Joseph Louis Lagrange (1736 - 1813)

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Lagrange, the brilliant 18th century mathematician, who made seminal contributions to diverse areas of mathematics, is best remembered for his work on number theory, algebra, calculus of variations and celestial mechanics. His work is also renowned for its almost poetic beauty and elegance. The impact of his reformulation of classical mechanics is considered to be on par with that of Sir Isaac Newton.

Joseph Louis Lagrange was born on 25 January 1736 in Turin, Sardinia – Piedmont (which is now a part of Italy) and was baptised with the name of Guiseppe Lodovico Lagrangia. He was the eldest of eleven children of his parents, but only two of them lived to adulthood.

Lagrange's family had French connections on his father's side and Lagrange always leaned towards his French ancestry. As a youth, he signed as Lodovico LaGrange or Luigi Lagrange, using the French form of his family name. Although Lagrange's father was in the service of the King of Sardinia, the family was not wealthy, as the father had lost large sums of money in speculative deals. He also chose that Lagrange become a lawyer and so Lagrange studied (among other subjects) classical Latin at the College of Turin. Lagrange's interest in mathematics was aroused when he read a copy of Halley's work on the use of algebra in optics. He was also inspired by the brilliant teaching of physics by Baccaria. He then decided to make a career in mathematics. He later claimed as the reason for his change of career, "If I had been rich, I probably would not have devoted myself to mathematics."

Lagrange was to a large extent self taught and did not have the advantage of attending lectures by masters in the field. In his first mathematical work, he drew an analogy between the binomial theorem and the successive derivatives of the product of functions. He began working on the tautochrone, the curve on which when a body moves under gravity, it will always arrive at the lowest point in the same time, independent of the initial position. By the end of 1754, he had obtained some important results on the tautochrone, which would contribute to the fledgling subject of the calculus of variations.

Lagrange sent his results on the tautochrone, containing his method of maxima and minima, to Euler, who at that time was working in Berlin. Euler was impressed with Lagrange's new ideas. Although he was only 19 years old, Lagrange was appointed Professor of Mathematics at the Royal Artillery School in Turin in 1755. Euler proposed Lagrange's name for election to the Berlin Academy and he was duly elected in 1756. His publications at this time included results on the calculus of variations and probabilities. In a work on the foundations of dynamics, Lagrange based his development on the principle of least action and kinetic energy.

Lagrange made a major study on the propagation of sound. He used a discrete mass model for the vibrating string problem, consisting of n masses joined by weightless strings. He solved the resulting $(n + 1)$ differential equations and then let n tend to infinity to obtain the same functional solution as Euler. His innovative approach showed that he was looking for methods other than those of Euler, for whom Lagrange had the greatest respect. Lagrange also worked on the application of differential equations to various areas, including classical mechanics and fluid mechanics, where he introduced the Lagrangian function. Another problem where he applied his methods was to the study of the orbits of Jupiter and Saturn.

D'Alembert, who had visited the Berlin Academy and was friendly with Frederick II of Prussia arranged for Lagrange to be offered a position in 1765 there. Despite the fact that Turin had made no improvement in Lagrange's position, he turned down the offer to go to Berlin, writing "It seems to me that Berlin would not be at all suitable while M. Euler is there." When Euler returned to St. Petersburg, he encouraged Lagrange to accept his post as Director of Mathematics at the Berlin Academy of Science in 1766. For the next 20 years in Berlin, Lagrange produced a number of papers on astronomy, the stability of the solar system, mechanics, probability and the foundations of calculus. He won prizes at the competition of the Académie des Sciences of Paris – he shared the 1772 prize on the three body problem with Euler, the 1774 prize, one on the motion of the moon and the 1780 prize on the perturbations of the orbit of comets by the planets. He also worked on number theory proving in 1770 that every positive integer is the sum of at most four squares. In the same year, he obtained results on why polynomial equations of degree up to 4 could be solved by radicals, an investigation for the first time considered roots of an equation as abstract quantities, rather than having numerical values. In 1771, he proved Wilson's theorem that n is prime if and only if $(n + 1)! + 1$ is divisible by n.

After the death of his wife in 1783 and the death of Frederick II in 1786, Lagrange left Berlin in 1787 to accept the offer to become a member of the Académie des Sciences in Paris, where he remained for the rest of his career.

In 1788 the Mécanique Analytique, written by Lagrange in Berlin, was published by the Académie and this summarised all the work done in the field of mechanics since the time of Newton. With its use of differential equations, the work transformed mechanics into a branch of mathematical analysis.

It was a time of political unrest in France with the French revolution followed by the Reign of Terror. All foreigners born in enemy countries were arrested. Lavoisier intervened for Lagrange and he was granted a reprieve, though Lavoisier himself and 27 others were put to death later.

In 1794, Lagrange was appointed the first professor of analysis, when the École Polytechnique opened. In 1795, when the École Normale was founded, Lagrange taught courses on elementary mathematics for training school teachers. During this time, he published two volumes of his calculus lectures. In 1797, the first theory of functions of a real variable to give "the principles of differential calculus, freed from all consideration of the infinitely small or vanishing quantities, of limits or fluxions, and reduced to the algebraic analysis of finite quantities" was published. The second work appeared in 1800.

Lagrange was awarded the Legion of Honour and the title of Count of the Empire by Napoleon in 1808. On 3 April 1813, he was named Grand Croix of the Ordre Impérial de le Reunion. He died a week later in Paris.

Suggested Reading

[1] The MacTutor History of Mathematics archive. http://www-history.mcs.st-andrews.ac.uk

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