

## Aryabhata

Aryabhata was perhaps the first mathematician and astronomer of India whose work and history are available to modern scholars. Often referred to as Aryabhata I (to distinguish him from a tenth century mathematician of the same name), he was born in 476 AD and flourished at the time when the Gupta Empire was at its peak. Although it is known that he spent most of his working life at a place named Kusumpura, which is thought to be Patliputra or close to it, the place of his birth has not been confirmed. The 15th century mathematician Nilakantha Somayaji wrote that Aryabhata was born in Asmaka region of the Vakataka Kingdom in South India, but by some historians believe that Nilakantha confused Aryabhata with the commentator (Bhaskara I) of Aryabhata's book, the *Aryabhatiya*. Almost nothing is known about his personal life.

*Aryabhatiya* is the only book written by Aryabhata that has survived. It has 121 verses summarizing the state of Indian mathematics and astronomy up to that time. In this book, the Earth was taken to be spinning on its axis (as elaborated in an article in this issue). Aryabhata gave the radius of the planetary orbits in terms of the radius of the Earth/Sun orbit as essentially their periods of rotation around the Sun – that is, his model was heliocentric. He also gave the correct explanation for the solar and lunar eclipses, and wrote that the Moon shines by reflection of sunlight. It is a pity that some of the later astronomer/mathematicians of India found it hard to believe that the Earth could spin and that the apparent rotation of the sky and the stars was due to the axial rotation of the Earth; these later astronomers even changed the text to save Aryabhata from what they thought were silly errors!

*Aryabhatiya* also gave a measure of the circumference of the Earth which is accurate within 1%. His approximation for  $\pi$  was remarkable. He wrote : “Add four to one hundred, multiply by eight and then add sixty-two thousand. The result is approximately the circumference of a circle of diameter twenty thousand. By this rule the relation of the circumference to diameter is given.” In other words  $\pi = 62832/20000 = 3.1416$  which is surprisingly accurate. Incidentally, he preferred to use  $\sqrt{10} = 3.1622$  for an approximation of  $\pi$  in practice; he did not explain though how he found this approximation.

*Aryabhatiya* contained tables of sines, with the approximate values of sines at intervals of  $90^\circ/24 = 3^\circ 4'$ . For this, he used a formula for  $\sin(n+1)x - \sin nx$  in terms of  $\sin nx$  and



$\sin (n-1)x$ . Interestingly, the word used for half-chords by Aryabhata was ‘jya-ardha’ or simply ‘jya’, which the Arabs wrote as ‘jiba’; but this was confused with another word in Arabic language, ‘jaib’ meaning ‘bay’ or ‘fold’, perhaps because the vowels were omitted. So when it was translated into European languages, the Latin word for ‘bay’ or ‘fold’ or ‘curve’, *sinus*, was used, which has given us the word ‘sine’ in modern mathematics. (see, *History of Mathematics*, Carl B Boyer, Wiley, 1991)

*Aryabhatiya* also contained discussions on algebra. For example, Aryabhata examined the integer solutions to equations of the form  $by=ax+c$  where  $a, b, c$  are integers. The problem was motivated by the determination of the periods of planets. Aryabhata used a method (called ‘Kuttaka’, which literally means ‘to pulverize’) to solve problems of this type; the method consisted of breaking the problem down into other problems where the coefficients became smaller and smaller with each step, and so the method is related to continued fractions.

Bhaskara I, writing a commentary of *Aryabhatiya* a century later, wrote, “Aryabhata is the master who, after reaching the furthest shores and plumbing the innermost depths of the sea of ultimate knowledge of mathematics, kinematics and innermost spherics, handed over the three sciences to the learned world.”

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Aryabhata seems to have used perimeters  $p^-$  and  $p^+$  of the inscribed and circumscribed regular polygons of 384 sides for the unit circle to approximate the value of  $\pi$ ,  $p^- < 2\pi < p^+$ . Archimedes used 96-sided regular polygons to find a similar approximation to  $\pi$ .

– B Sury

