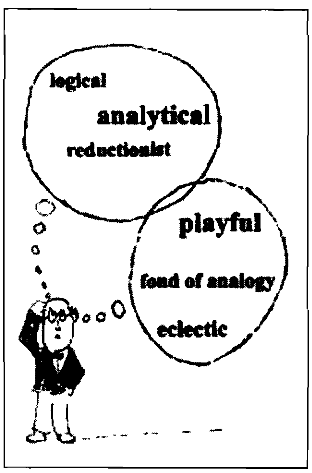
Here Smith's message was more directed towards the other side of the two cultures, pointing to the role of chance and inscrutable local circumstances in accomplishing change, and quoting Shakespeare (whose choice of the word *grains* must have made him chuckle):

If you can look into the seeds of time and say which grains will grow and which will not, speak then unto me.

Personal Advice

Smith disliked over-specialisation, although he firmly believed that one must first be deeply immersed in at least one subject. He advised that theorists should do experiments, and duly admonished me to that effect in a letter. To which instruction (*go out and buy a kitchen blender*) I quickly conformed, with happy consequences.

Figure 1. The author's personal conception of Smith at the time of their correspondence, which encouraged him to use both sides of his brain together.



He also thought we should be unashamed of a sense of fun and a propensity to play – hence my cartoon of him (*Figure* 1), conceived at the time of our correspondence.

A Dilettante?

All this could be open to criticism as tending to mere dilettantism and shallow speculation, and a return to the abandoned tradition of the early natural philosophers with their cabinets of curiosities and purely descriptive reports of this and that.

Perhaps that is why he was particularly proud of one achievement that was a piece of hard science cutting to the heart of many of the vague principles that he espoused, and epitomising many of the properties and principles that so preoccupied him in a single point of focus. It was his conscious choice as the only strictly scientific work that he retained in his essay collection [2].

The 2D Soap Froth

Even in the face of great complexity we must try to simplify, to strip out extraneous or incidental complications and retain the essence in an idealised model and/or prototypical physical system. That is part of the scientific method.

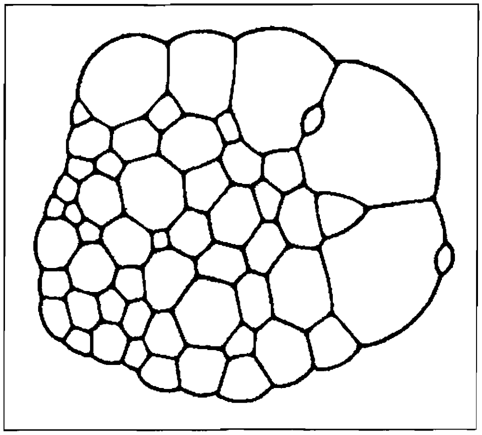


Figure 2. Smith's 2D soap froth.

Smith advanced the 2D soap froth (*Figure* 2) in that spirit, as the best representative of many cellular systems controlled by surface energy (or tension). Its merits include its two-dimensional nature, which was to facilitate measurement and analysis and eventually also simulation by computer. It is to be commended to teachers at all levels, being easy, cheap, safe and quite fascinating.

Made by squeezing an ordinary soap froth between two glass plates, it is first cousin to the soap bubble raft, with which Lawrence Bragg made great play at much the same time. But whereas Bragg's floating bubbles were to represent atoms (and were generally of the same size for that reason), Smith's packed, flattened bubbles, of random sizes, formed polygonal cells more akin to the grains of a polycrystalline metal, and indeed they behaved like those grains, especially in regard to grain growth.

Lawrence Bragg's "Bubble raft" was a prelude to Smith's soap froth for modelling metal structure and microstructures. The figure shows atoms being modelled as bubbles. Vacancies, dislocations and grain boundaries among the "bubble crystals" can be seen.
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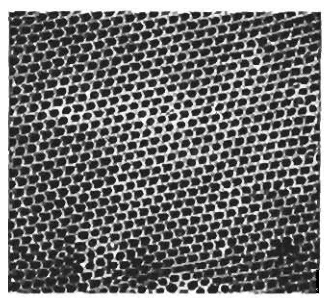


Grain Growth

Grain growth is an important process in metallurgy and is promoted by high temperatures. Atoms migrate between grains, with the effect that larger ones grow at the expense of the smaller ones. Hence grains progressively disappear, so that their average size increases. It is a telling metaphor for some aspects of society.

The process is driven by the reduction of (free) energy by reduction of surface area. In such terms, it is a little different from what happens in a soap froth. In that case molecules diffuse through the films between cells of different pressures. The details are different but the overall effect and the underlying laws are much the same; it is the reduction of surface energy that is at work.

Smith had a favorite trick to speed up such coarsening – he lowered the pressure. A simpler one (for 3D foam) is to use a bottle of fizzy mineral water. Recipe: add a dash of dishwashing fluid to half-filled bottle. Warm gently. Shake the open bottle and cap it as soon as the foam rises to fill it. Contrast with a foam made with plain water, which coarsens more slowly. Why?

Discussion on C S Smith's Paper

Cyril Stanley Smith gave a lecture on grain shape and growth to the American Society for Metals in 1952. John von Neumann attended this lecture and contributed a short but brilliant discussion. He arrived at the remarkable result that the growth or shrinkage rate of a 2D bubble in a dry foam does not depend on its shape or size. It is dependent only on the number of edges. This is a very well cited law for two dimensional grain growth. Its extension to three dimensions is a challenge and is attracting considerable attention now. It may be recalled that von Neumann was a major figure in the Manhattan Project and contributed to the computation of the implosion of the atomic device. He is also well known for his contributions to computer science, economics and game theory. He is credited with coining the word 'materials science' and gave it considerable support.

Ref: *J von Neumann* in *Metal Interfaces,* American Society for Metals, Cleveland, p.108, 1952.

Scaling

Where this process of coarsening will eventually take the foam is not at all self-evident.

Will it

- go to some *equilibrated state* (as some thought at the time), or
- continue to coarsen relentlessly (for an infinitely large sample) as a *scaling state* (explained below), or
- exhibit some other, more exotic, statistics with divergent length scales of the small and the large, that is, a *fractal*?

After a while, Smith observed the second alternative. All the (statistical) characteristics of the grain structure remain the same except that its length scale varies, and average cell diameter increases in proportion to the square root of time.

Those preliminary findings have survived as sound science, but not without alarms and diversions as well as some additions (an important one coming from an unexpected quarter when von Neumann took an interest in the matter).

I was responsible for some of the alarms, after my attention was drawn to the intriguing work of David Aboav. That was my first placement of my toe in those soapy waters, around 1973 in the Yale Library, about two decades after the 2D soap froth was born.

Aboav's Contribution

David Aboav conducted research at home in a private capacity after a career that had included an appointment as Assistant to Eduardo Andrade, the one time Director of the Royal Institution. Andrade was an eccentric figure, in some ways similar to Smith, but with a more self-conscious aspiration to be a Renaissance man. He wrote a good deal of flowery poetry, including one piece in praise and defence of humble bubbles: its condemnation of those fellow scholars who admire only "solidity" comes very close to what Smith declared more prosaically about the study of the solid state.

The exact line of descent from this to Aboav's work is something I have not tried to trace, though it surely exists. His first and most lasting insight was gained from polycrystalline sections, for which he found a statistical relation for the correlation of numbers of sides of neighbouring cells – Aboav's law. Realising that 2D froths provided an even better test-bed for his relation, he obtained a time series of photographs from Smith, and set to work on them. Eventually there emerged a refined form of the law, which seems to have rather general (and quite puzzling) validity.

But Aboav also analysed the coarsening of the Smith samples. One might describe the consequences as unfortunate, were it not that science, like exploration, usually profits from false trails and misconceptions. Certainly in this case I was driven to great efforts by the bizarre nature of Aboav's conclusions.

For it seemed that Smith's scaling state did not exist after all. The new analysis pointed (at least for me) to more exotic behaviour. Exactly what, and why?

Simulating Coarsening

It was primarily this question that led Paul Kermode and me to formulate an accurate computer simulation in 1984, the first full simulation of the standard idealised model of the 2D soap froth, generating pictures rather like *Figure* 2. While it worked better than we had any right to expect, the computational limitations of that time confined its results for coarsening to such short times and small samples that it remained inconclusive on the key point at issue. More approximate simulations by our group supported Smith, but we awaited a more definitive conclusion, and an explanation of the false trail innocently laid by Aboav.

The simulations went on to teach us many things (and in due course to confirm Smith's version of the analysis), but the immediate problem was to be settled by experiment, as Smith would have advocated, in the very Institute that he had founded in Chicago.

Two research students, Joel Stavans and James Glazier, working under Libchaber, became attracted to the issues that I had raised. They performed extensive experiments in the manner of Smith, conveniently using the newly invented photocopier to record their pictures. They showed that if one began with a sample consisting of nearly monodisperse (uniform size) bubbles, growth occurred only around defects. A long transient period was required to establish the eventual disordered scaling state.

It indeed transpired that Aboav had received from Smith a set of photographs from an experimental run much later than the original one, and these had never been analysed by Smith himself. The initial samples were indeed rather monodisperse.

Furthermore Aboav had (somewhat selectively) taken data only from the earliest part of the transient regime. Contrary to the advice that we regularly impart to the *ingenu* student but often do not follow ourselves, he had set aside that part of the data that did not fit a simple relation of his choosing. We are, of course, supposed to fit the theory to the facts and not the reverse.

Suggested Reading

[1] Denis Weaire, A Philomorph looks at Foam, *Proceedings of the American Philosophical Society*, Vol.145, p.564, 2001.

[2] http://www/tcd/ie/Physics/Foams; http://aia.org/SiteObjects/arup.pdf

[3] Cyril Stanley Smith, *A Search for Structure*, MIT Press, Cambridge MA, 1981.

[4] Tomaso Aste and Denis Weaire, *The Pursuit of Perfect Packing*, IOP Publishing, 2000. (Second Edition in preparation).

Soap Froth for its Own Sake

In recent decades the physics, chemistry and engineering of foams has developed a coherent community of adherents, with biennial European conferences reviving a venerable topic that attracted many great names in the distant past. For this foam community, the figurehead has been de Gennes, Nobel Prize winner for his many contributions to the physics of soft matter.

Smith's 2D froth is no longer a mere stand-in for a polycrystalline solid. It is the natural starting point for a comprehensive theory of foams and emulsions that links properties to structure. This takes off where the blind Belgian Joseph Plateau left the subject in the 19th century, with a clear understanding of local static equilibrium. It moves on to more global effects (coarsening, drainage, rheology) and eventually more dynamic ones.

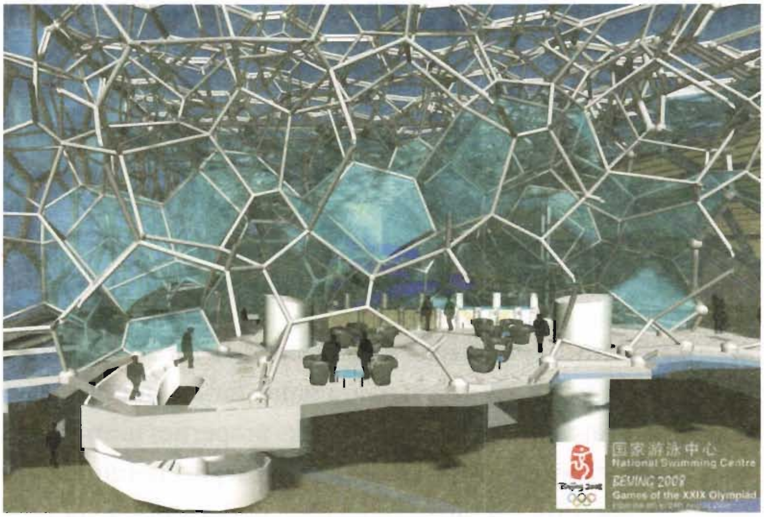
Without the experience of success in the research program that Smith launched, we would have been much more wary of attempting these things in three dimensions, as we do today.

Conclusion

As a renegade from more conventional condensed matter physics, and a dabbler in science history as well, I owe much to those philomorphic letters from Boston and New Hampshire. Their advice has taken me to interesting places, and continues to do so [4].

A year or so ago, we constructed a sculpture (*Figure* 3) that Smith would surely have liked. Crafted in stainless steel (itself a source of fascination for him), it represents the cells or bubbles of the Weaire–Phelan ideal foam structure. Originally this design was suggested for a building, such as a planetarium, exhibition centre or theatre,

Figure 3. Art meets science in a sculpture representing the Weaire–Phelan structure for foam. Included are Dave Grouse (technician, working skillfully in the metallurgical craft loved by Smith), the author (who conceived the original design with Robert Phelan) and Wiebke Drenckhan, who directed the project.



Foams present a fascinating variety of interdisciplinary challenges to our understanding, from the mathematics of minimal surfaces to the behaviour of the head on a pint of beer. They go back in history to Kelvin and Plateau (and beyond), and also forward to 2008 when the Beijing Olympics will feature an astonishing manifestation of the theory of foams – the Weaire–Phelan figure on a colossal scale. This bears comparison with the Buckminster Fuller Dome often used in stadia.

Courtesy:Consulting engineers Arup and partners,PTW Architects, the China State Construction and Engineering Corporation and the Shenzhen Design Institute, who provided the architectural images.

but it has never been realised in that form (although a different representation is now to be writ large on the Beijing skyline [2]...). A smaller version of the sculpture is now in the collection of the Science Museum, South Kensington.

If somebody wants to create the Smith Center for the Arts and Sciences (assuming it does not already exist), here is a ready-made concept. It can be extended in various ways, but the present combination of one 14-sided cell and two halves of 12-sided cells seems appropriate and practical.

It would indeed be good to commemorate him in a definite *form*, of whatever kind. We may need to be explicitly reminded of his subtle but pervasive influence by a bold building that brings to mind his *Search for Structure*.

Or, if you count yourself a philomorph, you could put it up in your garden.

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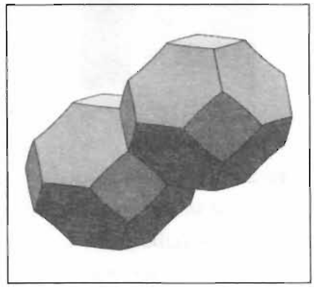
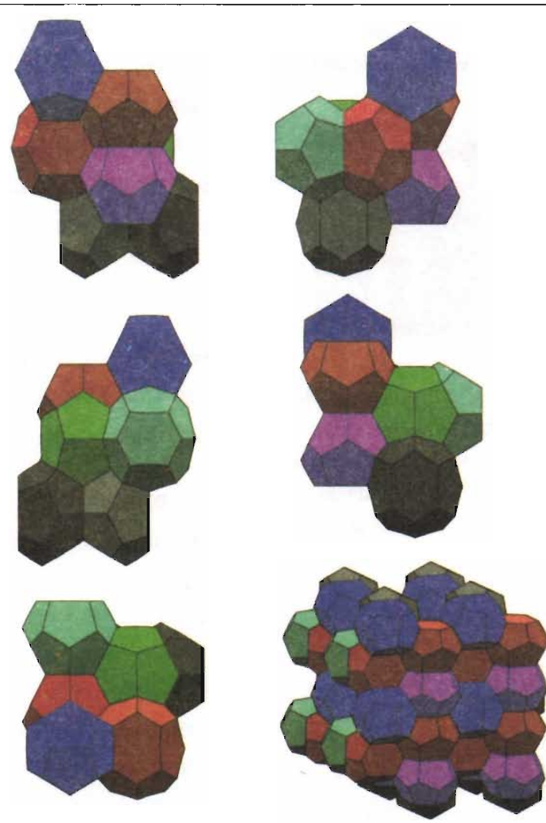
Lord Kelvin and Soap Froth

In 1887, during his research into the 'ether', Lord Kelvin proposed the following problem: *What is the most economical way of partitioning space with equally sized cells?* It is equivalent to the total area of the interfaces between the cells being a minimum. The best partition that Kelvin could come up with was made of slightly curved 14-sided polyhedra. Two of Kelvin's tetrakaidecahedra are pictured here:



For over a century, no one could improve upon Kelvin's partition. Then in 1993, Denis Weaire and Robert Phelan proposed a partition of space into two kinds of cells (of equal volume, of course) that beat Kelvin's partition by 0.3% in area. Several views of the Weaire–Phelan partition are shown below. A fundamental region of 8 different colored cells is shown. Two cells (green and blue) are dodecahedra, and the other six are 14-sided with two opposite hexagonal faces and 12 pentagonal faces. The 14-sided cells stack into three sets of orthogonal columns, and the dodecahedra fit into the interstices between the columns.



There is no proof that the Weaire–Phelan partition is optimal, or that Kelvin's partition is optimal for a single shape of cell.

Suggested Reading

[1] **W Thomson, (Lord Kelvin), On the division of space with minimum partitional area, *Phil. Mag.*, Vol. 24, p.503, 1887.**

[2] **D Weaire and R Phelan, A counterexample to Kelvin's conjecture on minimal surfaces, *Phil. Mag. Lett.*, Vol. 69, pp.107-110, 1994.**

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