Āryabhaṭa and Axial Rotation of Earth

1. Khagola (The Celestial Sphere)

Amartya Kumar Dutta

Āryabhaṭa (born 476 CE) is regarded as a pioneer of mathematical astronomy in ancient India. In this three-part article, I shall discuss one important contribution of Āryabhaṭa in astronomy involving the least technical background – the principle of axial rotation (Part 1) and computations on the sidereal day (Part 2). Portions of 4 verses, pertaining to Earth's rotation, from his famous treatise Āryabhaṭīya will be quoted. In Part 3, I shall give a brief historical account on the concept of axial rotation of Earth.

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The Āryabhaṭīya ('composition of Āryabhaṭa') is designed as a concise text of 121 verses briefly presenting important principles in astronomy and mathematics in the style of terse aphorisms. This highly condensed treatise is not meant to provide complete or detailed expositions. Various facts or methods, which were perhaps well-known in his time, are taken for granted in the text. Therefore, a major portion of this article will introduce certain basic features of spherical astronomy in modern language to facilitate discussions on the quoted verses. There will be incidental references to other statements of Āryabhaṭa.

The Āryabhaṭīya is divided into 4 sections (*pāda*). In the first and introductory section *Gītikā* (13 verses), there are ten verses called *Daśagītikā-sūtra* (ten aphorisms in the gītikā metre) summarising, for the beginner, the essential parameters of Āryabhaṭa's system. The remaining three sections (total 108 verses), that constitute the main text, are *Gaṇita* (mathematics): 33 verses,

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Box 1. Āryabhaṭa or Āryabhaṭṭa ?

The celebrated astronomer-mathematician is popularly known as 'Āryabhaṭṭa' in Indian languages. Early Indologists like H T Colebrooke (1817) too spelt the name with double 'ṭ'. However, as Bhau Daji (1865) pointed out ([7], p 518), the Sanskrit spelling of the name, as found in the manuscripts of Brahmagupta and other ancient Indian astronomers, corresponds to 'Āryabhaṭa'. The situation was aptly summarised by W E Clark (1930) in ([8], vii-viii):

"There has been much discussion as to whether the name of the author should be spelled Āryabhaṭa or Āryabhaṭṭa. *Bhaṭa* means 'hireling,' 'mercenary,' 'warrior,' and *bhaṭṭa* means 'learned man,' 'scholar'. 'Āryabhaṭṭa' is the spelling which would naturally be expected. However, all the metrical evidence seems to favour the spelling with one ṭ. It is claimed by some that the metrical evidence is inclusive, that *bhaṭa* has been substituted for *bhaṭṭa* for purely metrical reasons, and does not prove that Āryabhaṭa is the correct spelling.... However, until more definite historical or metrical evidence favoring the spelling Āryabhaṭṭa is produced I prefer to keep the form Āryabhaṭa."

Historians of Indian science have adopted 'Āryabhaṭa' as the official spelling.

Kālakriyā (reckoning of time): 25 verses and *Gola* (celestial sphere): 50 verses.

The original verses of Āryabhaṭīya (along with English translations) are given in ([1]). One can see ([2]) for a general student-friendly guidance and ([3],[4]) for overview articles on Indian astronomy, ([5]) for a compilation of ancient Sanskrit verses (with English translations) on various concepts, theories, techniques and instruments in astronomy, and ([6]) for a standard modern treatment on spherical astronomy.

The Principle of Rotation

The great discovery that the Earth rotates around its own axis from west to east is recorded in the Āryabhaṭīya (Gītikā 3,6; Kālakriyā 5; Gola 9,10). The young astronomer boldly declared that the apparent motion of the heavenly bodies round the Earth is only an illusion (Gola 10). He explained it using the following simile (Gola 9):

anulomagatirnausthaḥ paśyatyacalaṁ vilomagaṁ yadvat
acalāni bhāni tadvat samapaścimagāni laṅkāyām

[*anuloma* : in the natural direction; *gati* : moving; *nau* : boat; *stha* : being in; *paśyati* : sees; *acala* : stationary, mountain; *viloma* : against the natural direction; *ga* : moving; *yadvat* : just as; *acalāni* : stationary (plural); *bhāni* : stars; *tadvat* : likewise; *sama* : same, equal, exact, similar (also complete, whole, entire); *paścima* : west; *gāni* : moving (plural); *laṅkāyām* : at Laṅkā.]

Just as a passenger in a boat moving downstream sees the stationary (trees on the river banks) as traversing upstream, just so does an observer at Laṅkā see the fixed stars as moving towards the west at exactly the same speed (at which the Earth moves from west to east).

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Further, as we shall see in the next part of this article, he made an accurate estimate of the time taken by the Earth for one complete rotation.

Note that Laṅkā, in the quoted verse, does not refer to modern Sri Lanka. It denoted the point on the equator south of Ujjayinī (23.09 N, 75.43 E), i.e., the point at which the meridian through Ujjayinī (75.43 E) intersects the equator. (Thus the point is now on the Indian Ocean near the Maldive islands.) But why the reference to Laṅkā? We first define the celestial sphere and its relevant circles mentioning the ancient Indian analogoues.

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The Celestial Sphere

The sky appears to us as the upper half of a large hollow sphere. The stars and planets appear as luminous points on the inner surface of this upper hemisphere. The complete sphere seems to meet the Earth's surface in a circle which we call the (visible) horizon. This imaginary hollow sphere of an arbitrarily large radius with the observer at the centre is called the *celestial sphere.* The radius of the celestial sphere is usually conceived to be sufficiently large so that the entire Earth can be regarded as a point at the centre of the huge sphere.

Although the stars are scattered in space at different distances from the Earth, they all appear equally remote to an observer – the differences in their distances are not perceptible to ordinary observation. Thus the apparent position of a star on the imaginary celestial sphere signifies only the *direction* of the star from the observer; nothing else is indicated about its position in space. Two stars will be very close on this sphere when they have nearly the same direction even if one of the stars is physically much more remote than the other.

The apparent distance between two celestial bodies is simply the difference in their directions and is measured by the angle subtended at the observer by the two objects. 'Spherical astronomy' examines this angular distance between objects on the celestial sphere. Since the actual linear distances between celestial objects are not of much relevance here, the expression 'angular distance' is often abbreviated to 'distance' when there is no scope for confusion. The angular distance is measured in degrees, minutes and seconds. Recall: $1^0 = 60'$ and $1' = 60''$ Thus, "star X is at a distance of $30'$ from star Y" means that the directions of the two stars X and Y from a terrestrial observer make an angle of $\frac{1}{2}$ degree.

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The geocentric model of celestial sphere, presenting the universe *as it appears* to the observer, provides a convenient framework for the computational study of those astronomical phenomena which essentially involve the directions of celestial objects when viewed from the Earth. Modern texts on basic astronomy, therefore, begin with a chapter on the celestial sphere. In standard planetarium shows, the relative positions (i.e., directions) of celestial objects are depicted on a dome representing the celestial sphere.

Observers at different places on Earth get different pictures of the celestial sphere. This is because the horizontal planes at two distinct points on the Earth's surface are different. Two observers at different spots see different portions of the sky. Moreover, even if a star is seen by both observers, its orientations towards the two horizons are different.

For a simultaneous analysis of the situations at different places, it would be desirable to envisage a common centre of observation. The centre of the Earth is a natural choice. Therefore, while it is undoubtedly convenient to consider the sphere centred at the observer at a place O, it might also be necessary to reduce the observed directions of celestial bodies (as observed from O) to the corresponding *geocentric* directions – the directions of the celestial objects that would be observed by a hypothetical observer at the centre of the Earth C positioned along CO (i.e., parallel to the observer at O). The angle between the observed direction of a heavenly body X and its geocentric direction, i.e., the angle subtended by CO at X, is called the *geocentric parallax* of X (*Figure* 1).

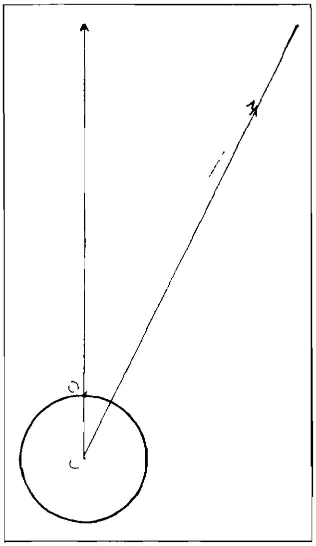

Figure 1. Geocentric parallax.
OX: Observed direction
CX: Geocentric direction
∠CXO: Geocentric parallax.

Now each star is at such an enormous distance from the Earth that its geocentric parallax is practically zero. Therefore, as mentioned in the first paragraph of this section, C and O may be treated as the same point while mapping the stars.[1] However, while considering objects which are relatively near – like the Moon or even the Sun – one cannot be so dismissive about the Earth's radius. At the horizon, the geocentric parallax of the Moon is about 57′ while that of the Sun is 8″ Thus, while studying the orbits of the Moon, the Sun and the planets, at different places of the Earth, corrections for

[1] For observers in distinct positions O_1 and O_2, even though the directions O_1X and O_2X of a star X coincide, the celestial coordinates of X are different. *Figure* 2 shows a star X to be simultaneously at the horizon of O_1, zenith of O_2 and nadir of O_3.

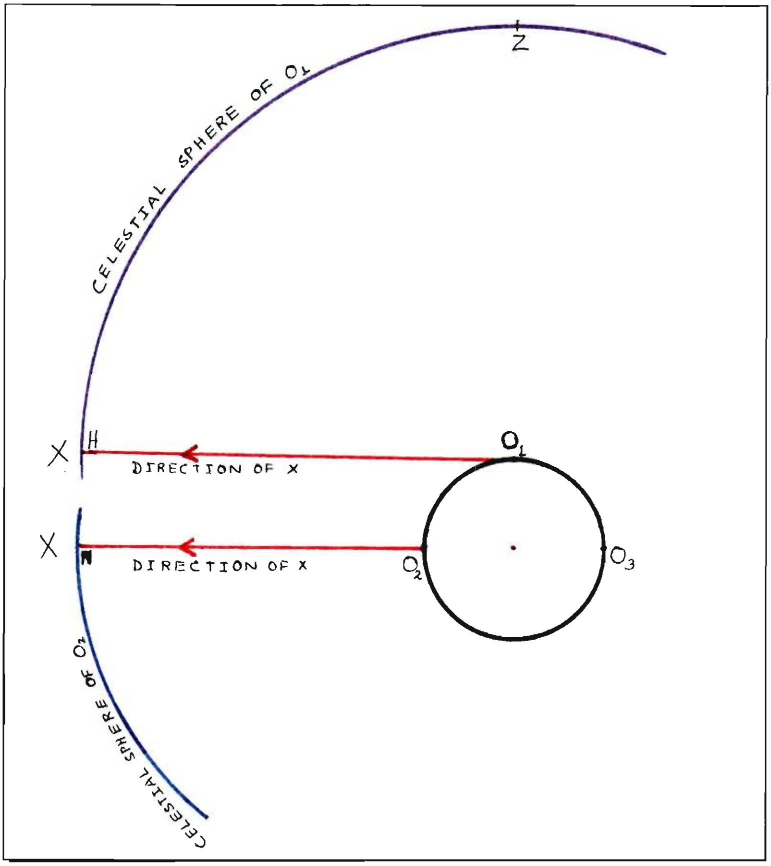

Figure 2. Coordinates of a star X for different observers O_1, O_2, O_3.
Z: Zenith of O_1
H: At horizon of O_1 (north point/south point)
N: zenith of O_2 and nadir of O_3.

the geocentric parallax have to be employed for accuracy.

The Indian astronomers conceived of the celestial sphere and analysed it in detail. Rotating spheres were constructed as concrete models for the celestial sphere. The Sanskrit terms for the celestial sphere are *bhagola*, i.e., sphere of the stars [*bha* : star or planet; *gola* : sphere], and *khagola*, i.e., hollow sphere or sphere of the sky or celestial sphere [*kha* : hollow, sky, heaven]. The term *bhagola* was used for the celestial sphere centred at the Earth's centre, while *khagola* denoted the sphere centred at the observer. Āryabhaṭa began the section "Gola" with a brief description of the *bhagola* and the *khagola* and used them to demonstrate the motion of the celestial bodies – the *bhagola* was used for describing the

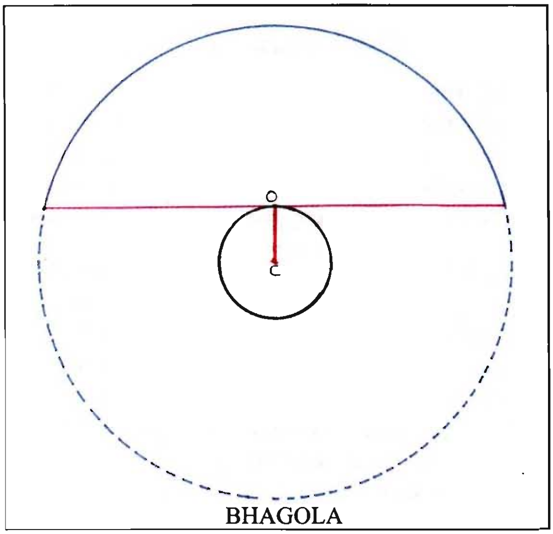

Figure 3. Bhagola.
O: Observer
C: Centre of the Earth as well as Bhagola
CO: radius of the Earth
__: Visible portion of Bhagola
- - - : Invisible portion of Bhagola.

motion of the Sun, the Moon, and the planets in their orbits; the *khagola* for the apparent daily motion of the heavenly bodies due to Earth's rotation ([1], p 113). In Gola 15, Āryabhaṭa explained: "One half of the *bhagola*, diminished by the Earth's radius, is visible from a level surface. The (view of) other half, increased by the Earth's radius, is cut off by the Earth." (see *Figure* 3). The focus on *bhagola* for the Sun, the Moon, and the planets, is perhaps a reflection of the concern for accuracy (recall geocentric parallax). For simplicity, we shall not distinguish between the two spheres.

Axis of Rotation and Celestial Poles

At a given place, the picture of the celestial sphere changes slowly but continuously with time. The Earth rotates from west to east around the line joining the geographical north and south poles. It is due to this rotation that an observer on Earth sees the system of "fixed stars" (see *Box* 2 for clarification of the term "fixed") as revolving from east to west around a line through the observer parallel to the Earth's axis of rotation (*Figure* 4). It is as though the celestial sphere is a rotating hollow globe with each of the fixed stars firmly stuck on its inner surface. Since the radius of Earth is negligible compared to

At a given place, the picture of the celestial sphere changes slowly but continuously with time. The Earth rotates from west to east around the line joining the geographical north and south poles.

Box 2. The Fixed Stars

To an observer on Earth, the relative positions of the stars (excluding the Sun) *appear* to be fixed; i.e., the angle subtended (at an observer) by two such stars does not seem to change with time. Hence, the stars (other than the Sun) are referred to as "fixed stars" to distinguish them from other star-like celestial objects (e.g., planets). Ancient Indian astronomers used the term *nakṣatra* for a fixed star and *tārāgraha* for a planet.

We add a caveat regarding the adjective "fixed". In reality, the stars travel in space in different directions at different velocities. However, all stars (other than the Sun) are several "light years" away from the Earth. Due to the enormous distances, the relative angular displacements between the stars (excluding the Sun) become too minute to be noticed by the unaided human eye. Only around 200 stars are known to have an angular motion (relative to the rest) at a rate exceeding 1 second (i.e., $\frac{1}{3600}$ degree) per year! But such tiny shifts in relative positions can no longer be ignored when one considers huge time intervals. For instance, the celebrated "Saptarṣi Maṇḍala" now appears in the shape of a question mark (?) or a hook (see [2], p 20). An observer is unlikely to discern any change in this shape during a lifetime; but after, say, a million years, the shape would become considerably different from what it is now!

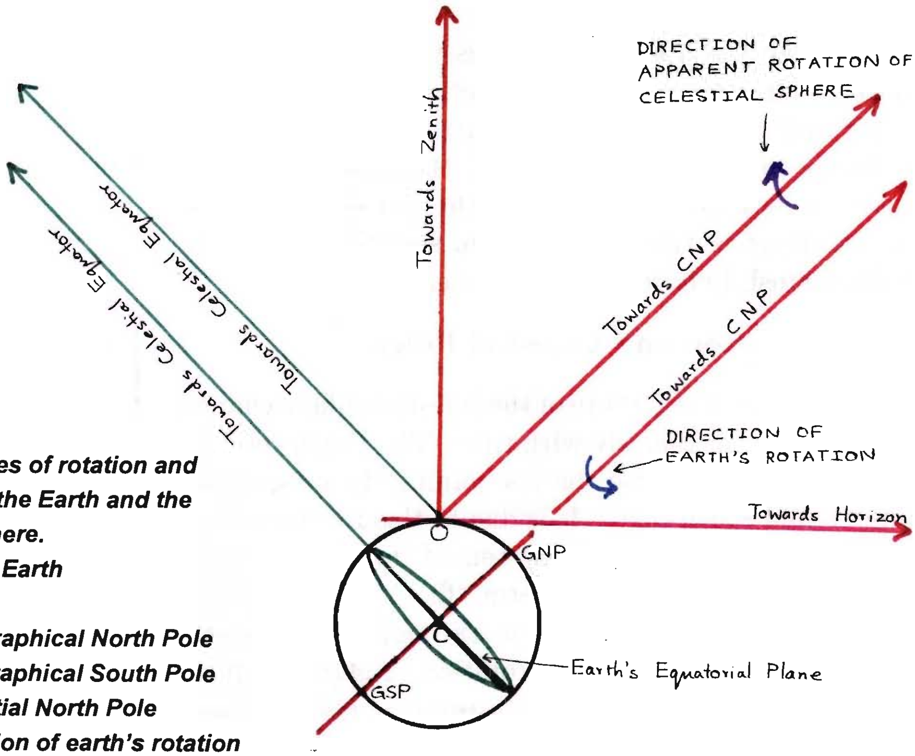

Figure 4. Axes of rotation and equators of the Earth and the celestial sphere.
C: Centre of Earth
O: Observer
GNP: Geographical North Pole
GSP: Geographical South Pole
CNP: Celestial North Pole
↪: Direction of earth's rotation
↖: Direction of apparent rotation of the celestial sphere.

Box 3. The Pole Star

The Pole Star is a visible star whose direction from a terrestrial observer is so close to that of the celestial north pole that it appears to play the role of the pole itself. To an observer in the northern hemisphere, other "fixed stars" appear to revolve around the "Pole Star" while this distinguished star appears to remain fixed in the same position even after a long time. In Indian astronomy, the Pole Star was called *dhruvanakṣatra* or *dhruvatārā* (*dhruva*: fixed, firm, immovable, unchangeable, constant, lasting, permanent, eternal). At present, Polaris, a moderately bright star belonging to the Ursa Minor constellation, is called the Pole Star. In reality, Polaris is now about 1^0 away from the celestial north pole. There is presently no analogous star near the celestial south pole bright enough to catch the eye of an observer in the southern hemisphere.

No individual star remains the "Pole Star" for ever. For, due to a phenomenon called "precession", the celestial north pole executes a slow circular orbit around a certain point on the celestial sphere it takes about 25,800 years to complete an orbit. Consequently, different epochs witness different Pole Stars. At present, the pole is approaching Polaris the current "Pole Star" and will continue to do so till about the year 2100 CE when they will be at their nearest but still $27.5'$ apart. There will be long periods when the celestial north pole will be unmarked by any bright star in its vicinity. Around 14000 CE, Vega (Alpha Lyrae), the brightest star of the northern hemisphere, will have the distiction of being the Pole Star just as it was around 12000 BCE. However, it will then be twice as far from the true pole as our present Pole Star is today. Around 3000 BCE at the time of the early astronomers of the Egyptian, Mesopotamian and Indus valley civilisations the star Alpha Draconis was the Pole Star.

From ancient times, navigators and travellers have relied on the Pole Star for determining directions during journeys at night. The concept has inspired a literary metaphor for the firm, ever-present, shining, eternal Guide whose constant unfailing luminous radiation alone can be trusted by man for orienting his course through the darkness of his perilous voyage across the sombre ocean of life.

the distance of any star from Earth, the axis of rotation of the celestial sphere and the axis of Earth's rotation may be regarded as coincident.

The axis of rotation, when extended bothways, meets the celestial sphere in two diametrically opposite points. Of the two points, the point in the direction of the Earth's geographical north pole is called the *celestial north pole* (*uttara dhruva*); the other is called *celestial south pole* (*dakṣiṇa dhruva*). For practical purposes, the "Pole Star" (see *Box* 3) is identified with the celestial north pole and used to locate the approximate north direction.

Some Great Circles of the Celestial Sphere

A *great circle* on a sphere is a circle on its surface (inner or outer) of maximum possible radius – the radius of the sphere. It is formed by the intersection of the sphere with a plane passing through the centre of the sphere. Given two distinct points on the sphere which are not diametrically opposite, there is a unique great circle passing through the two points. Any two distinct great circles bisect each other. The line, through the centre of a great circle, and perpendicular to the plane containing the great circle, intersects the sphere in two points – these points are called *poles* of the great circle. A circle on the sphere which is not a great circle is called a *small circle*.

In a plane, the position of a point is referred to in terms of two fixed lines and a point called origin. Analogously, in astronomy, the co-ordinate of a heavenly body on the celestial sphere is described with reference to fixed great circles and an origin. We now define a few important great circles on the (inner surface of) celestial sphere (*Figure* 5).

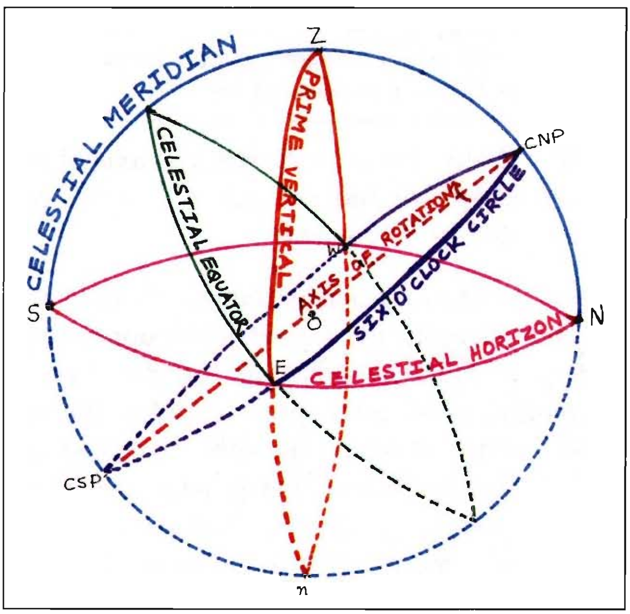

Figure 5. Some great circles on the Khagola (celestial sphere); The portions of the great circles lying below the horizon (and hence invisible to the observer) are indicated by dotted lines.
O: Observer
Z: Zenith
n: Nadir
CNP: Celestial north pole
CSP: Celestial south pole
N: North point
S: South point
E: East point
W: West point.

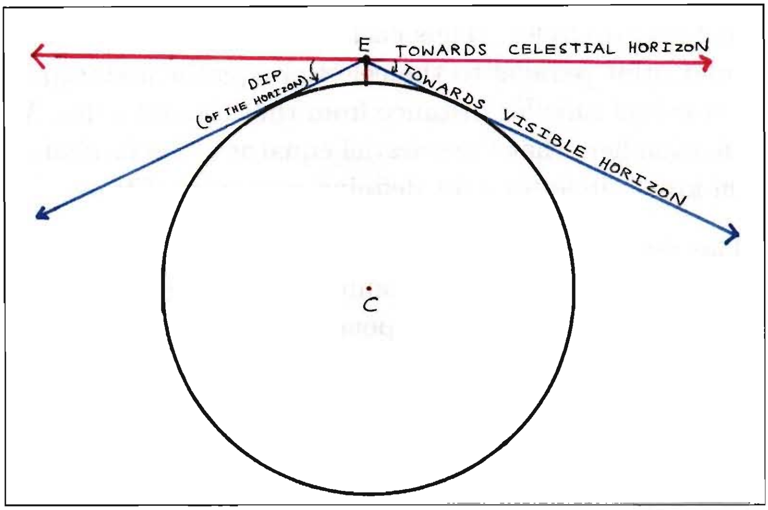

Figure 6. Dip of the horizon. C: Centre of the Earth E: Exact location of the observer's eye.

The great circle of the celestial sphere in which the (horizontal) tangent plane to the Earth's surface, at the position of the observer, meets the celestial sphere, is called the *celestial horizon* (*kṣitija*). As the observer's eye is at some distance above the sea-level, the "visible horizon" appears somewhat below the celestial horizon as a small circle (*Figure* 6). However, to minimise technicality, we shall ignore the difference[2] in this article and regard the celestial horizon itself to be the visible horizon and call it horizon. The observer sees only the portion of the celestial sphere above the horizon. In *Figures* 3 and 5, the invisible portion of the celestial sphere and its circles are indicated by dotted lines. The rising and setting of the stars and planets take place on the horizon towards the east and west respectively.

[2] To avoid a possible confusion, we clarify here that if the dip can be neglected, it would be not because of the smallness of the Earth's radius relative to stellar distances but rather the largeness of the Earth's radius relative to the height of the observer. While the Earth's radius can be regarded as negligible for mapping the stars, the dip of the visible horizon has nothing to do with the distances of the stars. It depends solely on the height of the observer's eye from the Earth's surface and the Earth's radius.

The great circle of the celestial sphere whose plane is perpendicular to the axis of the celestial sphere (i.e., to the line joining the celestial poles) is called the *celestial equator* (*viṣuvadvṛtta* or *nāḍivṛtta*). Thus, as with the axes of rotation (*Figure* 4), the plane of the celestial equator is parallel to the plane of the Earth's equator (*nirakṣa*) – the two planes practically coincide. Half the celestial equator is above the celestial horizon (both being great circles). The celestial poles are the "poles" of

The great circle of the celestial sphere whose plane is perpendicular to the axis of the celestial sphere is called the celestial equator.

this "great circle" Thus each fixed star traverses a circular orbit parallel to the celestial equator maintaining a constant angular distance from the celestial poles. We mention here that the celestial equator is the fundamental great circle used for defining measures of time.

The *zenith* is the point on the celestial sphere vertically overhead. Thus the line joining the zenith and the observer is a vertical line perpendicular to the plane of the horizon. Any great circle through the zenith is called a *vertical* great circle. The vertical great circle through the celestial poles is called the *observer's meridian* or *celestial meridian* (*yāmyottara maṇḍala*).

The point below the celestial north pole where the observer's meridian meets the horizon is called the North point. The point on the horizon opposite the North point is called the South point. The meridian is thus the vertical circle through the North and South points. The points of intersection of the celestial equator and the horizon are the East and West points – the East point lies to the right of the North-South line for an observer facing North; the West point lies to the left. These four points are called cardinal points. The vertical great circle through the East and West points is called the *prime vertical* (*samamaṇḍala*). The great circle through the celestial poles and the East and West points is called the *six o'clock circle* (*unmaṇḍala*).

Āryabhaṭa defined the prime vertical, meridian and horizon in Gola 18, the six o'clock circle in Gola 19; while the equator is implicit in Gola 1. The horizon is pictured as an encircling cord around the prime vertical and the meridian (as if fastening them). The suggestive imagery is interesting. Amidst the apparent rotation of the celestial sphere, the horizon remains firmly fixed in the middle of the sphere at right angles to the prime vertical and the meridian.

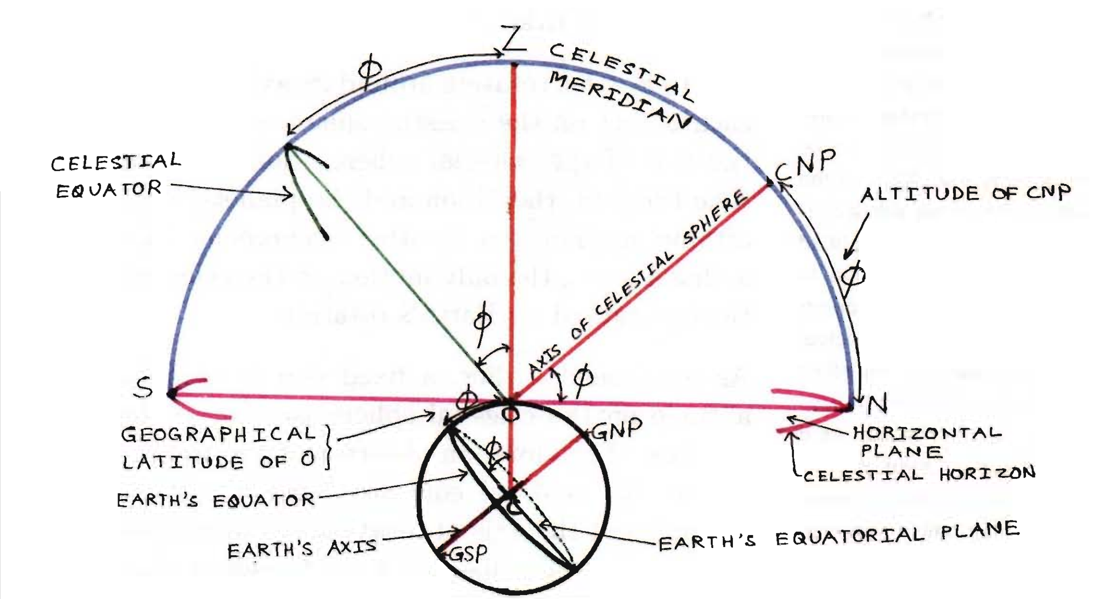

Figure 7. Altitude of CNP = Geographical Latitutde of O.

GNP: Geographical North Pole

GSP: Geographical South Pole

C: Centre of Earth

CNP: Celestial North Pole

N: North point

S: South point

O: Observer

Z: Zenith.

Geographical Latitude and Altitude of Celestial North Pole

The angular distance between the celestial north pole and the North point (i.e., the elevation of the celestial north pole from the horizon) is precisely the geographical latitude of the place of the observer (see *Figure* 7).[3] It is also the depression of the celestial south pole from the horizon as also the angle between the celestial equator and the prime meridian. The equality of the latitude with the altitude of the celestial north pole (or the depression of the celestial south pole) is implicit in Gola 19. In particular, for an observer on the Earth's equator, the celestial north pole coincides with the North point, the celestial south pole with the South point and the celestial equator with the prime vertical; while for an observer in the North Pole, the celestial north pole is at the zenith and the celestial equator becomes the horizon! At the equator, the six o'clock circle coincides with the celestial horizon and hence *unmaṇḍala* was sometimes referred to as the "horizon at Laṅkā" ([8], p 69).

[3] Thus the latitude of a place in the northern hemisphere would be approximately the altitude of the Pole Star at that place. For an observer at Bangalore (12.58N, 77.38E) or Chennai (13.04N, 80.17E), the Pole Star appears about 13^0 above the northern horizon, while for an observer in Kolkata (22.34N, 88.24E), the elevation of the Pole Star is about 22 1/2^0.

Choice of Laṅkā

Due to Earth's rotation around its axis, the observer sees each object on the celestial sphere as revolving around the axis of the celestial sphere. Some of these objects (like the Sun, the Moon and the planets) also have additional motions due to other phenomena. However, for a "fixed star" the only motion on the celestial sphere is the one caused by Earth's rotation.

As mentioned earlier, a fixed star is seen to describe a circle on the celestial sphere *parallel* to the celestial equator. Therefore, an observer at the Earth's equator (where the celestial equator coincides with the prime vertical and the axis of rotation lies in the plane of the horizon) sees the fixed star as travelling from east to west in a *vertical* circle (parallel to the prime vertical and perpendicular to the horizon). At an intermediate latitude between the equator and a pole, the orbit of any fixed star is oblique and thus the fact that the star is moving from east to west could be less obvious to a casual observer. In the extreme case of the poles, the stars are seen to traverse in horizontal circles (parallel to the horizon).

It is only at the equator that *all* fixed stars rise and set. Their orbits cross the horizon at right angles and are bisected by it, so that every fixed star remains above the horizon for 12 hours daily (if we neglect refraction[4] and other subtleties).

An observer in the northern hemisphere sees that northward from the celestial equator, the daily orbits of the stars come up more and more above the horizon till they are entirely above it; southward from the celestial equator, they are depressed more and more until they disappear completely from the observer's view. Thus while some of the stars close to the celestial north pole remain visible all the time, some of the stars close to the celestial south pole have orbits completely below the horizon

[4] We are restricting ourselves to theoretical explanations based essentially on the geometrical aspects of the celestial sphere. But, in reality, one has to take into account the effects of the Earth's atmosphere which considerably modify our simplified picture. For instance, due to refraction, a star can appear higher in the sky than its actual geometric position. The effect of refraction is most pronounced at the horizon. As a result, the actual duration of the day is several minutes longer than the theoretical duration. At a place on the equator, where day and night should be of equal duration from geometric consideration, the actual durations are 12 hours 5 minutes and 11 hours 55 minutes respectively. In this article we shall not discuss the visibility corrections employed in ancient India.

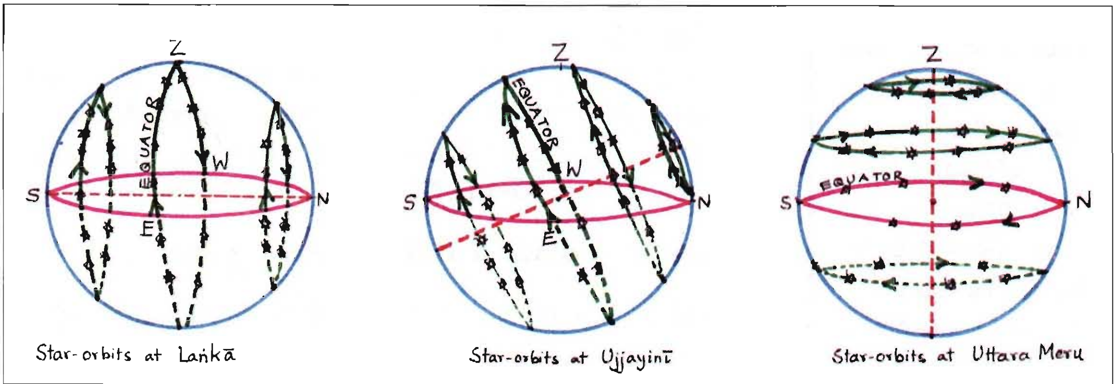

Figure 8. Star-orbits at 3 locations.
N: North point
S: South point
E: East point
W: West point
Z: Zenith
__ : Horizon
__ : Portions of star-orbits above horizon
---- : Portions of star-orbits below horizon
—> : Direction of an orbit.

and remain invisible all the time. For instance, at a place of latitude 23N, there is a circular area around the celestial north pole of radius 23^0 containing stars that never set and a similar area around the celestial south pole containing stars that never come into view. The remaining band of the sky contains the stars that rise and set. This band is symmetrical with respect to the celestial equator and extends 67^0 on either side of it. (See *Figure* 8.) The situation is exactly reversed in the southern hemisphere.

As described by Āryabhaṭa in Gola 16, an observer at the North Pole sees one half of the celestial sphere as revolving from left to right (i.e., clockwise) while an observer at the South Pole sees the other half as revolving from right to left (i.e., anticlockwise).

Thus the equator is the only part of the Earth where each and every portion of the *entire* sky is brought into view by the daily rotation for some time during 24 hours. At any other place, some portion of the sky remains permanently hidden from the observer. The proportion of the invisible portion increases as one approaches the poles; at each pole, one half of the sky remains visible all the time while the other half remains permanently invisible.

The visibility of the whole sky and the vertical nature of the stellar orbits must have been among the reasons

In Gola 14, Arybhata conveyed a rough idea of the distance of Ujjayini from Lanka (i.e., the latitude of Ujjayini) by mentioning that it is one-sixteenth of the Earth's circumference.

for Āryabhaṭa specifying a point on the geographical equator (Laṅkā) in the quoted verse (Gola 9). *Figure* 8 shows how the orbits of the stars appear before an observer at (i) Laṅkā (on equator), (ii) Ujjayinī (23.09N) and (iii) Uttara Meru (North Pole).

Prime Meridian in Indian Astronomy

The visual impact of vertical orbits would be same at *any* point on Earth's equator. However Laṅkā was situated on a special geographical meridian – the meridian through Ujjayinī. This meridian was chosen by Indian astronomers as the "Prime Meridian" (just as the meridian through Greenwich is now taken to be the prime meridian). The acceptance of one prime meridian by all Indian astronomers reflects the cultural unity of ancient India. Ujjayinī was a great centre of learning, especially in astronomy. The academic, cultural and political eminence of the city, as well as its central location, must have influenced the choice of the prime meridian.

It was then natural for Āryabhaṭa to select Laṅkā to represent an equatorial point. In fact, Laṅkā used to be a standard reference point in Indian astronomy. For instance, in Āryabhaṭīya (Gītikā 4), the commencement of the current "*yuga*" is reckoned from sunrise at Laṅkā.

The determination of geographical latitude and longitude of a place on Earth from observations of celestial bodies is one of the most important applications of spherical astronomy. In Gola 14, Āryabhaṭa conveyed a rough idea of the distance of Ujjayinī from Laṅkā (i.e., the latitude of Ujjayinī) by mentioning that it is one-sixteenth of the Earth's circumference. As the reader can see, $\frac{1}{16}$ $(= \frac{22\frac{1}{2}}{360})$ is a *convenient* fraction fairly close to the actual ratio of the distance of a place at 23.09N from the equator to the length of the circumference.

Gola-Yantra

Gola-Yantra denotes a spherical apparatus representing the *bhagola*; it is designed to rotate uniformly at the rate of one rotation per day, i.e., at the rate of Earth's rotation. In a single verse (Gola 22), Āryabhaṭa prescribed the construction of this rotating sphere with a cryptic brevity characteristic of Āryabhaṭīya:

> kāṣṭhamayaṁ samavṛttaṁ samantataḥ samaguruṁ laghuṁ golam
> pārada tailajalaistaṁ bhramayet svadhiyā ca kālasamam

> *(Make) a wooden gola (globe) which is perfectly spherical and uniformly dense all around (but is) light in weight. Using mercury, oil and water, and applying one's own intellect, (make the globe) rotate (at the required rate) to keep pace with time.*

Excerpts from a passage of the astronomer Lalla: *How can the ignorant, who know neither mathematics nor the fundamental principles of the celestial sphere, ever detect the motion of the planets? An astronomer without the knowledge of this sphere is like a disputant without the knowledge of grammar, a sacrificer without the light of the Vedas and a physician without the experience of an active practitioner.*

This emphasis on the creative use of the intellect for working out the details from terse aphorismic indications is a recurrent feature in the original works of the ancient Indian Masters. Sometimes the commentators provide more elaborate expositions. In his commentary on Āryabhaṭīya, the 12th century astronomer Sūryadeva described in detail how mercury, oil and water can be used "intelligently" to rotate the sphere at the required rate (see [1], p 129-130 or [5], p 86).

Importance of the Gola

Indian astronomers laid special emphasis on the importance of the sphere Gola (referring to both the concept and the physical model of the celestial sphere). To quote (translations of) excerpts from a long passage of the astronomer Lalla (around 8th century CE) on the indispensability and profundity of this study ([5], p 74): "No astronomy treatise is complete without a section on the sphere of the universe. The astronomers stress that this sphere is specifically needed for mathematical computations and that those who wish to study the planets must be experts on the sphere. The mean motions of the planets and other celestial objects are clearly perceptible on this sphere, especially to the scholar who has mastered the science of their geometrical representation. How can the ignorant, who know neither mathematics nor the fundamental principles of the celestial sphere, ever detect the motion of the planets? An astronomer without the knowledge of this sphere is like a disputant without the knowledge of grammar, a sacrificer without the light of the Vedas and a physician without the experience of an active practitioner. He who acquires a comprehensive knowledge of the celestial sphere containing the Sun etc. sees, in front of his eyes as it were, the whole universe; he beholds it decorated beautifully with a wide variety of exquisite phenomena. He gets spiritually enriched and attains *mokṣa* (Liberation); he becomes famous."

Suggested Reading

[1] K S Shukla and K V Sarma, *Āryabhaṭīya of Āryabhaṭa*, Indian National Science Academy, New Delhi, 1976.

[2] S Balachandra Rao, *Indian Astronomy—An Introduction*, Universities Press, Hyderabad, 2000.

[3] P R Ray and S N Sen (ed), *The Cultural Heritage of India Vol VI: Science and Technology*, The Ramakrishna Mission Institute of Culture, Calcutta, 1986; reprinted, 2002.

[4] D M Bose, S N Sen and B V Subbarayappa (ed), *A Concise History of Science in India*, Indian National Science Academy, New Delhi, 1971; reprinted, 1989.

[5] B V Subbarayappa and K V Sarma (ed), *Indian Astronomy; A Source-Book*, Nehru Centre, Bombay, 1985.

[6] W M Smart, *Text-Book on Spherical Astronomy*, Cambridge University Press, 1956.

[7] D Chattopadhyaya (ed), *Studies in the History of Science in India*, Vol.II, Editorial Enterprise, New Delhi, 1982.

[8] W E Clark (ed), *The Āryabhaṭīya of Āryabhaṭa*, University of Chicago, Illinois, 1930.

[9] M Monier Williams, *Sanskrit-English Dictionary*, Clarendon Press, Oxford (1899); reprinted Munshiram Manoharlal, New Delhi, 2002.

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