Nucleosynthesis and Energy Production in Stars: Bethe's Crowning Achievement

Indranil Mazumdar

On the 6th of March this year, the seemingly immortal Prof. Hans Bethe passed away, at the ripe age of ninety eight. His mighty mathematical prowess, coupled with his amazing virtuosity in applying the tools of theoretical physics, helped him make original contributions to an astonishingly large number of topics in theoretical and applied physics. In this age of hyper-specialization and super computers, Bethe steered his path through the myriad problems at the frontiers of physics and kept on calculating numbers on his trusted slide rule. In this article we will discuss one of his great contributions that fetched him utmost satisfaction, in addition to the Nobel Prize.

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Introduction

The moonless night sky, studded with twinkling stars has captivated and inspired the human mind from the beginning of civilization. The quest to understand stars has come a long way from the time of early philosophers and star watchers. Today we understand that the study of the structure and evolution of the stars is essential to answer a large number of questions about the universe, including the chemical composition of our galaxy. Despite being astronomically far away from us, the brightness of stars observed from the Earth proves that stars are losing energy at a prodigious rate. The luminosity, i.e. the total amount of energy radiated per second by the Sun, our own and nearest star, is about 3.84×10^{26} Js^{-1}. This is equivalent to releasing the world's annual energy production in one ten-millionth of a second. In

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comparison, for another star belonging to the spectral class of O3 and with a hundred or so solar masses ($M_\odot$) and surface temperature of about 50,000 K the luminosity would be a million times that of the Sun. How does the star replenish this loss of energy so as to burn so bright for millions of years without cooling down?

The obvious answer is a continuous generation of energy inside the star. Thanks to the pioneering contributions of Hans Bethe and many other physicists over the last seven decades, we now have a very clear picture of the mechanisms for energy production in stars. At this stage, we must understand and appreciate the important distinction between stable and unstable stars. For a steady star there is an equilibrium between the amount of energy radiated and the amount of energy generated inside the star. A star would erupt catastrophically or periodically if it is unable to maintain this balance. Creating more energy than it is able to get rid of would warm up the star, whereas it would be cooling down if it keeps radiating without sufficient internal generation of energy. Of course, the stars do cool down or warm up over millions of years which is the problem of stellar evolution. But observed over centuries or thousands of years the luminosity and the surface temperature remain steady with minor fluctuations. It is now well accepted that the present luminosity of the Sun has roughly been constant for the last 2 billion years. Such constancy in its brightness and temperature enables astronomers to apply the principle of equilibrium and calculate the amount of energy generated inside the star that would balance the observed luminosity. The principal aim of this article is to discuss the mechanisms responsible for the energy generation process in stars in their steady state.

The source of the energy that sustains the luminosity of the Sun was one of the great mysteries of the late nineteenth and early twentieth centuries. By then, the defining solar parameters of mass, radius, and luminosity were known with sufficient precision and the resolution of the problem required a suitable model calculation to reproduce the observed luminosity. It was obvious, even at that time, that the chemical processes, the common source of energy in our day to day life, could not account for the huge rate of energy release from the Sun. If the Sun consisted of carbon and oxygen only and the solar energy were generated due to the burning of carbon, then all of it would burn up in only about fifteen hundred years. This follows from the following considerations: burning or oxidation is a chemical process that leads to the release of energy by rearrangement of chemical bonds. The burning of coal ($C + O_2 = CO_2$) releases 4 eV of energy per atom of carbon. If the Sun consisted of carbon and oxygen only and the solar energy were generated due to the burning of carbon, then for the given solar mass (2×10^{30} kg) and luminosity (3.84×10^{26} Js^{-1}), it is straight forward to check that all of it would burn up in only 1500 years.

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We are now left with two other possible sources of solar energy, namely, gravitational and nuclear. While nuclear energy became a promising candidate only after the emergence of nuclear physics in the 1930s, gravitation as a source of solar energy was established by the late nineteenth century. The essential idea arose from the knowledge of transformation of potential energy from the shrinking of a large body under gravitational attraction towards its centre. It was found that such a source of energy could only keep the Sun shining at the observed luminosity for no more than ten million years. The inadequacy of this theory was pointed out both by biologists and geologists. They argued that both the biological and geological evolution of the Earth demand that the Sun must have shone at its present rate for much longer than ten million years. The discovery of radioactivity and the analysis of rocks for their

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uranium/lead ratio showed this time to be more than 10^9 years.

Today, we understand that both gravitational contraction and thermonuclear reactions play important roles in producing energy over the lifetime of a star. Of course, thermonuclear fusion is the primary source of energy for the long stable period in the life of a star. The formation of a star is initiated by gravitational contraction of massive interstellar gas clouds, predominantly composed of hydrogen. The condensation of the dusty interstellar gases is counteracted by the outward pressure generated by the translational kinetic energy of each gas particle. It was shown by the English physicist James Jeans that for a given temperature and particle number density the mass of the cloud has to be above a certain value so that the force of gravity can overcome the gas pressure and contraction can occur. This critical mass, known as Jeans mass is given by the expression

$$\mathrm{M}_j = \frac{9}{4} \left(\frac{1}{2\pi n}\right)^{1/2} \frac{1}{m^2} \left(\frac{kT}{G}\right)^{3/2}$$

where n is the particle number density, m is the mass of the average gas particle in the cloud, T is the gas temperature, G is Newton's gravitational constant, and k is Boltzmann constant. The contraction of the cloud leads to the formation of smaller fragments which also continue to contract independently, provided they satisfy the Jeans criterion. The fragments, called protostars, keep condensing until the temperature in the core rises sufficiently to trigger thermonuclear fusion (also called 'burning') of hydrogen to form helium. The energy released from the hydrogen burning process halts the gravitational contraction of the star which now radiates energy at a rate equal to that liberated by the nuclear reactions. The beginning of hydrogen burning marks the birth of a full fledged star and the protostar joins the so-called main sequence. The time taken for a fragment to reach this stage depends upon the mass of the fragment and generally varies from 10^5 to 10^9 years. The gravitational contraction resumes after the hydrogen content in the core of the star gets sufficiently depleted and nuclear reactions can no longer supply enough energy to support the star against gravity. The internal temperature starts rising again till the burning of the next available fuel, helium, begins and further gravitational contraction is arrested. This self regulatory mechanism takes the star through a sequence of nuclear burning stages, prolongs its life and synthesises heavier and more tightly bound atomic nuclei. The number of stages a star goes through in the thermonuclear burning process is governed by its mass. For a low mass star, the hydrogen burning is ignited only if the mass is $0.08M_\odot$ ($M_\odot$ symbolises the solar mass). For the ignition of helium burning the initial mass of the star should be greater than $0.5M_\odot$. Massive stars with masses between 8 to 11 $M_\odot$ progress to carbon burning to produce heavier elements like magnesium, sodium and neon. Stars with masses greater than $11M_\odot$ progress through every stage of thermonuclear fusion upto the synthesis of elements near iron. The binding energy per nucleon of the atomic nuclei increases with the mass and reaches a broad maximum around the iron region. Thermonuclear fusion involving isotopes of iron, nickel, cobalt, etc. are not exothermic and do not produce energy. Any further synthesis of elements beyond the iron region takes place by neutron capture processes at the final stage of stellar evolution. However, we will not discuss these advanced burning processes any further in this article.

Evidently, the process of stellar evolution is intimately connected with the process of nucleosynthesis. In fact, of the more than three hundred known stable isotopes, barring the nine lightest ones, all are produced inside the stars. Five of the nine lightest nuclei, namely, ^1H, ^2D, ^3He, ^4He, and ^7Li were produced in the Big Bang. These light isotopes together with the microwave background

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radiation are the relics from the distant past that provide evidence in favour of the standard big bang model of cosmology. The remaining isotopes of ^{6}Li, ^{9}Be, and ^{10,11}B are believed to be produced in spallation reactions of high energy protons on ^{12}C and ^{16}O. According to the most modern work it is the high energy C and O which are ejected from massive stars and supernovae that bombard the proton.

Hydrogen Burning in Stars

In this section we will discuss, in some detail, the hydrogen burning process. Historically, Jean Perrin and Arthur Eddington were the first to point out, independently, that the fusion of four atoms of hydrogen into one atom of helium could release enough energy to resolve the problem of energy production in stars. However, a formal theoretical explanation and clearer picture emerged only after the maturing of quantum mechanics and nuclear physics. Robert Atkinson and Fritz Houtermans were the first to suggest that in the very hot interior of a star, atomic nuclei could penetrate into other nuclei and release energy. They suggested the idea of a cyclic nuclear reaction leading to the fusion of four protons to form an α-particle. This was indeed a major breakthrough in the right direction. The key to energy production from the fusion of atomic nuclei lies in the mass dependence of the binding energy per nucleon for atomic nuclei. This is best understood from the characteristic shape of the binding energy curve as shown in *Figure* 1. The binding energy per nucleon keeps increasing with mass number and reaches a broad maximum around mass 60 and then decreases gradually with higher values of mass. The nucleus ^{56}Fe has the highest binding energy per nucleon and so is most tightly bound. A nuclear reaction will be exothermic or release energy if a heavy nucleus from the right side of the iron peak fissions into smaller fragments with masses around 60. The other obvious alternative would be to fuse two

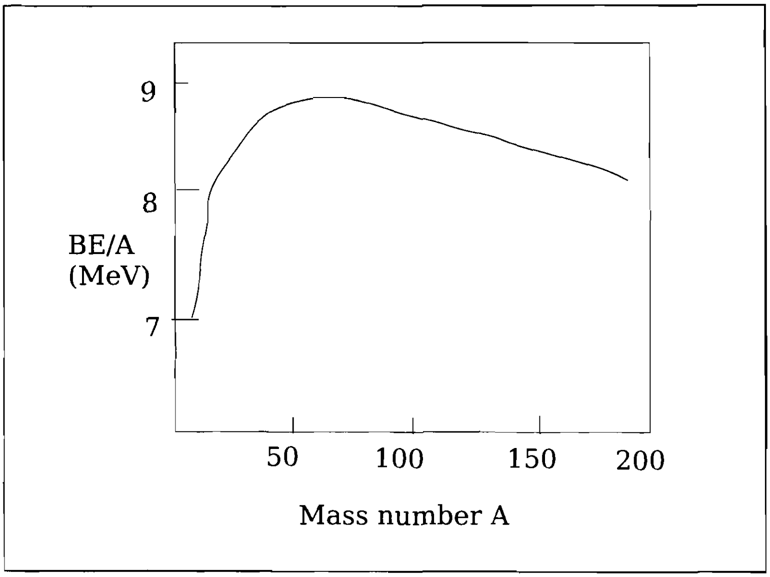

Figure 1. The binding energy curve showing the variation of binding energy per nucleon with the mass number.

of the lightest nuclei and form a heavier one and release energy. Most of the stars are known to be remarkably similar in composition and are predominantly made up of hydrogen and helium and a few per cents of heavier elements. This is why the fusion of hydrogen nuclei to form helium appears to be the most promising source of energy inside the star, as concluded by Atkins and Houtermans.

The possibility of fusion of charged particles inside the star met with initial skepticism. The reason for this being that the temperature inside a star, say, the Sun, is not high enough for the positively charged nuclei to gain enough kinetic energy and overcome the mutual Coulomb barrier and fuse. For the interior temperature of the Sun to be $\sim 10^7$ K the mean thermal energy E= kT is of the order of a keV. This is much below the potential barrier height between two light nuclei like deuteron or hydrogen for a distance of a few fms (1 fm = 10^{-15}m). However, quantum mechanically there may be tunneling through the barrier enabling the reaction to take place. Of course, the probability of penetration through the barrier is higher at higher energies. The quantum mechanical theory of barrier penetration was worked out by George Gamow and was applied by Atkinson and Houtermans in their calculations for thermonuclear reaction rates in stellar interiors. While the probability for barrier penetration goes up with increasing energy the possibility of having a particle of high energy at a given temperature decreases rapidly with increasing energy. The net result of these two opposing effects give rise to an energy window for the fusion of charged particles. Mathematically, the reaction rate between two nuclei x and y is given by the expression

$$R_{xy} = n_x n_y \left(\frac{8}{\pi m_r}\right)^{1/2} \left(\frac{1}{kT}\right)^{3/2} \int_0^\infty S(E) \exp\left[-\frac{E}{KT} - \left(\frac{E_G}{E}\right)^{1/2}\right] dE \qquad (1)$$

where the function $S(E)$ is determined by the nuclear physics of fusion, E_G is called the Gamow energy and depends on the charge on the nuclei and their reduced mass m_r and the exponential term is the product of the Coulomb barrier penetration probability and the Boltzmann factor.

Bethe and the Solar Cycle

The concept of hydrogen burning and the work on barrier penetration mechanism and thermonuclear reaction rates in the stellar atmosphere provided sufficient conceptual and mathematical foundation for future and decisive work by Bethe, Charles Critchfield and Carl von Weizsacker.

The hydrogen burning process cannot take place by simple fusion of two protons or α-particle or the fusion of a proton with α-particle. This is because of the absence of any stable mass-5 or mass-8 nuclide. It was Weizsacker who suggested in 1937 that the only possible reaction for the fusion of two protons is

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{d} + \mathrm{e}^{+} + \nu_e.$$

It was at this stage Hans Bethe entered the picture and wrote a landmark paper with Critchfield on the formation of deuterons by proton combining. The reaction under consideration is exceedingly improbable as it involves a weak β-decay process and was considered to be too rare to account for energy production in stars. That this belief was unfounded was proved by Bethe and Critchfield who calculated the probability of the reaction and showed that this reaction gives the correct rate of energy production in the Sun. They used Fermi's theory of β-decay for positron emission and calculated the penetration of protons through their mutual potential barrier and the transition probability to the deuteron state exactly. This is the main nuclear reaction in the Sun and paves the way for much faster reactions to synthesize ^{4}He nuclei. Once the deuteron is formed, synthesis of ^{4}He takes place via three competing branches shown in *Figure* 2. The main branch has three steps. The formation of the deuteron is followed by the fusion

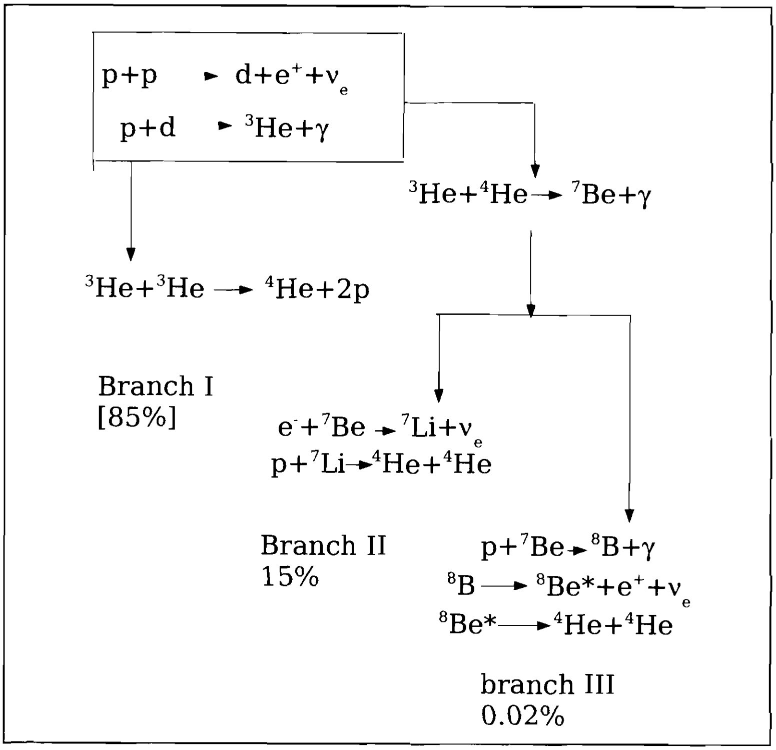

Figure 2. The three branches of the proton-proton cycle in the hydrogen burning phase.

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of a proton and the deuteron to produce ^{3}He and a γ-ray. Two ^{3}He nuclei, so produced, fuse to form a ^{4}He and two protons. The net result is the consumption of 4 protons to form a ^{4}He nucleus with the energy released being carried away by the particles and the γ-rays at each step of the sequence. In the other two branches II and III, the ^{3}He nucleus fuses with a pre-existing ^{4}He nucleus to form ^{7}Be. The ^{4}He nucleus actually acts as a catalyst and the end result of both the sequences is the formation of two new ^{4}He nuclei.

The proton–proton chain was successful in predicting the rate of energy production in stars with masses comparable to that of the Sun, but failed to explain the enormous increase in luminosity in more massive stars. Once again Bethe played the lead role in resolving this puzzle. It was known from the earlier work of Eddington that central temperature of the stars increase slowly with the mass of the star. The p–p chain which has a moderate T^4 dependence on temperature, could not therefore, account for the luminosity in heavier stars with a moderate increase in internal temperature. Evidently, a mechanism with a much stronger dependence on temperature was required. The temperature dependence is governed by the Coulomb barrier and Bethe correctly concluded the need for heavier nuclei in the burning process. However, since the abundance of heavy elements is very small in stars it was necessary for Bethe to consider reactions that would regenerate the heavy elements at the end of the cycle and would prolong the hydrogen burning process. In a monumental paper published in March 1939 in *Physical Review*, Bethe established the CNO cycle as the source of energy of the heavier stars in the main sequence. Bethe found that for all nuclei lighter than carbon, the reaction with proton generates the α-particle without recovering the original nucleus. While for all nuclei heavier than fluorine leads to the radiative capture of proton and destroys the original nucleus.

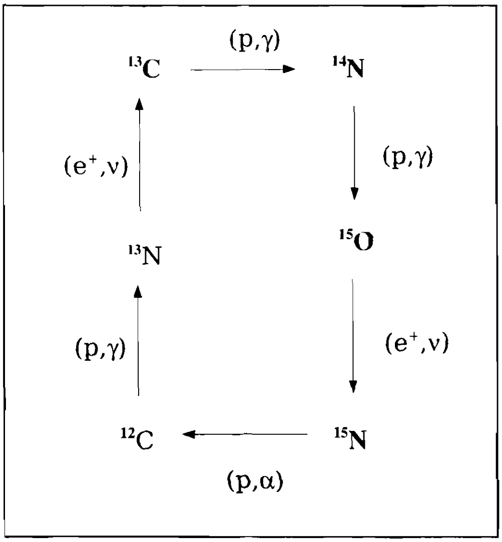

Figure 3. The set of reactions in the CNO cycle converting hydrogen into helium.

The carbon-nitrogen-oxygen or CNO cycle is illustrated in *Figure* 3. The net effect is to produce a ^{4}He nucleus from four protons with the participation of the carbon, oxygen and nitrogen nuclei in the reactions as catalyzers so as to reappear at the end of each cycle. As in the p–p chain the energy released in the CNO cycle appears in the form of the γ-rays and the kinetic energy of the particles. The rate of energy production is governed by the slowest reaction in the cycle, namely, $p + {}^{14}N \rightarrow {}^{15}O + \gamma$.

Bethe's seminal work on the CNO cycle originated from a small conference in Washington in early 1938. It was only at this conference that he had his formal initiation in astrophysics. By that time Bethe had established himself as one of the founders of modern nuclear physics. His deep insight and profound knowledge of atomic nuclei together with his amazing skill to calculate led him to the solution of the problem of energy production in stars. He can truly be credited for founding the subject of modern nuclear astrophysics. Bethe remained at the forefront of nuclear astrophysics till the very end and continued to write seminal papers on solar neutrino problem, supernovae, etc. Those are topics for another article.

Suggested Reading

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