Dynamics of the Sun-Earth-Moon System

S M Alladin and G M Ballabh

S M Alladin is a retired professor of Astronomy, Osmania University, Hyderabad. He specialized in dynamical astronomy and did research on the dynamics of colliding and merging galaxies. He is interested in the interaction between religion and science.

G M Ballabh is a retired professor of Astronomy, Osmania University, Hyderabad. His field of research includes dynamical astronomy and positional astronomy. At present he is the Chairman of the Standing Advisory Committee for the Positional Astronomy Centre, Kolkata

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The dynamics of the Sun-Earth-Moon system is discussed with special attention to the effects of Sun's perturbations on the Moon's orbit around the Earth. Important secular effects are the regression of the nodes, the advance of the perigee and the increase in the Moon's mean longitude. We discuss the relationship of the first with the occurrence of eclipses, the second with the fluctuations in the moon's synodic period, and the third with the slowing down of the Earth's rotation due to tidal friction. The Sun-Earth-Moon system is compared with other triple celestial systems with regard to the intensity of tidal effects.

1. Introduction

The Sun-Earth-Moon system (*Figure* 1) is of vital importance for several reasons. It is our native system and is the system with which we are most intimately associated. It has stimulated the intellect of man from time immemorial and continues to do so. It was the first three-body problem to be studied with Newtonian mechanics. It is now a very well studied system and hence it can be used as a standard system with which other triple systems may be compared. Abhyankar [1] has given a good account of the Indian contributions to this problem.

The Holy Quran has drawn attention to the Sun-Earth-Moon system. In one place (Chapter 11, verse 6) it says:

"He (Allah) it is Who made the Sun radiate a brilliant light and the Moon reflect a lustre and ordained for it stages that you might know the number of years and

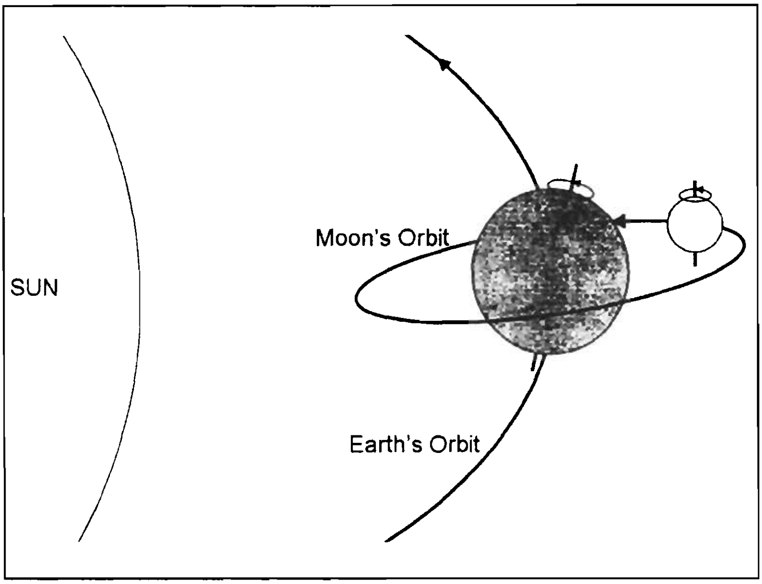

Figure 1. The Sun-Earth-Moon system.

mathematics. Allah has not created this but with truth. He details the signs for a people who have knowledge."

As indicated in the above verse, the study of the Moon has been particularly useful in the development of mathematics. The motion of the Moon is very complicated. Sir Isaac Newton is supposed to have told his friend Halley that lunar theory "made his head ache and kept him awake so often that he would think of it no more" [2].

The Earth and the Moon are sufficiently close to the Sun and hence the Moon's orbit around the Earth is appreciably perturbed by the gravitational attraction of the Sun. Smaller perturbations arise due to the attraction of other planets and also due to the deviations of the shape of the Earth and the Moon from perfect spherical symmetry. Non-gravitational effects also affect the motion.

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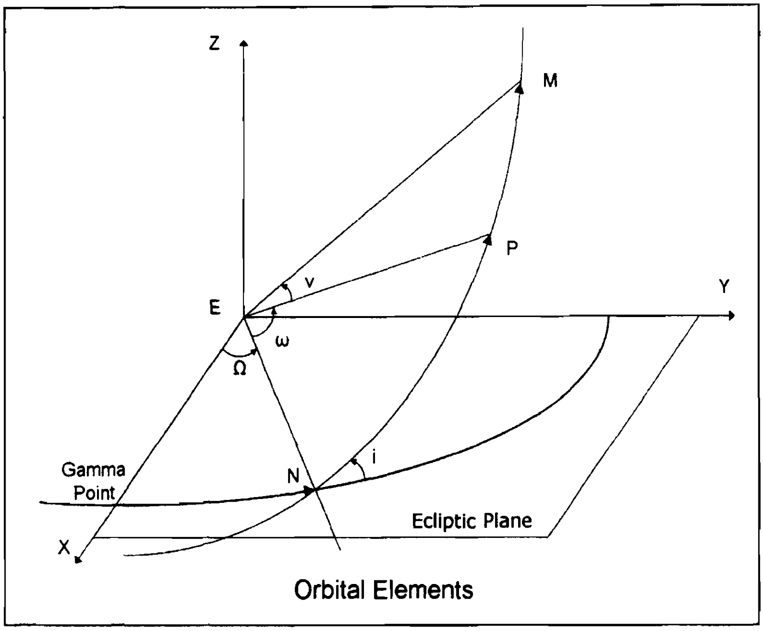
The Moon moves around the Earth in an approximately elliptic orbit which can be described by the six orbital elements shown in *Figure* 2, namely: semi-major axis 'a' which gives the size of the orbit, eccentricity 'e' which

Figure 2. Orbital elements.



describes the shape of the orbit, inclination 'i' which gives the orientation of the orbit with respect to the ecliptic, longitude of the node 'Ω' which gives the position of the intersection point of the ascending node of the lunar orbit and the ecliptic, longitude of perigee 'ω' which defines the point in the orbit at which the Moon is closest to the Earth and the time of perigee passage 'T' which is the time at which the Moon is at perigee. On account of the perturbations mentioned earlier all these elements change with time.

At present when powerful computers are available the motion of the Moon is determined accurately by numerical integration of its equations of motion.

Lunar theory has been developed analytically by very eminent mathematicians such as Newton, Euler, Laplace, Poisson, Hansen, Delauny, Hill and Brown. At present when powerful computers are available the motion of the Moon is determined accurately by numerical integration of its equations of motion. A good textbook on the motion of the Moon has been written by Cook [3].

In this article we shall briefly review the salient features of the lunar theory arising out of the Sun's perturbation on the lunar orbit. We shall also mention non-gravitational affects arising from the slowing down of the Earth's rotation due to tidal effects.

2. Sun's Perturbations on the Moon's Orbit

If we consider the Earth, the Moon and the Sun as a system of three mass points, then the equation of motion of the Moon with respect to the Earth as origin is given by (see [2]):

$$\ddot{\vec{r}} + \frac{G(E+m)\vec{r}}{r^3} = GM_\odot \left(\frac{\vec{\Delta}}{\Delta^3} - \frac{\vec{r}\,'}{r'^3} \right), \qquad (1)$$

where $\vec{r}$ is the radius vector of the Moon, $\vec{r}\,'$ is the radius vector of the Sun and $\vec{\Delta} = \vec{r}\,' - \vec{r}$ is the radius vector of the Sun from the Moon (*Figure* 3); E, m and $M_\odot$ are the masses of the Earth, Moon and Sun respectively and G is the gravitational constant. The second term represents the principal force on the Moon due to the attraction of the Earth while the term on the right hand side represents the tidal force on the Moon due to the Sun. It is the difference between the force with which the Sun attracts the Moon and the force with which the Sun attracts the Earth. The Sun is about 3,30,000 times more massive than the Earth while its distance is about 400 times the distance of the Moon. If we put these numeral values we find that the principal force is about 70 times the maximum tidal force.

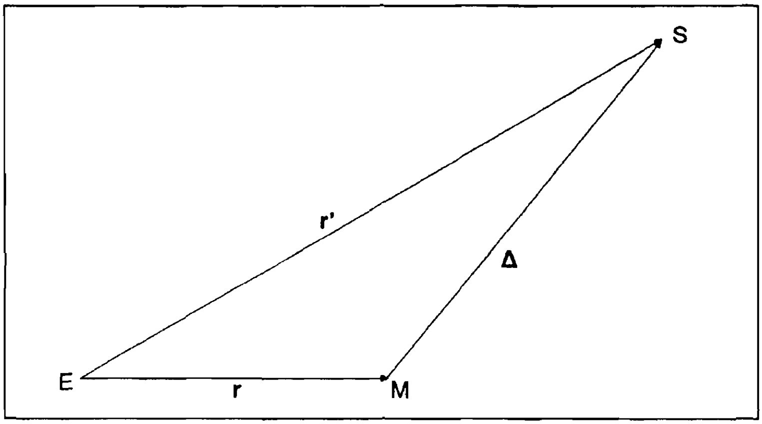


Figure 3.

The right hand side of (1) can be expressed as the gradient of the disturbing function, R, given by:

$$R = GM_{\odot}\left(\frac{1}{\Delta} - \frac{\bar{r}.\bar{r}'}{r'^3}\right) \tag{2}$$

Since $(r/r') \cong 1/400$ we obtain an expansion for R in terms of r/r'. We then express R in terms of the orbital elements. Some of the important terms are as follows:

$$R = \left(\frac{k^2 M a^2_{\text{moon}}}{a^3_{\text{sun}}}\right)[\frac{1}{4} + \frac{3}{8}e^2 - \frac{1}{2}e\cos M - \frac{1}{8}e^2\cos 2M +$$
$$\frac{15}{8}e^2\cos\{2(\Omega+\omega-\psi')\} - \frac{9}{4}e^2\cos\{2(\Omega+\omega-\psi')+M\}+$$
$$\frac{3}{4}\cos\{2(\Omega+\omega-\psi')+2M\} - \frac{15}{8}e^2\cos\{2(\Omega+\omega-\psi')+2M\}+$$
$$\frac{3}{4}e\cos\{2(\Omega+\omega-\psi')+3M\} + \frac{3}{4}e^2\cos\{2(\Omega+\omega-\psi')+4M\}]. \tag{3}$$

In the above expansion terms of higher order than two in e have been ignored.

Here a, e, Ω, ω are the elements of the Moon's orbit and M is its mean anomaly[1]. The angle ψ' is the mean longitude[2] of the Sun.

We can make estimates for the changes in the orbital elements with time by using Lagrange's planetary equations which relate the time derivatives of the elements with the derivatives of the disturbing function with respect to the elements. We get:

$$\dot{a} = \frac{2}{na}\frac{\partial R}{\partial \sigma},$$

$$\dot{e} = \left(\frac{1-e^2}{na^2 e}\right)\frac{\partial R}{\partial \sigma} - \left(\frac{\sqrt{1-e^2}}{na^2 e}\right)\frac{\partial R}{\partial \omega},$$

[1] The mean anomaly M is the angle described by a radius vector with constant velocity. At any time t it is given by:

$$M = n(t-T),$$

where $n(=2\pi/\text{period of revolution of the body})$ is the constant angular velocity and T is the time of perihelion passage.

[2] The mean Sun is a fictitious body which is supposed to move along the celestial equator with the Sun's mean angular velocity n. Then the mean longitude, ψ', of the Sun at any time equals the right ascension of the mean Sun and is given by:
$\psi' = n(t$-the time when the Sun was at the vernal equinox).

$$\dot{\sigma} = \left(-\frac{1-e^2}{na^2 e}\right)\frac{\partial R}{\partial e} - \frac{2}{na}\frac{\partial R}{\partial a},$$

$$\dot{\Omega} = \left(\frac{1}{na^2 \sin i\sqrt{1-e^2}}\right)\frac{\partial R}{\partial i},$$

$$\dot{\omega} = \left(\frac{-\cos i}{na^2 \sin i\sqrt{1-e^2}}\right)\frac{\partial R}{\partial i} + \left(\frac{\sqrt{1-e^2}}{na^2 e}\right)\frac{\partial R}{\partial e},$$

$$\frac{di}{dt} = \left(\frac{\cos i}{na^2 \sin i\sqrt{1-e^2}}\right)\frac{\partial R}{\partial \omega} - \left(\frac{1}{na^2 \sin i\sqrt{1-e^2}}\right)\frac{\partial R}{\partial \Omega}.$$

It may be convenient to use the mean anomaly $M = n(t-T) = nt + \sigma$ instead of σ itself, which is the coefficient of the secular acceleration. This implies that the perturbation function can be written in the form $R(a, e, i, \Omega, \omega, M)$.

The solution of these equations shows that the elements a, e, i, and T undergo only periodic changes, while the elements Ω, ω undergo both periodic and secular changes. The mean values of a, e, and i are:

$$a = 384400\text{km}, \quad e = 0.05490, \quad i = 5^o 09'$$

The following important results of analytical studies may be noted [1,2]

(a) 'a' varies periodically by $\pm 0.9\%$, being maximum at the new moon and full moon and minimum at quadratures. Thus the period of the perturbation is half a synodic month. There is also an overall increase in 'a' causing an increase in the period of the Moon by about one hour, i.e. if the Sun was not there, the month would have been shorter by about one hour. Changes in 'a' over longer time will be discussed later.

(b) From celestial mechanics, the integral of area h is given by:[3]

$$h^2 = \mu p = G(E+m)a(1-e^2). \tag{4}$$

[3] In equation (4), h is a constant which is equal to twice the rate of the description of area by the radius vector, μ is defined as the product of the gravitational constant G and sum of the masses of the Earth and the Moon ($E+m$) while p is equal to $a\,(1-e^2)$, a being the semi-major axis and e the eccentricity of the orbit.

In general, the eccentricity varies by ± 0.0117 i.e., by ± 20%. Eccentricity is the most seriously perturbed element in the periodic perturbations.

Due to solar perturbation 'h' varies periodically by 0.5% in phase with 'a' Equation (4) may be written as:

$$\log(1-e^2) = 2\log h - \log a - \log\mu.$$

Differentiating we get:

$$\frac{-2e\Delta e}{(1-e^2)} = \frac{2\Delta h}{h} - \frac{\Delta a}{a} = 0.001.$$

As $e = 0.054$, we get $\Delta e = -0.009$. The negative sign indicates that the variation of 'e' is opposite in phase with that of 'a' and 'h'. Thus the orbit becomes more elliptical at quadratures. This effect is known as the 'variation' and was discovered by Tycho Brahe in the 15th century AD. The period of this variation is also half a synodic month.

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(c) The inclination i varies periodically by small amounts with three different periods of half year, half sidereal month and half synodic month. The variation in inclination is $\pm 9'$. Therefore $\Delta i/i = 9'/5^o 9' = 0.03$ i.e., the inclination varies by 3%.

(d) The node Ω regresses continuously making one round of 360^0 in 18.6 years. The variation from the average value is $\pm 1^0 40'$. This regression of the node is related with the recurrence of the eclipses.

(e) The perigee advances continuously and takes 8.85 years for a complete revolution of 360^o. The variation from the average value in this case is $\pm 12^o 20'$. The advance of perigee is related with the periodicity in the fluctuations of the Moon's synodic period [4].

(f) The mean longitude of the Moon, $l = \Omega + \omega + n(t-T)$, also varies periodically. The 'evection' is the largest periodic perturbation in Moon's longitude. This effect was known to Hipparchus in the second century BC. The complete coefficient of the term according to Brown's theory is $1^o 16' 26''.4$. It has a period of 31.8 days. This period corresponds to the time interval between two successive passages of the Moon's perigee with respect to the Sun.

According to Brown's calculations there are 155 periodic terms in the expression for the Moon's longitude with coefficients exceeding $0.''1$, and more than 500 smaller ones which though insignificant, may at times add up to a sum which is not negligible, so that they must all be considered if we wish to compute the longitude to $0.''1$. Each of these terms has its own period [5].

An eclipse of the Sun and the Moon can occur only at new and full moon respectively when the Sun, Earth and Moon are nearly in a straight line. This alignment can happen when the Moon is near one of its nodes and the Sun is near the lines of nodes.

3. Secular Effects

3.1 *Regression of the Nodes and its Relationship with the Periodicity of Eclipses*

An eclipse of the Sun and the Moon can occur only at new and full moon respectively when the Sun, Earth and Moon are nearly in a straight line. This alignment can happen when the Moon is near one of its nodes and the Sun is near the lines of nodes. Hence together with the interval between two new moons (synodic month), the interval between two successive passages of the Sun and the Moon through the same lunar node are important. Due to the regression of the nodes these periods are shorter than the complete 360 degrees motion of these bodies in the sky. The interval between two successive passages through the same node by the Moon is called a nodical or a draconic[4] month and is equal to 27.212 days and that by the Sun is called an eclipse year and is equal to 346.620 days.

[4] The term draconic is due to the theory held at one time that the Sun is swallowed by a dragon at the time of a solar eclipse.

Various multiples of the above three periods give:
223 Synodic months = 6585.32 days
242 Nodical months = 6585.35 days
19 Eclipse years = 6585.78 days.

Just as the regression of the nodes is related with the periodicity of the eclipses, the advance of perigee is related with the periodicity of fluctuations in Moon's orbital period.

Therefore the sequence of eclipses will repeat after about 6585 1/3 days or 18 years and 10 1/3 or 11 1/3 days (depending upon leap years). This interval is called a Saros and was known to ancient astronomers who used it in predicting the eclipses. It may be noted that the Saros can be more elegantly expressed in the Lunar Calendar as it has only integral number of months (18 years and 7 months) in contrast to the Julian Calendar where even fraction of a day comes.

A calendar year of 365.242 days exceeds the eclipse year by 18.6 days and 12 synodic months by 10.8 days. This results in the minimum number of eclipses in a calendar year to be two which will be both solar and maximum number as seven in which case 4 solar and 3 lunar or 5 solar and 3 lunar eclipses can take place.

3.2 *Advance of Perigee and its relationship with Periodicity of Fluctuations in Moon's Period:*

As the semi-major axis 'a' and the orbital period 'P' are related by Kepler's third law $P^2/a^3 =$ constant, the period P of the Moon's orbit also undergoes periodic changes on account of solar perturbations. The synodic period (i.e. time taken by the Moon to return to the same phase as seen from Earth, i.e. from new moon to new moon) fluctuates between 29 and 30 days, the average being 29.530588 days. Just as the regression of the nodes is related with the periodicity of the eclipses, the advance of perigee is related with the periodicity of fluctuations in Moon's orbital period.

To illustrate the fluctuations in period of the Moon's synodic months, we have taken the times of 475 new moons from 1975 to 2015. The difference between the actual synodic period and the mean synodic period i.e. amplitude of fluctuation is shown as a function of synodic month in *Figure* 4.

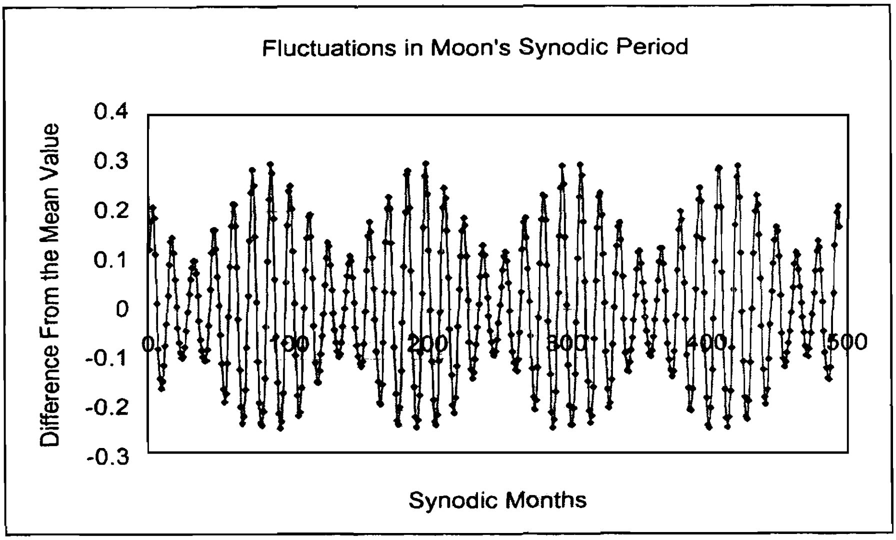
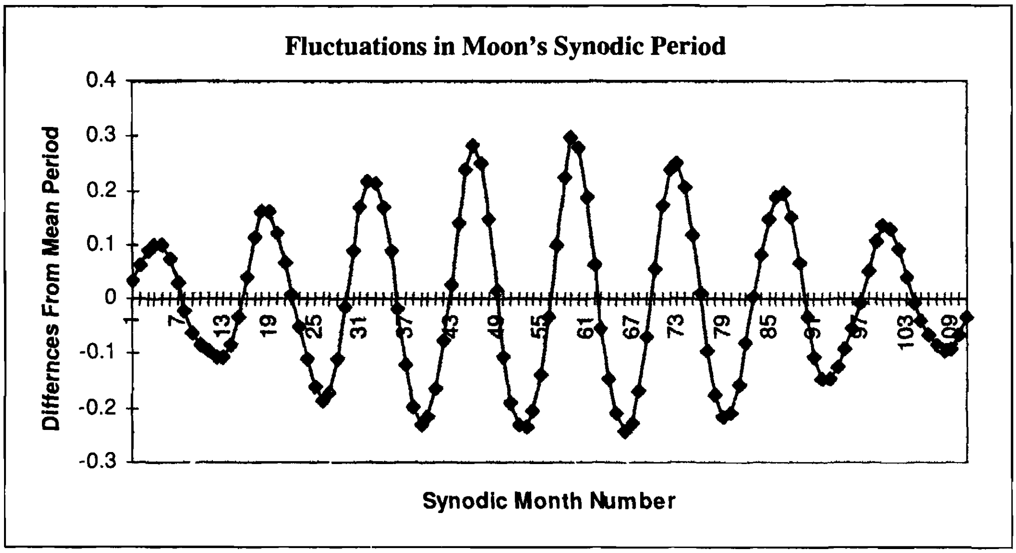


Figure 4.

The following characteristics may be noted. The gross fluctuation pattern repeats itself after about 112 synodic months. This is close to 109.5 synodic months, the period of one complete revolution of Moon's perigee. Within each gross fluctuation pattern there are 8 smaller cycles of about 14 synodic months. We shall discuss these features qualitatively. The first of such patterns is shown in detail in *Figure* 5.

Figure 5.



The gross fluctuation cycle depends upon the phase difference between the perigee of the Moon and the perigee of the Sun. The smaller cycle depends on the phase difference between the perigee of the Moon and the Sun. In one synodic period the perigee moves ahead of the new moon by about 26°. Hence it requires about 14 synodic months for the new moon to come to the same position with respect to its perigee. This corresponds to 13 months of 31.81 days, which is the period of the evection term. That is we have $13 \times 31.81 \approx 14 \times 29.53$. Therefore one complete cycle of fluctuation is about 14 synodic months.

It is noted that the angular separation between the new moon and the perigee of the Sun is less important than the angular separation between the new moon and the perigee of the Moon. Nevertheless it has some effect as can be seen from the 4th and 5th peaks. It may be noted that 13×31.81 days (period of evection term) $\approx$ 14 synodic months. Therefore this is the time interval for the recurrence of lunar perigee (or apogee) at new moon. Further it may be noted that 8×14 synodic months ≈ 9 solar years. Therefore this is the time interval for the recurrence of both lunar and solar perigees (or lunar apogee and solar perigee) at new moon. These recurrences explain *Figure* 4.

Increase in the period means increase in the semi-major axis and hence increase in energy. Maximum increase in energy occurs when the Sun is at perigee (since the tidal force is then maximum) and the Moon is at apogee (since then the Moon moves slowly and has appreciable time for interaction with the Sun). Conversely the maximum decrease in energy with respect to the average occurs when the Sun is at apogee and the Moon is at perigee. The amplitude of various peaks on the positive and the negative side of the mean can be understood by this underlying physics. Four consecutive 112 synodic month cycles were superimposed and it was noted that the repetition is close but not exact over the time scale of 112 synodic months.

From the data studied, $\Delta P/P$ over a period of 112 lunation is $0.3/27.3 = 0.011$. Therefore $\Delta a/a = 2/3(\Delta P/P) = 0.007$. Thus on the larger time scale of about 9 years $\Delta E/E$ varies by about 0.7 percent. This may be compared with the value of 0.9 percent mentioned earlier in connection with variation over one synodic month [4].

4. Secular Acceleration of the Moon

A secular effect of considerable research interest is related with the motion of the mean longitude, l, of the moon. Using the records of ancient eclipses, Halley found that 'l' can be represented empirically by:

$$l = l_o + nt + \sigma t^2 + \text{periodic terms.}$$

Here n is the mean motion of the Moon, t is time measured in Julian centuries and σ is the coefficient of secular acceleration; its observed value is approximately $11''$. Since $\sigma = \dot{n}/2$ hence the mean motion is increasing at the rate of $2 \times 11'' = 22''$ per century.

An outstanding problem in the 18th century was to give a theoretical explanation for this secular acceleration of the Moon. In 1787, Laplace announced that this was due to a slow variation in the eccentricity of the Earth's orbit around the Sun arising on account of planetary perturbations. His theoretical determination of $\sigma(= 10.4'')$ agreed almost exactly with the value deduced from observations. Laplace's value remained unquestioned for over 60 years.

An outstanding problem in the 18th century was to give a theoretical explanation for this secular acceleration of the Moon. In 1787, Laplace announced that this was due to a slow variation in the eccentricity of the Earth's orbit around the Sun arising on account of planetary perturbations.

But in 1853 J C Adams studied the problem in greater detail and modified Laplace's value. Laplace and his followers had investigated the equation of motion as if the Earth's eccentricity was constant, substituting the variable value in the results and determined the acceleration. Adams showed that in a more exact treatment it

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is necessary to introduce the variability of eccentricity in the differential equations themselves when proceeding to higher orders. Adams' method was confirmed by several investigators. Adams obtained $\sigma = 5.7''$ which is lower than the observed value. Hence the problem of the secular acceleration of the Moon was resurrected.

Actually Earth's eccentricity fluctuates periodically, the principal period being 24000 years. Hence the value obtained by Adams arises from a periodic perturbation and not from a secular perturbation. Lyttleton [6] comments as follows: "The periodic term, although highly important in the lunar theory, averages out over long periods of time and so have no secular effect. Now the phenomenon that ancient eclipse records reveal is the small accelerative term ... depending on t^2 and this is quite inexplicable on Newtonian dynamics."

It is now generally believed that the secular acceleration of the Moon chiefly arises from the secular slowing down of the Earth's rotation on account of tidal friction in the seas. But the discussion on the subject is open.

Cook [3] states as follows: "Recent results from laser ranging and occultation observations indicate that the present anomalous acceleration of the Moon is about $-22.8'' \pm 1.5''$ arc/century^2 while the tidal part estimated in various ways seems to be about $-28.8'' \pm 1.5''$ arc/century^2." Huaguan *et al* [7] report a value of $\dot{n} = -25.4'' \pm 0.1''$ arc/ century^2 obtained by a weighted least square fit from lunar ranging data (August 1969 to December 1987).

Let us get an idea of the effect of $\dot{n}$ on eclipse observations. If we take $\dot{n} = -25''$per century^2, then in one year the longitude will be displaced by $\{(1/2)\dot{n}(1/100)^2\} = (1/2) \times 25 \times (1/100)^2 = 0.00125''$. In 1000 years, Δl would be $(1/2) \times 25 \times (1000/100)^2 = 1250'' \approx 1/3$ of a degree, a little less than the diameter of the full Moon or the Sun as seen from the Earth, which is half a degree.

In 3000 years , Δl would be $(1/2) \times 25 \times (3000/100)^2 = 11250'' \approx 3$ degrees. This is about 6 times the diameter of the Moon. The diameter of the Moon is about 3500 km, so the effect would be that the shadow of the Moon on the Earth would be shifted by a huge distance. An eclipse expected at one place would be seen at another place.

Taking the total angular momentum of the Earth-Moon system as constant, we can deduce the deceleration of the Earth's rotation from the acceleration of the Moon's mean motion n by equating the loss of rotational angular momentum of the Moon to the gain in the orbital angular momentum of the Earth-Moon system.

Stephenson and Morrison [8] have made a detailed study of the long-term changes in the rotation of the Earth from 700 BC to 1980 AD. They state as follows:

"Recording of the time of eclipses has been very useful to astrophysicists and geophysicists as from this information we have derived the change in the Earth's rotation and the secular acceleration of the Moon. The ancient timings of lunar eclipses by Babylonian astronomers and the medieval timings of lunar and solar eclipses by Arab astronomers, have been very valuable to us. It is believed that on account of the tidal friction, the Earth's rotation is slowing down at the rate of about 2 milliseconds per century and the Moon is receding from the Earth at the rate of about 4 cms a year."

The ancient timings of lunar eclipses by Babylonian astronomers and the medieval timings of lunar and solar eclipses by Arab astronomers, have been very valuable to us. It is believed that on account of the tidal friction, the Earth's rotation is slowing down at the rate of about 2 milli-seconds per century and the Moon is receding from the Earth at the rate of about 4 cms a year.

Laser ranging can measure distances up to an accuracy of 1 cm. Hence a close comparison between theory and observations can be made in our age. Note that:
4 cm/year=4 km $/10^5$ years $= 4 \times 10^4$ km / 10^9 years $\approx$ (1/10) Moon's distance/10^9 years, i.e. the Moon would recede from the Earth by a distance equal to (1/10) of its present distance from the Earth in about 109 years if we assume constant rate of recession.

Change in constant of gravitation G can also affect n but pulsar observations seem to rule out change in G.

5. Comparison of Sun-Earth-Moon System with Other Celestial Systems

Let us compare the intensity of tidal effects of the Sun on the Moon's orbit with those on orbits of other satellites of planets in the solar system. We can conveniently do so by making use of the concept of tidal radius which is extensively used in stellar dynamics. King [9] showed that a star in a star cluster of mass m moving in a circular orbit of radius R around a galaxy of mass M at a distance D from its centre, would be removed from the cluster if its tidal force due to the galaxy F_t exceeded the main force F_n due to the star cluster, i.e. if

$$\frac{3M}{D^3} \geq \frac{m}{R^3},$$

or

$$R \geq \left(\frac{m}{3M}\right)^{\frac{1}{3}} D \equiv R_t,$$

where R_t is the tidal radius of the cluster. If the cluster was considered stationary we would have 2 instead of 3 in the formula.

If in the place of the galaxy, cluster and star we substitute the Sun, Earth and Moon we get for the tidal radius of the Earth

$$R_t = \left(\frac{E+m}{3M}\right)^{\frac{1}{3}} D,$$

where M, E and m are the masses of the Sun, Earth and Moon respectively and D is the distance of the Earth from the Sun. The sense of rotation of the satellite around the planet affects the tidal limit. As the tidal effect depends upon the time of interaction it is enhanced for a direct satellite and reduced for a retrograde one. Innanen [10] has given the following modification of the tidal radius for direct and retrograde motions in circular orbits :

$$R_{td} = \left(\frac{m}{9M}\right)^{\frac{1}{3}} R \approx 0.7R_t,$$
$$R_{tr} = 2.08R_{td} \approx 1.4R_t.$$

It may be noted that the ratio of the tidal force to the main force is:

$$\frac{F_{\text{tidal}}}{F_{\text{main}}} = \left(\frac{R}{R_t}\right)^3$$

The ratio R_t/R is simply related to the ratios of the orbital periods as can be seen from the following:

For the Sun-Earth-Moon system we have from Kepler's third law :

$$\frac{P^2_{\text{moon}}}{R^3_{\text{moon}}} = \frac{4\pi^2}{G\,(E+m)},$$
$$\frac{P^2_{\text{sun}}}{D^3} = \frac{4\pi^2}{G\,(M_\odot + E + m)}.$$

Therefore

$$\frac{P^2_{\text{sun}}}{P^2_{\text{moon}}} = \left(\frac{D^3}{R^3_{\text{moon}}}\right) \frac{E+m}{M_\odot + E + m}$$
$$= \left(\frac{3R_t^3}{R^3_{\text{moon}}}\right) \frac{M_\odot}{M_\odot + E + m} \approx \frac{R_t^3}{R^3_{\text{moon}}}.$$

We have $P_{\text{sun}}/P_{\text{moon}} \approx 12.4$. Hence $R_t/R_{\text{moon}} \approx 4$.

Therefore the moon is well within R_t as well as within R_{td} and hence is quite safe.

In *Table* 1 we give R_t/R for other satellites of the solar system. It may be noted that *all other satellites are less perturbed by the Sun in comparison with the Moon.*

As another example consider the galaxy, globular cluster and a star in the globular cluster as a triple system. Taking the mass of the galaxy within the distance of

Planet	R_t	R_{td}	R_{tr}	Satellite	R_t/R	Satellite position and Direction of motion	
Mercury	0.23	0.16	0.33	---	---	---	
Venus	1.0	0.69	1.4	---	---	---	
Earth	1.5	1.0	2.2	Moon	3.9		Direct
Mars	1.1	0.79	1.6	Phobes	115.6	Nearest	Direct
				Deimos	46.2	Farthest	Direct
Jupiter	53.0	37.0	76.0	Metis	4151.9	Nearest	Direct
				Elara	4.5	Farthest	Direct
				Ananke	2.5	Nearest	Retrograde
				Sinope	2.2	Farthest	Retrograde
Saturn	66.0	46.0	95.0	Pan	488.7	Nearest	Direct
				Iapetus	18.3	Farthest	Direct
				Phoebe	5.0		Retrograde
Uranus	69.0	48.0	100.0	Cordelia	1410.8	Nearest	Direct
				Oberon	120.4	Farthest	Direct
				Calibon *	9.8	Nearest	Retrograde
				Sycorax *	5.7	Farthest	Retrograde
Neptune	117.0	81.0	169.0	Naiad	2413.7	Nearest	Direct
				Nereid	21.1	Farthest	Direct
				Triton	328.1		Retrograde
Pluto	6.0	4.2	8.6	Charon	393.1		Retrograde

* Provisional names

Table 1. Ratio of tidal radius to distance of satellite from the planet.

the cluster as $10^{11}M_{\rm sun}$, mass of the cluster as $10^5 M_{\rm sun}$ and D and R as 10 Kpc and 10 pc respectively, we get:

$$R_t/R = [10^5/(3 \times 10^{11})]^{1/3} 10\text{Kpc}/10\text{pc} \approx 7.$$

This shows that the stars in the globular cluster are less perturbed by the galaxy than the Moon is perturbed by the Sun. However for clusters closer to the galactic centre, the perturbation on the star could be comparable or even more than the perturbation experienced by the Moon.

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6. Concluding Remarks

The Sun-Earth-Moon system provides a splendid example of the various manifestations of the tidal force. The Sun is not only making the Earth-Moon system go around it in a regular elliptic orbit but it is also actively engaged in modifying this orbit into an orbit of such a high degree of complexity that even a scientist as great as Newton had headache and insomnia when he pondered over it. The Sun brings about numerous changes of various amplitudes and various periods in the six elements of the orbit. The various elements of the orbit are affected differently. The eccentricity fluctuates by about 20% from the average value while the semi-major axis fluctuates by only about 1%. The period of revolution of the Moon around the Earth is also permanently increased by about one hour due to the presence of the Sun. The perigee and the nodes undergo secular changes in such a way that the perigee advances while the node regresses. The two elements are affected in opposite ways by the same tidal force. The bewildering diversity in the effects of one and the same tidal force is highly illuminating.

The dynamics of the Sun-Earth-Moon system also teaches us that all tidal effects are not due to gravitation. The slowing down of the Earth's rotation and the consequent secular acceleration of the Moon are due to tides in the seas. To make progress in this area we need to learn from geophysicists or collaborate with them. The study of the Earth-Moon system thus encourages interdisciplinary research and emphasizes the unity of knowledge.

Let us conclude with the following words of the great celestial mathematician Henri Poincaré[11]:

"The stars send us not only the visible and gross light which strikes our bodily eyes, but from them also comes to us a light, more subtle, which illuminates our minds"

The Sun is a star and Poincaré's inspiring words certainly hold for the Sun-Earth-Moon system.

Suggested Reading

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Address for Correspondence
S M Alladin and G M Ballabh
Department of Astronomy
Osmania University
Hyderabad 500 007, India.
Email:gm_ballabh@yahoo.com