

Bethe's Contributions to Solid State Theory A Tribute

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This article attempts to acquaint the reader with the seminal contributions made by Hans Albrecht Bethe to the area of solid state theory. It should be read in conjunction with the other tributes to Bethe's work that appeared in the October 2005 issue of *Resonance*.

1. Introduction

Hans Bethe actually started his physics career as a 'Solid-State Theorist'! He joined the University of Munich in 1926 as a protégé of Arnold Sommerfeld. He quickly mastered the newly founded quantum mechanics (because, as he later recalled, he was not burdened by knowing the recipes of the 'old' quantum theory!) and started applying it to cutting edge problems in solid state and atomic physics. He obtained his doctorate in theoretical physics in 1928. For his thesis research he studied the passage of electrons through crystals. This subject was of great interest at that time since quantum theory had surprisingly predicted that electrons, formerly thought to be particles, could, like light, also behave as waves, and Davisson and Germer had just then performed their pioneering experiments which verified this by demonstrating that crystals diffracted electrons in much the same way as X-rays. Bethe developed the detailed theory to explain these experiments. This work led to his first published paper [1], on the 'Theory of diffraction of electrons by crystals'.

During the very next year he cemented his place in the history of solid state theory with his seminal work entitled 'Splitting of terms in crystals' [2], one of the first



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Keywords

Crystal field theory, Bethe's Ansatz, Bethe-Peierls method.



Of the first 18 papers of Bethe, 11 are in the area of solid state physics. These helped set up the foundations of the modern Quantum Theory of Solids.

applications of group theory to quantum mechanics (see the next section for a more detailed account of this work). Overall, although only 14 out of his 300 or so papers are in the area of solid state physics, they comprised the majority of his initial papers (7 out of the first 11, 11 out of the first 18, the rest being in what one might call 'atomic and molecular physics') and helped to set up the foundations of the modern Quantum Theory of Solids. In what follows, I sketch briefly those of Bethe's contributions to solid state theory which are *readily available* in English translations [3].

2. Crystal Field theory

An atom or ion placed in vacuum is in an environment which is believed to be 'isotropic'; i.e., all directions in space are equivalent. Correspondingly, its Hamiltonian (i.e., its energy expressed as a function of the coordinates and momenta of its electrons) is symmetric or invariant with respect to rotations (eg., of the coordinate axes). Mathematically, in the quantum mechanics of electrons in an atom/ion, the set of all possible three-dimensional rotations is represented by a 'continuous symmetry group' [4] under whose action the Hamiltonian is invariant. This has the consequence that the net spin angular momentum (S) and orbital angular momentum (L) of all the electrons, and their z components (unless spin-orbit interactions are important, in which case the total angular momentum (J) and its z component) are 'good quantum numbers' that can be used to label the atomic energy eigenstates. When the quantum numbers are non-zero, the energy eigenstates exhibit characteristic degeneracies.

The Hamiltonian of an isolated atom/ion is invariant with respect to the continuous group of rotations in 3 dimensions.

The simplest such example shows up in the quantum mechanics of the hydrogen atom. Just from the fact that the potential seen by the electron moving around a fixed nucleus is spherically symmetric or isotropic, one can show that the electronic orbitals or quantum states



can be characterized by a set of four quantum numbers. These are the principal quantum number n (which takes values 1,2,....) which determines the 'size' of the orbital, the azimuthal (or orbital angular momentum) quantum number ℓ (which takes integer values from 0 to $n - 1$) which determines the magnitude of the orbital angular momentum, the 'magnetic' quantum number m (which takes values from $-\ell$ to $+\ell$) which determines the orientation of the orbital or the z-component of the orbital angular momentum, and the spin quantum number s (which takes values $\pm 1/2$) which determines the z-component of the spin angular momentum. Furthermore, one can show that the energy must be independent of m and s and can depend only on n and ℓ . Hence each energy level $E(n, \ell)$ must have a degeneracy of $2(2\ell + 1)$. [In the hydrogen atom, the energies, given by the Bohr formula $E_n = -me^4/(8\epsilon_0^2 h^2 n^2)$, are also independent of ℓ , leading to a higher degeneracy, of $2n^2$, but this is due to an additional special symmetry of the hydrogen atom problem, and is not generic.]

In case of atoms with higher atomic number, one must include the effects of electron-electron interactions [4, 5]. If this is done approximately, in the form of an average, self-consistent potential seen by the electrons, which is still taken to be spherically symmetric, then the same degeneracy structure as outlined above for the hydrogen atom emerges, but with the energies dependent on ℓ as well. The levels must now be filled with the requisite number of electrons (equal to the atomic number) respecting the Pauli-exclusion-principle (according to which no two electrons can occupy the same orbital, i.e., have all the same quantum numbers). The existence of the degeneracies mentioned above then leads [5] to the so called 'shell structure' of atoms which is responsible for the systematic and periodic alternation in the physical and chemical properties of the elements as brought out by the periodic table. When the electron

The symmetry under rotations in 3d is responsible for the 'shell structure' of atoms.



numbers (equal to the atomic numbers) have the special values of 2, 10, 18, 36, 54, and 86, corresponding to the noble gases He, Ne, Ar, Kr, Xe, and Rn respectively, all the degenerate levels corresponding to a set of $\{n, \ell\}$ are completely filled, corresponding to 'closed shells', which are energetically very stable and hence inert. For other atoms, one gets partially filled shells, whence the remainder of the electron-electron interaction effects forces the electrons in the outermost, 'valence' shell, to co-operatively arrange their motion according to the Hund's rules; i.e., in such a way as to first maximize their total spin S , and then (consistent with this) their total orbital angular momentum L , (and, when spin-orbit effects are important, their total angular momentum J). Accordingly, the ground state will have a degeneracy of $(2S + 1)(2L + 1)$ (reduced to $(2J + 1)$ if spin-orbit interactions are included). This structure is responsible for all the wonderful variety of chemical, magnetic, optical, and other properties of different atoms and ions in 'isotropic' (i.e. in gaseous, and, for many purposes, in liquid-like) environments [5].

In his monumental work on the 'Splitting of terms in crystals' mentioned above [2, 3], Bethe raised, and answered completely, with his characteristic thoroughness, the question as to in what way the above features change when the same atom/ion is in a crystalline environment [6]. The change is necessitated by the presence of the crystalline electric field (CEF), arising from the electrostatic forces on the electrons of the specific atom/ion under consideration due to the other ions surrounding it. The symmetry (group) under which the atomic/ionic Hamiltonian which includes the CEF is invariant is the smaller, discrete, point (group) symmetry of the crystal, one of 32 possibilities (the 'Crystallographic Point Groups') [4, 6] depending on the crystal type. Using his knowledge of the theory of representations of the crystallographic point groups [4], Bethe completely sorted

An atom or ion in a crystalline environment is symmetric only under the point group of that crystal.



out this problem, covering the cases of small, intermediate and large CEF. The work has been extensively used ever since, and is one of the cornerstones of modern solid state theory [4]. It is of direct experimental relevance in understanding the spectra (especially electron paramagnetic resonance [7]) of ions and impurities in crystals, and is a starting point for the modelling of many crystalline solids.

In a cubic crystal, the 5 '3d orbitals' are split into one group of 3 levels and another group of 2 levels.

As an illustrative example, consider an atom/ion in a cubic (simple-cubic(SC), face centered cubic(FCC), or body centered cubic(BCC)) crystal. Then the CEF has the point group symmetry of a cube, i.e., it is invariant under all the symmetry operations that take a cube into itself, eg., rotations by multiples of 90° around the x, y or z axes. As a consequence, as Bethe showed, the 5 '3d-orbitals' (corresponding to $n = 3$ and $\ell = 2$) which would be 'degenerate' in an isolated ion as explained above, get split into one group of three levels, referred to as t_{2g} orbitals, (corresponding to the orbitals labelled d_{xy} , d_{yz} and d_{zx} according to their angular dependence) and another group of two levels, referred to as e_g orbitals, (corresponding to the orbitals labelled $d_{x^2-y^2}$ and $d_{3z^2-r^2}$ according to their angular dependence). In a crystal of even lower symmetry, eg., a tetragonal crystal, these would get split further. These 'split levels' have now to be filled with electrons according to (a modified version of the) 'Hund's rules', and the consequent electronic configurations play a crucial role in determining the properties of crystals containing transition metal ions in which the 3d orbitals are partially filled. Apart from the elemental transition metal crystals, instances of such crystals include famous recent examples like the high- T_c cuprate superconductors, and the 'manganites' which show 'colossal magneto-resistance'. *Figure 1* depicts the above in the context of the doped manganite material $La_{1-x}Ca_xMnO_3$, where for $x = 0$, all the Mn ions have a charge of +3, with four d electrons, and

Crystal field effects are essential for understanding properties of crystals, especially those containing atoms/ions with partially filled shells.



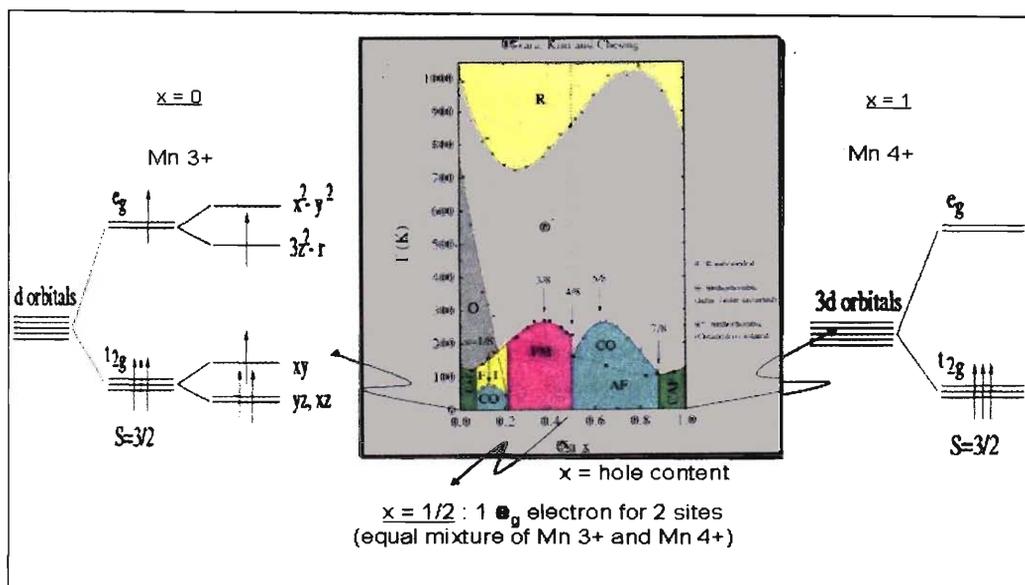


Figure 1. The phase diagram of $\text{La}_{1-x}\text{Ca}_x\text{MnO}_3$ in the doping x and temperature T plane (courtesy M Uehara, K H Kim and S W Cheong). Various kinds of anti-ferromagnetic insulator (AF), paramagnetic insulator (PI), ferromagnetic insulator (FI), ferromagnetic metal (FM) and charge/orbitally ordered insulator (CO/O *) are the phases shown. O and O * are orthorhombic (Jahn–Teller distorted) and orthorhombic (octahedron rotated) structural phases. The figure also illustrates the splitting of the 3d levels of Mn due to the CEF and how these are occupied for $x=0$ and $x=1$.

for $x = 1$, all the Mn ions have a charge of +4, with three d electrons. For intermediate values of x , the Mn ions fluctuate between the two configurations, and this is responsible for the extraordinarily complicated phase diagram that is also shown in Figure 1. The crystal field effects that Bethe drew our attention to nearly 75 years ago have to be included from the outset in constructing models for understanding the phase diagram and the properties of the phases [8].

3. Passage of Fast Particles through Matter

A year after Bethe's work on crystal field theory came the seminal paper 'Theory of the passage of fast corpuscular rays through matter' [9] In this paper Bethe studied the problem of how charged particles travelling at high speeds lose energy as they pass through matter. This was an important problem at the time because the relationship between the energy and the 'range' of the particles (i.e. the average distance travelled by them before they are stopped) was an important tool for measuring the energy of the particles. Bethe simplified considerably the quantum theory of atomic collisions proposed

by Max Born in 1926, and used it to calculate the cross sections for all the different scattering mechanisms experienced by the charged particles (electrons, protons, α particles,...) as they collide with the electrons and nuclei comprising the matter, such as elastic collisions, inelastic collisions, ionization processes, etc, and obtained, in his own modest words, 'satisfactory to good' agreement with experiment. Some idea of the importance of this work can be gleaned from the amazing statistics that this article was cited [10] 545 times during the period 1961-1975, and 91 times in 1975-76, forty-five years after its publication!

Bethe calculated the cross sections for all the different scattering mechanisms experienced by charged particles moving through matter.

4. The Bethe Ansatz

In the very next year, while visiting Enrico Fermi at Rome, Bethe wrote the famous paper 'On the theory of metals I: Eigenvalues and eigenfunctions of the linear atomic chain' [11] where he proposed the celebrated 'Bethe's Ansatz' Although his paper is phrased in terms of the Heitler-London theory of exchange processes applied to a chain of atoms, in terms of modern terminology Bethe obtained in his paper *all* the 2^N eigenstates and eigenvalues of the spin-1/2 nearest neighbour Heisenberg Hamiltonian on a 'ring' (i.e., a chain with periodic boundary conditions) of N sites (with coupling J) exactly analytically for arbitrary N ! This was an astounding accomplishment that was way ahead of his times, and forms the foundation for the bulk of the extensive research work on 'exactly soluble 1-d models of quantum many-body theory' that has since followed. It is difficult to appreciate this work without some exposure to the quantum mechanics of spin-1/2 operators, so some more details about this work are given separately, in *Box 1*.

5. Modern Theory of Metals

Two years after the 'Bethe Ansatz', in 1933, came two



Box 1. Some Details about the Bethe Ansatz

The Heisenberg Hamiltonian for an N-site ring of spin-1/2 atoms can be written as

$$\begin{aligned} \mathcal{H} &= -J \sum_{n=1}^N \vec{S}_n \cdot \vec{S}_{n+1} = -J \sum_{n=1}^N (S_n^z S_{n+1}^z + S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \\ &= -J \sum_{n=1}^N [S_n^z S_{n+1}^z + \frac{1}{2}(S_n^+ S_{n+1}^- + S_n^- S_{n+1}^+)], \end{aligned}$$

where \vec{S}_n is a spin-1/2 operator at site n , and $\vec{S}_{N+1} \equiv \vec{S}_1$.

The z-component of the total spin operator, $S^z \equiv \sum_n S_n^z$ commutes with \mathcal{H} , and the values it takes (from $N/2$ to $-N/2$ in steps of 1) can hence be used as a 'good quantum number' for labelling its eigenstates. The state in which all spins are up, which we will denote as $|\Omega\rangle \equiv |\uparrow, \uparrow, \dots, \uparrow\rangle$ has $S^z = N/2$, and is easily verified to be an eigenstate of \mathcal{H} with eigenvalue $E_\Omega = -NJ/4$. If J is positive, corresponding to the ferromagnetic case, this state is the lowest energy state; if J is negative, corresponding to the anti-ferromagnetic case, it is the highest energy state.

States with smaller values of S^z are obtainable by starting from this reference state and flipping spins down, say D of them, corresponding to $S^z = N/2 - D$, at an arbitrary subset of sites $\{m_1, m_2, \dots, m_D\}$. We will denote this state, which can be obtained for example by letting D spin lowering operators act on $|\Omega\rangle$, as $|m_1, m_2, \dots, m_D\rangle \equiv S_{m_1}^- S_{m_2}^- \dots S_{m_D}^- |\Omega\rangle$. There are as many distinct such states as the binomial coefficient

$${}^N C_D \equiv \frac{N!}{D!(N-D)!}.$$

The eigenstates corresponding to D spins down should hence be expressible as the linear combination

$$\sum_{m_1, m_2, \dots, m_D} |m_1, m_2, \dots, m_D\rangle \psi(m_1, m_2, \dots, m_D).$$

It is useful to think of the down spins as 'particles' with a hard-core (so that no more than one of them can be placed at a site), whence $\psi(m_1, m_2, \dots, m_D)$ is their 'wave-function' on the discrete lattice on which these particles move. It is not hard to see that the (x-x, y-y) or transverse part of the Heisenberg Hamiltonian makes the particles move to the left or to the right with an amplitude of $J/2$. Whereas the z-z part of \mathcal{H} gives a site energy of J to each particle, and an interaction energy of $-J$ (i.e., attractive in the ferromagnetic case and repulsive in the anti-ferromagnetic case) when a pair of particles come onto adjacent sites.

Box 1. continued...



When $D=1$, i.e, there is only one down spin, there are no interaction effects and it is not hard to see that $\psi(m)$ satisfies the 'Schrödinger equation'

$$E\psi(m) = E_{\Omega}\psi(m) + J[-\psi(m) + \frac{1}{2}(\psi(m-1) + \psi(m+1))].$$

This has the solution $\psi(m) \propto \exp(ikm)$, i.e., the eigenfunctions are 'plane waves' with wave-vector k . The periodic boundary conditions, requiring that $\psi(m) = \psi(m+N)$ imply that k can only take the values $k = 2\pi\nu/N$ where ν is an integer. The corresponding energy eigenvalues are

$$E_k = E_{\Omega} + (J - J\cos(k)).$$

When $D=2$, i.e., there are two down-spin particles, they move in the form of the same independent plane waves when they are far away. But because of the interactions that become operative when they come on near neighbour sites, they can exchange their momenta or wave-vectors, and also have a scattering phase-shift in their wave-functions. Indeed, Bethe showed that in this case the solution for the wave-function can be written in the form

$$\psi(m_1, m_2) \propto [e^{i(k_1 m_1 + k_2 m_2 + \frac{1}{2}\phi_{1,2})} + e^{i(k_2 m_1 + k_1 m_2 - \frac{1}{2}\phi_{1,2})}], \quad m_1 < m_2;$$

with $2\cot(\phi_{1,2}/2) = \cot(k_1/2) - \cot(k_2/2)$ determining a 'two-particle phase shift' as a function of the wave-vectors k_1 and k_2 of the particles, and the energy eigenvalue being given by

$$E(k_1, k_2) = E_{\Omega} + (J - J\cos(k_1)) + (J - J\cos(k_2)).$$

The requirement of periodic boundary conditions, namely $\psi(m_1, m_2) = \psi(m_2, m_1 + N)$ now implies that the wave-vectors are determined in terms of two integers ν_1 and ν_2 as

$$Nk_1 - \phi_{1,2} = 2\pi\nu_1; \quad Nk_2 - \phi_{1,2} = 2\pi\nu_2.$$

Bethe showed that this pair of coupled equations for the wave-vectors k_1 and k_2 has $(N-1)(N-2)/2$ real solutions, corresponding to 'scattering states', for $0 \leq \nu_1 \leq (\nu_2 - 2) \leq N - 3$ and $(N-1)$ 'complex conjugate pair' solutions, corresponding to 'bound states' (or 'anti-bound states' depending on the sign of J), for $\nu_1 = \nu_2 - 1$ or for $\nu_1 = \nu_2$, accounting for all of the total of $N(N-1)/2$ eigenstates corresponding to $D = 2$.

Next came the great leap; the 'Bethe's Ansatz', that for all $D \geq 2$, the solution for the wave-function (say for $m_1 < m_2 < m_3 \dots < m_D$) can be found in the form

$$\psi(m_1, m_2, \dots, m_D) \propto \sum_P e^{i[\sum_{j=1}^D k_{Pj} m_j + \frac{1}{2} \sum_{j=1}^{D-1} \sum_{j'=j+1}^D \phi_{Pj, Pj'}]},$$

Box 1. continued...



involving the same ‘two-particle phase shift functions’ as defined above (i.e., $2\cot(\phi_{j,j'}/2) = \cot(k_j/2) - \cot(k_{j'}/2)$). Here P denotes an index that identifies and runs over the $D!$ permutations of the numbers $1, 2, \dots, D$, and Pj denotes the number that replaces j under the permutation P . After guessing the form of the solution, Bethe showed that it does satisfy the Schrödinger equation for $\psi(m_1, m_2, \dots, m_D)$, the energy eigen-values being now given by

$$E\{k_j\} = E_\Omega + \sum_{j=1}^D (J - J\cos(k_j)).$$

The periodic boundary condition now determines the wave-vectors k_1, \dots, k_D in terms of D integers ν_1, \dots, ν_D via the set of D coupled equations:

$$Nk_j - \sum_{j' \neq j} \phi_{j,j'} = 2\pi\nu_j$$

and the solutions for these (including scattering states that correspond to real values of k and bound states that correspond to complex k) exactly account for all the eigenstates with D down spins.

The Bethe Ansatz wavefunction and its ramifications have many beautiful mathematical properties, which have been explored in great detail in the enormous amount of research in the area of ‘exactly integrable quantum many body problems’ that followed in the wake of Bethe’s work. For example the wave function can be shown to correspond to a description of an arbitrary scattering as a succession of pair-wise scatterings, leading to the ‘factorizability of the S matrix’ into a product of two-body S matrices [13]. In the thermodynamic limit ($N, D \rightarrow \infty$), it embodies an infinite number of conserved quantities. An incredible variety of problems in condensed matter physics and quantum field theory in one (space) dimension [13, 14, 17] such as gases of Fermi or Bose particles with delta function interactions, the Hubbard model, etc., as well as models of magnetic impurities in metals [16], have been shown to be soluble using the Bethe Ansatz. It is also profoundly related to exactly solved models of statistical mechanics [12], and to the ideas of quantum inverse scattering and conformal field theory [15]. For more information, see references [12-17].

celebrated monographs in the *Handbuch. der Physik* 24: the first on the ‘Quantum Mechanics of 1 and 2-electron problems’ by Bethe alone, and the second entitled ‘Electron theory of metals’, co-authored with A Sommerfeld. Except for the first chapter of about 35 pages where Sommerfeld elaborated his ‘free electron theory of metals’ (see chapter 2 of [6]) the rest of the 300 page article was written by Bethe. It reviewed the papers already published in the field, such as the seminal work by Bloch which showed that electrons moving in the pe-



riodic potential arising from a crystal lattice of ions essentially continued to propagate like free electrons but with an altered energy-momentum relation; the work of Brillouin on the reciprocal lattice, etc. In addition, it set out a host of new ideas, laying the foundations of the modern theory of solids, and very soon achieved the status of a classic. A delightful interview of Bethe regarding this work by N D Mermin was published in *Physics Today* in June 2004 and was reproduced in the previous issue of *Resonance*¹.

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6. The Bethe–Peierls Method

In 1935, while visiting the H H Wills Physical Laboratory at the University of Bristol, Bethe's attention was drawn by W L Bragg to the problem of the order-disorder transition in binary alloys such as β -brass. Here, in a bipartite crystal lattice (i.e, a lattice which is made up of two sublattices '1' and '2' whose sites are each other's nearest neighbours, eg., a BCC lattice which is made up of two SC sublattices) each site/cell is randomly occupied with equal probability by either of two types of atoms A and B (Cu and Zn in case of brass) at high temperatures. But below a sharp transition temperature, a super-lattice with a doubled unit cell forms, with one sublattice (say '1') preferentially occupied by A atoms, and the other sublattice ('2') preferentially occupied by B atoms.

At the simplest level of modelling of such alloys, the energetics of an arbitrary configuration of A and B atoms can be written in terms of a variable σ_j , which takes the value +1 if the site j is occupied by an A atom, and the value -1 if it is occupied by a B atom, as the sum

$$\mathcal{E} = \frac{1}{2}V \sum_{\langle j,j' \rangle} \sigma_j \sigma_{j'}, \quad (1)$$

where $\langle j, j' \rangle$ indicates that the sum is only over pairs of sites which are nearest neighbours. According to the

In binary alloys such as β -brass, below a sharp transition temperature, a super-lattice with a doubled unit cell forms.



Bragg and Williams published the simplest 'mean-field theory of the order-disorder transition in binary alloys.

rules of statistical mechanics, the probability of occurrence of different configurations of A and B atoms is proportional to the Boltzmann weight $\exp(-\mathcal{E}/k_B T)$ where T is the absolute temperature of the system and k_B is the Boltzmann constant. This leads to a competition between energy and entropy which is responsible for the transition.

At high temperatures, the entropy of the many possible (2^N if N is the number of lattice sites) configurations dominates and $S_j \equiv \langle \sigma_j \rangle = 0$ (for a 50% alloy), corresponding to the equal probability of occupancy of sites by A or B atoms. But at low temperatures, the lowest energy configurations in which A atoms are surrounded by B atoms and vice-versa dominate (for positive V) leading to $S_j = \pm S \neq 0$ alternately on sublattices '1' and '2'.

Bragg and Williams had just then published [18] the simplest 'mean-field theory' of this transition, in analogy with Weiss's molecular-field theory of ferromagnetism, which assumes that the spontaneously generated self-consistent molecular 'field' that produces the 'ordering' in a given unit cell is determined completely and uniquely by the 'average order S ' in the crystal as a whole. Accordingly, the energetics of the occupancy of the sites of sublattices '1' and '2' are simply approximated by

$$E_1 = -zV\sigma_1 S; E_2 = zV\sigma_2 S \quad (2)$$

with z being the co-ordination number (i.e., the number of nearest neighbours) of the lattice (6 for the SC lattice, 8 for the BCC lattice, etc). A calculation of S using this energetics leads to the simple self consistent equation

$$S = \tanh\left(\frac{zVS}{k_B T}\right) \quad (3)$$

which has a nonzero solution for S which first appears



below a critical temperature given by $k_B T_c = zV$ and grows to the maximum value of 1 as $T \rightarrow 0$. However this approximation has the unsatisfactory feature that for *all* $T > T_c$, the energy, specific heat, etc are all zero, and the entropy is the maximum possible, namely $Nk_B \ln(2)$!

Bethe's challenge was to go beyond this simple Bragg-Williams(BW) theory, and include the effects of 'fluctuation corrections' to the above. This investigation led to his paper entitled 'Statistical theory of super-lattices' [19] In this paper Bethe introduced the important concepts of 'short range order', characterized for example by the nearest neighbour correlation $C \equiv \langle \sigma_j \sigma_{j'} \rangle$ (referred to as 'order of neighbours' in the paper) as distinct from the 'long range order' (referred to as 'long-distance order' in the paper) characterized by S (in the BW theory, $C = S^2$) and showed that only the latter disappears above the critical temperature. The paper is astounding in that it contained the germs of many of the ideas that were reinvented in later developments culminating in the modern theory of critical phenomena. Furthermore, Bethe introduced a series of ingenious approximations such that, in the limit of large coordination number, $z \rightarrow \infty$ keeping zV fixed, the BW theory reemerged, but for any finite z , the results systematically improved upon the BW theory. In particular, sensible results were obtained for physical quantities for $T > T_c$ as well, as illustrated in *Figure 2*, taken from the above paper. The approximations were further developed in follow-up work by Peierls [20], and are referred to as the 'Bethe-Peierls Method' or the 'Bethe-Peierls Approximation' and have been widely used ever since. This work was also the precursor to later work by Kikuchi, Suzuki and others on self-consistent cluster approximations for the statistical mechanics of lattice models.

Bethe developed a theory of order-disorder transitions that included 'fluctuation corrections' to the Bragg-Williams theory.

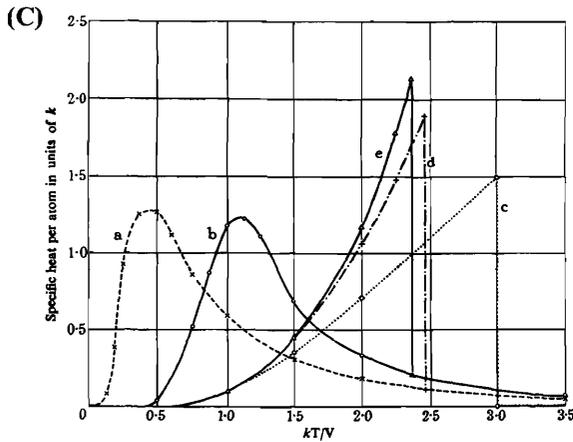
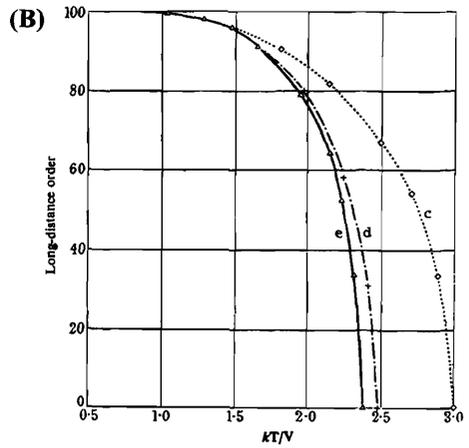
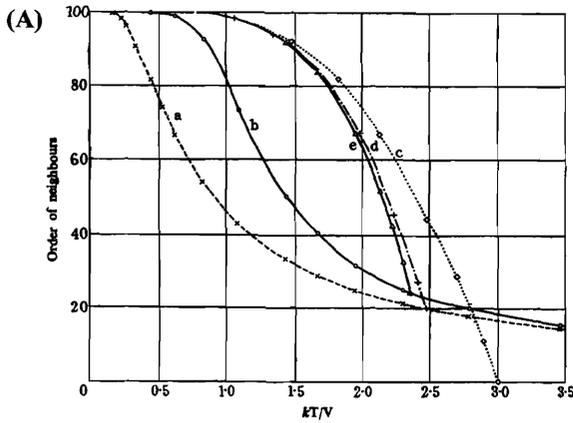


Figure 2. Results for the 'order of neighbours' (A), long distance order (B) and the specific heat (C) obtained by Bethe. The curves labelled (a)- (e) are obtained respectively using : (a) the linear chain approximation (equation (4) of [19]); (b) the second approximation with no long-distance order (§ 5 of [19]); (c) the approximation of Bragg and Williams; (d) and (e) the first and second approximation including order at long distances (§§ 6 and 8 of [19]).

7. Concluding Comments

Around 1934 Bethe's interests had already started shifting towards nuclear physics. The above paper, and another paper on the same subject, entitled 'Order and disorder in alloys', published in 1938, were the last of his contributions to solid state theory. Solid state theory's loss, however, was to the gain of many other fields of physics, as elaborated by the other tributes published in the previous issue of *Resonance* to one of the most extraordinary and gifted minds of the last century.



Suggested Reading

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