Synchronization

A Heuristic Approach

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This article introduces the concept of synchronization with the help of simple examples of oscillators. Towards the end some real life examples are also discussed.

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1. Introduction

Synchronization, in simple terms, is the adjustment of rhythms of two mutually interacting systems, such as a pair of coupled oscillators. Synchronization was discovered in the seventeenth century by Christiaan Huygens who observed it when working with clocks (see *Box* 1). He saw that two clocks (pendulums) suspended from a common wooden support had the same rhythmic motion. The oscillation of the clocks coincided perfectly and they always moved in opposite directions. Even if the two clocks were disturbed, they managed to reestablish their rhythms. Later on, Lord Rayleigh observed synchronization of oscillating air columns in pipes.

The twentieth century witnessed the growth of electrical and radio engineering. This led to the study of oscillations in electrical circuits. W H Eccles and J H Vincent discovered the synchronization property of a triode generator. They coupled two generators with slightly different frequencies and showed that the system, owing to the interaction, vibrated with a common frequency. In another few years, Edward Appleton and Balthasar van der Pol carried out theoretical studies of this effect.

For a long time, synchronization has also been known to occur in living systems. Examples of such systems abound. Synchronous flashing of fireflies, singing crickets,

Box 1. Synchronization

The earliest accounts on synchronization are by the Dutch researcher Christiaan Huygens. He studied the motion of two identical clocks (two pendulums of almost same time period) suspended from the same wooden beam. He observed that the motion of the two pendulums in opposite directions were very much in agreement and that the rhythm was maintained without getting spoilt. Even when this rhythmic motion was disturbed by some external means, the pendulums readjusted in a short time. This is credited to the phenomenon of synchronization. He attributed this synchronous motion to the interaction of the two pendulums through the wooden beam supporting them.

Though this was one of the earliest written accounts of synchronization between two systems, for a very long time the physics behind it was not properly understood. For instance, it was not clear why the pendulums always settled for an anti-phase synchronization. Physicists from Georgia Tech, Kurt Wiesenfeld and Michael Schatz along with undergraduate student Matthew Bennett (now a graduate student) faithfully reconstructed Huygens experiment in an effort to understand the physics behind it. The apparatus consists of two spring-powered pendulum clocks attached to a wooden platform with metal weights added as shown in the photograph below. The platform is set on wheels, free to move on a level metal track.

The motion of the pendulums in this experiment were also synchronized in anti-phase. The center of mass of the system is not stationary during in-phase motion leading to very small platform movement, which drains energy out of the system owing to friction between the platform and its support. The anti-phase motion keeps the platform fixed reducing energy losses, and thus is favored in nature. The theoretical model suggested by this group predicts correctly the existence of this anti-phase oscillation as the only *stable* motion possible.

Matthew Bennett adjusts one of the pendulums of the recreated Huygens apparatus.

Photo by Gary Meek.

and firing neurons are some of them. In recent years, the idea of synchronization has been extended to systems which are not oscillatory. Synchronization of systems showing aperiodic behavior, such as chaotic systems, is one of the new fields of study.

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In the following section, we will discuss some simple examples of oscillators and show how synchronized behavior differs from non-synchronized motion. Most of this will require only of knowledge of high school science and mathematics. Based on the study of these examples, we will make some generalizations regarding the nature of synchronization. In the last section, we will see some real-life examples of synchronization.

2. Synchronization

Synchronization is a concept which finds mention in science, engineering, biology, and even behavioral sciences. We can talk of synchronization of something as simple as a pair of harmonic oscillators, or as complex as two chaotic systems. This section intends to take you on a journey exploring and unraveling the beauty of synchronization. We will study simple examples in trying to understand this phenomenon, make comments on interesting observations, and finally highlight the important aspects of synchronization. These examples will be based on the harmonic oscillator.

Identical Uncoupled Undamped Oscillators

Let us start by observing two identical undamped simple pendulums kept in different enclosures. For small amplitudes of oscillation, the system can be described by the following equations for the evolution of the amplitudes of the pendulums, whose solutions are oscillatory.

$$\begin{aligned} \ddot{x} + \omega^2 x &= 0 \\ \ddot{y} + \omega^2 y &= 0. \end{aligned} \tag{1}$$

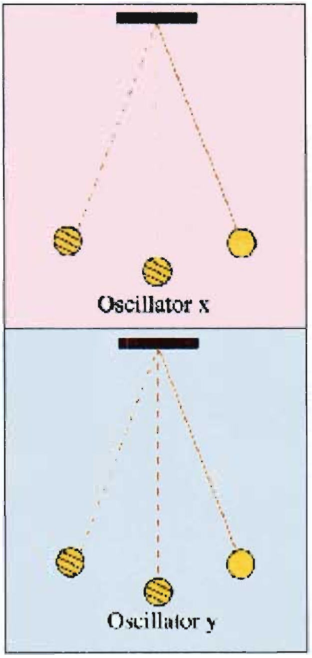


Figure 1. A pair of mutually isolated (uncoupled) identical pendulums oscillating with identical initial conditions.

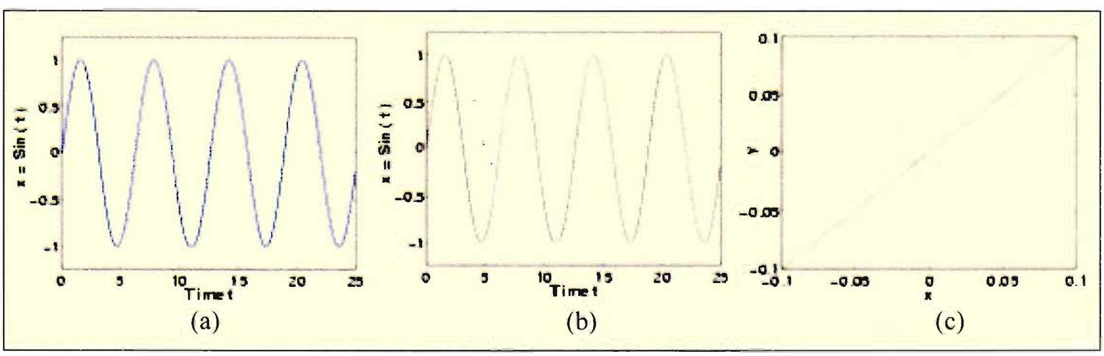
Figure 2. For a pair of identical uncoupled pendulums plot of (a) angular displacement x as a function of time t, (b) angular displacement y as a function of time t and (c) y vs x.

Watching two such independent pendulums is not going to be very interesting. Though they have the same frequency, they could be oscillating with some phase difference and even with different amplitudes. Now, instead of having these pendulums oscillate arbitrarily, let us displace them identically at the start and release them. This is shown in *Figure* 1 for $\omega = 1$. The pendulums will oscillate in phase and with the same amplitude. The temporal variation of the angular displacements of the two pendulums, x and y, have been shown in *Figures* 2(a) and 2(b). The plot of x versus y (also known as a Lissajous figure) is shown in *Figure* 2(c). This is a straight line of slope unity, hinting at the equality of x and y at every time instant. You would have realized that the equality of these two systems is a coincidence and any perturbation given to this system will spoil this equality. This is discussed in detail in *Box* 2.

Identical Coupled Oscillators

Next, let us take the case of two pendulums which 'interact' mutually. Physically, the interaction can be introduced by suspending them from a common support. Mathematically, the interaction can be captured by bringing in coupling terms on the right hand side of equation (1). We will discuss a simple example of two oscillators with mutual coupling given as

$$\begin{aligned} \ddot{x} + \omega^2 x &= k(\dot{y} - \dot{x}) \\ \ddot{y} + \omega^2 y &= k(\dot{x} - \dot{y}). \end{aligned} \tag{2}$$



Box 2. Uncoupled Oscillators

Two oscillators which are equal to each other at every instant of time cannot be said to be synchronized. Synchronization has more to it than just equality of states. This will be clear from the discussion of two identical uncoupled oscillators given as

$$\begin{aligned} \ddot{x} + \omega^2 x &= 0 \\ \ddot{y} + \omega^2 y &= 0. \end{aligned} \tag{a}$$

The solutions to these two differential equations are oscillatory and for identical initial conditions are equal to each other for all time. Let us the look at the evolution of this system in the *phase space*[1]. The undamped harmonic oscillators will move in closed orbits in the phase space. When the two oscillators x and y start off with the same initial conditions they will evolve in a circular path as shown in *Figure* A(a). The two disks representing x and y are on top of one another denoting the equality of states. Now when one of the oscillators, say y is displaced to a larger amplitude we see that the green disk representing y starts to move in a circle of larger radius as shown in *Figure* A(b). In this case y will continue to move along the bigger circle and will not come back to its initial state of equality until an external force acts upon it. Hence the equality of states is lost. Instead of pushing the two apart we could try to retard the motion of one of the oscillators causing a phase difference between the two oscillators. In this case the picture looks as shown in *Figure* A(c). Here the phase difference that is once established persists without reducing at all. Once again the equality of states is lost. Thus for two systems to be called synchronized it is just not enough if the states are equal for all time; they should also be resilient enough to revert back to the state of equality in the face of external disturbances.

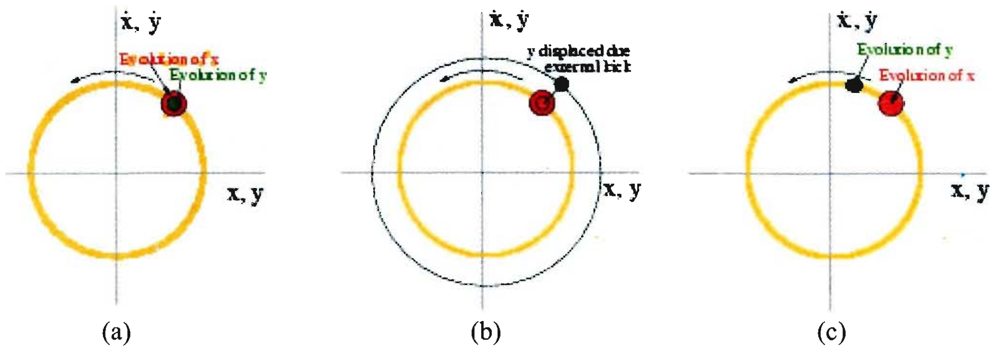


Figure A. Phase portrait of a pair of uncoupled identical oscillators (a) oscillating in phase with the same amplitude, (b) oscillating in phase with varying amplitude and (c) oscillating with a phase difference but with same amplitude.

[1] The evolution of a system described by a second order differential equation can be visualized as the evolution of points in a hypothetical space described by the state and its derivative called the *phase space*. Here, the oscillators, x and y, can be visualized to be moving in a space of $(x, \dot{x})$ and $(y, \dot{y})$ respectively.

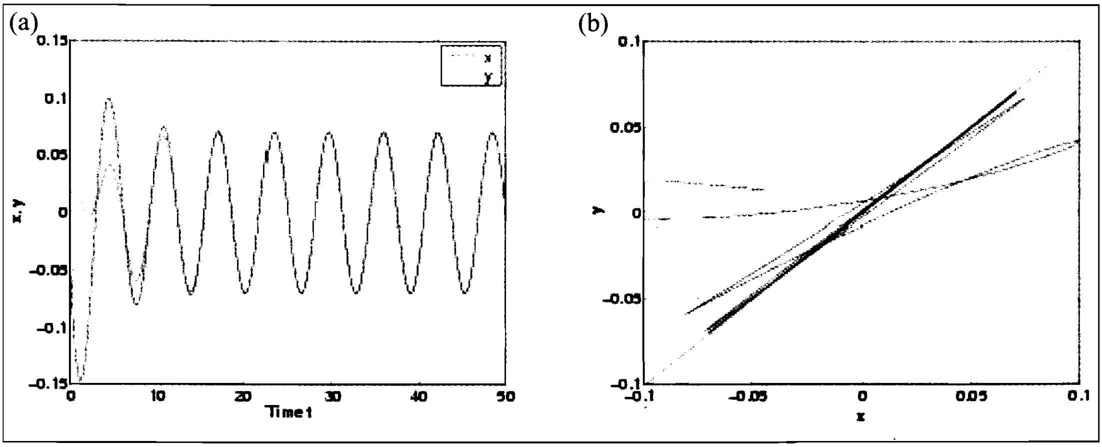


Figure 3. For a pair of identical coupled oscillators plot of (a) angular displacement x and y as a function of time t and (b) y vs x.

The positive constant k is called the coupling constant. Unlike the previous case, we will not put any restrictions on the initial conditions of these systems. We will let each oscillator start with an arbitrary initial condition and try to see the temporal variation of the two oscillators. This has been plotted in *Figure* 3(a) for $\omega = 1$ and $k = 0.4$. It is interesting to see that the two oscillators start off at two different positions, but adjust their rhythms and start oscillating in phase and with the same amplitude. The plot of x versus y shown in *Figure* 3(b) settles on a line of slope unity (green line) barring a few initial points. We will look at the error dynamics of the two equations to understand why the oscillators readjust their rhythms. The error equation is obtained by subtracting the equation of y from the equation of x given in (2). This gives us

The two oscillators start off at two different positions, but adjust their rhythms and start oscillating in phase and with the same amplitude.

$$(\ddot{x} - \ddot{y}) + \omega^2(x - y) = 2k(\dot{y} - \dot{x}). \tag{3}$$

Defining $e = x - y$, the error equation appears as

$$\ddot{e} + 2k\dot{e} + \omega^2 e = 0, \tag{4}$$

which is the equation for a linear damped oscillator. Hence, for positive values of k, the error must go to zero asymptotically, and the displacements of the two oscillators will readjust and become equal. If you try to write the error equation for the previous uncoupled oscillator case, you will realize that the error is oscillatory and does not decay.

The phenomenon of two coupled oscillators with different natural frequencies beginning to oscillate at a common frequency owing to coupling is called *frequency locking* or *entrainment.*

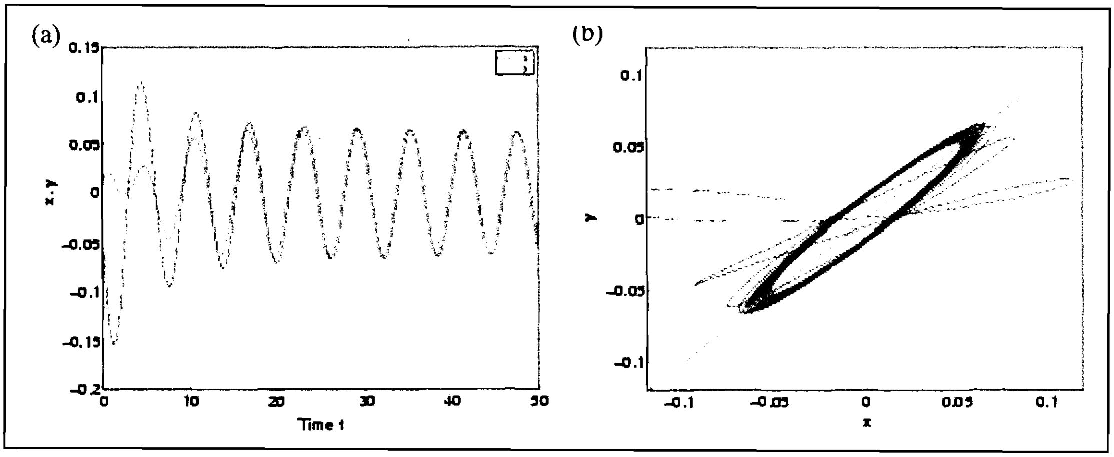
Nonidentical Coupled Oscillators

We have discussed two instances of identical oscillators, one without coupling and the other with coupling. Here we will discuss the effect of coupling in a pair of nonidentical oscillators differing only in their natural frequencies. We look at an example of the form

$$\begin{aligned} \ddot{x} + {\omega_x}^2 x &= k(\dot{y} - \dot{x}) \\ \ddot{y} + {\omega_y}^2 y &= k(\dot{x} - \dot{y}). \end{aligned} \tag{5}$$

The difference between the two frequencies $\omega_x - \omega_y$ is called *frequency detuning* or just *detuning.* We have taken frequency detuning of the order of 0.05. The plot of the temporal variation of x and y is shown in *Figure* 4(a) for $\omega_x = 1$, $\omega_y = 1.05$, and $k = 0.2$. There are two very important observations that can be made from this plot. One is that the frequencies of x and y match in spite of having different natural frequencies. The phenomenon of two coupled oscillators with different natu-

Figure 4. For a pair of nonidentical coupled oscillators plot of (a) angular displacement x and y as a function of time t and (b) y vs x.



The phenomenon of two oscillators oscillating with a certain relationship between their phases is termed as *phase locking.*

ral frequencies beginning to oscillate at a common frequency owing to coupling is called *frequency locking* or *entrainment*, and this common frequency of oscillation is called *locking frequency.* This frequency calculated numerically turns out to be $\Omega = 1.025$, correct to three decimal places. Secondly, the phase difference of the two oscillators settles to a constant value different from zero (this may not be obvious from the figure). This phenomenon of two oscillators oscillating with a certain relationship between their phases is termed as *phase locking.* For a simple case, as we saw in this example, the phase relationship could be a constant phase difference between the two oscillators, in which case the Lissajous figure shown in *Figure* 4(b) is a loop about a line of slope unity rather than a straight line. The green line marks the line of slope unity.

The phenomenon of frequency locking is a key factor behind synchronization of systems. This can be understood on the basis of the following argument. Coupling between the two oscillators tries to make their phases equal while detuning tries to drag the phases apart. Hence, the effects of coupling and detuning are counteractive. So, we get two qualitatively different situations based on the relative strengths of coupling and detuning. When the detuning is small in comparison to the coupling strength, the oscillators settle into a common frequency and a stable relationship between the phases of the two oscillators is established. We then call the two oscillators as synchronized. For relatively larger values of detuning, the effect of the coupling is not good enough to force a relation between the phases of the two oscillators. This leads to loss of synchrony.

Based on the examples we have seen so far, we can make the following general comments about the properties that a pair of oscillators may exhibit and which we may use in order to accept them as synchronized:

1. For nonzero detuning, the frequencies of the oscillators should be entrained. The zero detuning (or identical frequency) case is trivial.
2. The phases of the two oscillators should be locked.
3. The error between the two oscillator variables should asymptotically go to zero.

The rhythm generated by the sino-atrial node is due to synchronization of the rhythms of the collection of pacemaker cells.

Though these points have been discussed for a system of two oscillators, they hold good for a larger collection of oscillators as well. When all these three points are satisfied, then the system can be said to be *identically synchronized*. This is seen, for example, in the case of identical coupled oscillators in *Figure* 3(b). However, if only the first two of the above properties are satisfied, i.e., if the frequencies are entrained and the phases are locked but the error does not tend to zero with time, then the system of oscillators is said to be *phase synchronized*. An example of this is the nonidentical coupled system in *Figure* 4(b).

3. Synchronization in Real Life

In this section, we will discuss different instances of synchronization that we come across in our day to day life. This discussion will be non-mathematical and is intended to give you a feel for the breadth of the subject.

Cardiac Pacemakers

The heart muscle has its own pacemaker which transmits electrical impulses, signaling the heart to beat. The *sino-atrial* node, a bundle of muscle fibers located in the right atrium, can be considered as an assembly of a large number of pacemaker cells whose rhythms are mutually coupled. A signal fired by one of these cells forces the other cells to fire earlier than they normally would. Thus, the rhythm generated by the sino-atrial

Forcing of the heart by the electrical pacemaker entrains the heart to beat at the frequency of the pacemaker.

node is due to synchronization of the rhythms of the collection of pacemaker cells. In a pathological case, the pacemaker cells become faulty which results in an abnormally low heart rate known as *bradycardia*. This results in symptoms like fatigue, dizziness and fainting. In such cases, artificial pacemakers are attached to the heart. These electronic pacemakers consist of a battery-powered generator and wires that are attached to the heart. They continuously monitor the activity of the heart, and when the heart rate goes below a certain level, they take control by firing an electrical signal at a predetermined level. This forcing of the heart by the electrical pacemaker entrains the heart to beat at the frequency of the pacemaker, thus bringing the heart to a normal heart beat rate. Other conditions which require pacemakers include heart blocks in which the heart stops beating altogether for several seconds, and *tachyarrhythmia*, an overly rapid heartbeat.

Flashing Fireflies

When a swarm of fireflies gather, they have been found to flash in synchrony.

Flashing of fireflies in unison is one of the naturally occurring examples of synchronization. Male fireflies flash on and off to attract females. When a swarm of fireflies gather, they have been found to flash in synchrony. It is not that these fireflies start off flashing in synchrony. In fact, a non-synchronous flashing becomes more and more synchronous as the night progresses. A simple explanation of this phenomenon is as follows. These fireflies flash at a frequency specific to the individual. This flashing can also be stimulated or inhibited by external flashing light. Thus, when a swarm of fireflies gather in the dark, every individual firefly is influenced by the flash that it sees around it. That is, every individual firefly slows down or speeds up its pace of flashing so as to flash in phase with the other fireflies in the next cycle. Owing to this interaction, the flashing frequencies get entrained and the phases of the fireflies are locked. This causes synchronization of flashing fireflies.

Applauding Audience

The rhythmic hand clapping of an applauding audience is one of the examples of synchronization being manifested in social behavior. This has been studied experimentally by scientists by recording opera performances in Romania and Hungary. A microphone placed on the ceiling measured the global clapping rhythm while concealed microphones placed in the vicinity of the spectator measured the individual clapping rhythms. The global clapping rhythm was observed to undergo a transition from a noisy state to a synchronous clapping rhythm. It was also observed that this transition to the synchronous rhythm was preceded by an approximate period doubling of the individual clapping rhythms. One of the plausible explanations is that at lower clapping speeds the individuals are more likely to maintain a stable rhythm thus ensuring the decrease in frequency fluctuations. This decrease ushers the transition to synchronous clappping rhythm. It is believed that the doubling of the period of clapping is a voluntary act of the individual. This collective behavior has been observed more often in culturally homogeneous Eastern European communities and happens rarely in Western European and North American audiences.

4. Conclusion

Though the phenomenon of synchronization has been known to exist for a long time, a proper theoretical study of this phenomenon is only a few decades old. New theoretical tools developed in the study of nonlinear science have led to better understanding of this phenomenon. In this article, we discussed phase synchronization and identical synchronization for a pair of simple oscillatory systems. In recent years, synchronization has also been observed in chaotic systems. This discovery is known to have interesting applications in cryptography.

Suggested Reading

[1] **A Pikovsky, M Rosenblum and J Kurths, *Synchronization – A Universal Concept in Nonlinear Science*, Cambridge University Press, 2001.**

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