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The Mechanical Black Box
A Challenge from the 35th International Physics Olympiad

This article describes an interesting challenge concerning a mechanical black box that was posed as a five hour experimental examination in the 35th International Physics Olympiad[1] held in Pohang, Korea from July 15-23 2004. It marked India's seventh foray into this exciting event where seventy one nations participated. As a leader of the Indian team at Korea, one of us (RBK) was privileged to be in the thick of action. Our performance was a success and we secured one gold, two silver and two bronze medals.

[1] See articles listed in Suggested Reading for details on IPhO.

The Problem

The problem is presented in an abbreviated form to make it suitable for presentation as an article. The solution is discussed in a formal fashion without getting into distracting 'numerics' The readers are however encouraged to set up the experiment and then they will perforce get involved in the detailed numerics.

Keywords
Black box, mechanical, springs, rotation, centre of mass, moment of inertia, work–energy theorem, Olympiad.

The mechanical 'black box' consists of a solid spherical ball attached to two springs fixed in a cylindrical tube as shown in *Figure* (1). The tube is sealed with two identical end caps. The purpose of the experiment is to determine the mass m of the ball and the spring constants k_1 and k_2 of the springs without of course opening the black box. Here we discuss the determination

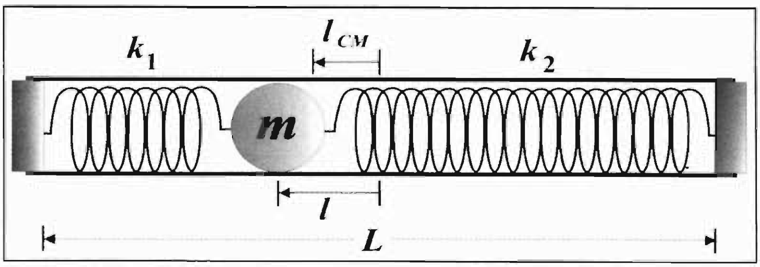

Figure 1. The mechanical 'black box' in its equilibrium horizontal position. The box consists of a rigid cylindrical tube. It encloses a spherical ball of unknown mass m which is attached to two soft springs of unknown spring constants k_1 and k_2.

of m by the method suggested by the Scientific Committee of the 35th International Physics Olympiad. We however discovered that the spring constants could be determined by cleverly employing the results used to determine m. We could thus dispense with the elaborate observational exercise suggested by the Committee to determine the spring constants. We present this 'short-cut' method here.

In the analysis of the experiment the following aspects must be kept in mind:

- The cylindrical tube is rigid and homogeneous.
- The lengths of the springs when they are not extended can be ignored. The springs are soft.
- The diameter of the ball (2.2 cm) is slightly smaller than the inner diameter of the tube (2.3 cm).
- There is finite friction between the ball and the inner walls of the tube.
- The energy dissipation due to the kinetic friction between the ball and inner walls of the tube can be ignored.
- The pulleys (see *Figure* 2) are frictionless. We also assume that their rotational kinetic energy can be ignored.

The experimental determination of the ball mass m and the spring constants k_1 and k_2 proceeds in several stages:

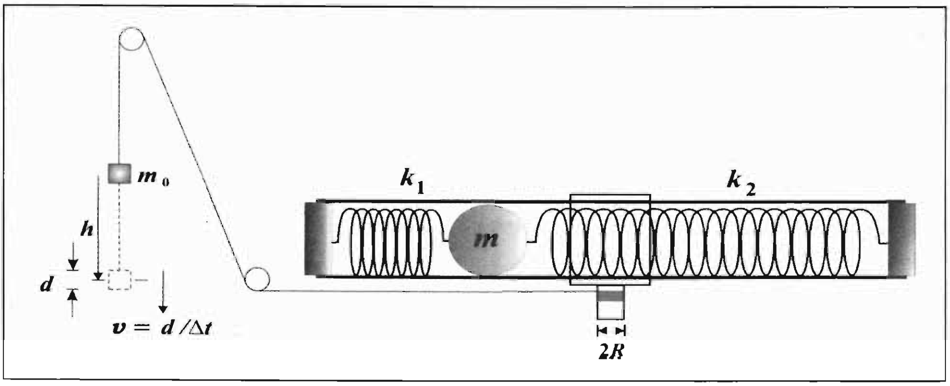

Figure 2. The rotation of the black box is accomplished by mounting it on a rotating stub. A thread wound around the stub passes over an arrangement of pulleys. As the known mass m_0 descends, the black box rotates.

Product of the mass and the position of the ball: Let l be the position of the centre of the ball relative to that of the tube when the black box lies horizontally as shown in *Figure* 1. Obtain the product ml.

Slow rotation: *Figure* 2 depicts the black box being fixed onto a rotating stub. One end of a string is wound around the stub while the other end goes over a pulley and has a mass m_0 attached to it. As m_0 falls under gravity the black box rotates. We first consider slow rotation regime. In this regime friction dominates and the ball does not slip from its static equilibrium position. Relate the speed v of the attached mass m_0 to the height h of the fall.

Fast rotation: When the black box rotates sufficiently fast, friction is overcome, and the ball moves to the end of the tube. It stays at this far end since the springs are soft. Once again relate v to h.

Determination of mass of the ball: Determine the mass m of the ball inside the black box.

Determination of the spring constants: Determine the two spring constants k_1 and k_2. Hint: Employ the results of 'fast rotation'.

The Solution

Product of the mass and the position of the ball: Let l_{CM} be the distance between the centre of the tube and the centre of mass (CM) of the entire assembly of tube, springs, and the ball (see *Figure* 1). Then using the lever rule

$$ml = (M+m)l_{CM}. \qquad (1)$$

Here M is the mass of the assembly without the ball. The CM is easily determined by balancing the assembly on a knife edge or by suspending and balancing the assembly horizontally using a thread. A simple balance is sufficient to determine $(M + m)$. In the experiment it was found that

$$ml = 296 \text{ g-cm.} \qquad (2)$$

Slow rotation: The change in kinetic energy of the system is

$$\begin{aligned}\Delta K_s &= \frac{1}{2}\left[m_0 v^2 + I\omega^2 + m(l^2 + 2r^2/5)\omega^2\right] \\ &= \frac{1}{2}\left[m_0 + I/R^2 + m(l^2 + 2r^2/5)/R^2\right] v^2. \quad (3)\end{aligned}$$

Here R is the radius of the rotating stub and I the effective moment of inertia (MI) of the whole system except the ball. The radius of the ball is r. We have used the parallel axis theorem for the MI of the ball. For no slipping $v = \omega R$. As mentioned earlier we have ignored the rotational kinetic energy of the pulley.

The mass falls through a height h and from the work-energy theorem (or equivalently the principle of conservation of mechanical energy) we have

$$\begin{aligned}\Delta U_g &= m_0 g h \\ \Delta U_g &= \Delta K_s \\ h &= A_s v^2, \qquad \text{where} \\ A_s &= \frac{[m_0 + I/R^2 + m(l^2 + 2r^2/5)/R^2]}{2m_0 g}. \quad (4)\end{aligned}$$

A close approximation to the instantaneous speed of the ball v is obtained by a combination of photogate and timer which measures the passage time of m_0 across the photogate after its fall through h. A plot of h versus v^2 in the slow rotation regime ($v^2 < 300$ cm^2-s^{-2}) is linear as shown in *Figure* 3.

Fast rotation: In this regime static friction is overcome. Since the springs are soft the ball goes to one end of the tube and stays put. Its position with respect to the centre of the tube is $L/2 - r$ where r is its radius. The change in kinetic energy is

$$\Delta K_f = \frac{1}{2}\left[m_0 + I/R^2 + m((L/2-r)^2 + 2r^2/5)/R^2\right]v^2. \tag{5}$$

The change in elastic energy is (*Figure* 1)

$$\Delta U_e = \frac{1}{2}\left[k_2(L-2r)^2 - k_1(L/2-l-r)^2 - k_2(L/2+l-r)^2\right] \tag{6}$$

As mentioned before we ignore the rotational kinetic energy of the pulley as well as the work done against kinetic friction. From the work-energy theorem, $\Delta K_s = \Delta U_g + \Delta U_e$. This yields

$$h = A_f v^2 + B, \tag{7}$$

where

$$A_f = \frac{1}{2m_0 g}\left[m_0 + I/R^2 + m((L/2-r)^2 + 2r^2/5)/R^2\right] \tag{8}$$

$$B = \Delta U_e/2m_0 g = \frac{1}{2m_0 g}\left[k_2(L-2r)^2 - k_1(L/2-l-r)^2 - k_2(L/2+l-r)^2\right] \tag{9}$$

The plot of h versus v^2 is once again a straight line but this time with a finite intercept on the h axis. There is a distinct discontinuity when one crosses over from a regime where static friction dominates to one where it does not. *Figure* 3 depicts this. The crossover speed is given by $v^2 = (h_2 - h_1 - B)/(A_f - A_s)$.

Determination of mass of the ball: Employing equations (4) and (8) we have

$$A_f - A_s = \frac{m}{2m_0 g R^2}[(L/2-r)^2 - l^2]. \tag{10}$$

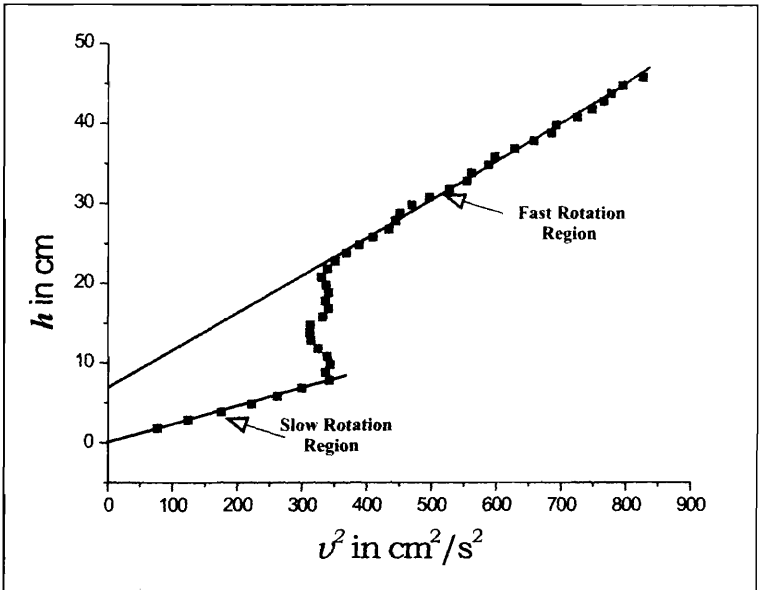

Figure 3. The plot of the 'instantaneous' speed v^2 of the mass m_0 versus distance of its descent h. Note that there are two clearly defined regimes: 'slow' and 'fast' rotations. The crossover which occurs at v^2 ~330 cm^2–s^{-2} is reminiscent of a phase transition. An understanding of this plot is the key to unraveling the mysteries of the black box.

One can measure the length of the tube L, and the radius of the ball is given as $r = 1.1$ cm. One obtains $(L/2 - r)^2 = 338.6$ cm^2. Further simple measurements yield $2m_0gR^2 = 756 \times 10^3$ g - cm^3 s^{-2}. The unknown quantity ml^2 may be written using equation (2) as

$$\begin{aligned} ml^2 &= [(m+M)l_{CM}]^2/m \\ &= (296)^2/m. \end{aligned}$$

Finally the slopes in *Figure* 3 yield $A_f - A_s = 0.026$ s^2-cm^{-1}. Inserting these numerical values in equation (10) we obtain a quadratic equation for m.

$$338.6m^2 - 19655m - 87616 = 0, \qquad (11)$$

where m is expressed in units of grams (g). The physically valid positive root yields $m = 62$ g.

Determination of the spring constants: When the black box is in the horizontal position the force balance equation yields (see *Figure* 1)

$$k_2(L/2 + l) - k_1(L/2 - l) = 0. \qquad (12)$$

Employing equation (9) we have

$$\left[k_2(L-2r)^2 - k_1(L/2-l-r)^2 - k_2(L/2+l-r)^2\right] = 2m_0 gB. \quad (13)$$

Equations (12) and (13) constitute two linear simultaneous equations in the two unknowns k_1 and k_2. We note that the numerical values of the remaining physical quantities in these two equations can be established. From the intercept on the h axis in *Figure* 3 we have B = 7.2 cm. The value of the 'extension' l in the horizontal position is estimated from equation (2) and from the determination of the mass m = 62 g in the previous subsection, namely

$$\begin{aligned} ml &= 296 \quad \text{g-cm} \\ l &= 296/62 \\ &= 4.8 \quad \text{cm}. \end{aligned}$$

Note that the mass m_0 = 100.4 g, the length of the black box L = 39.0 cm, and the radius of the ball r = 1.1 cm. The coefficients and constants of the two linear simultaneous equations in the two unknowns k_1 and k_2 are thus unambiguously specified. Solving these we obtain $k_1 = 4.6$ N/m and $k_2 = 2.8$ N/m.

Postscript

As mentioned earlier we discovered that the spring constants could be determined by ingeniously employing the results used to determine m. In the Olympiad problem it was suggested that we measure the time period of small oscillations of the black box suspending it from either end. In effect, we treat it as a compound pendulum. We note that it is not necessary to carry out this elaborate observational exercise.

The black box experiment is uncommon at the higher secondary school level. In the event it is posed, it tests the student's knowledge of basic electronics. A box encloses three to four electronic components (resistors, capacitors, diodes, etc). The student measures the voltage-current responses across the terminals and must identify the components and the circuit enclosed by the black box. The mechanical black box experiment is a rarity. This is the first time in the thirty-five year history of the Physics Olympiad that the mechanical black box has been posed as a full-fledged five hour experimental problem.

One can make an argument for including black box physics experiments at the school and university level. These experiments have a detective theme and can be exciting to students. Further they are in the same genre as non-destructive testing. Oil logging and water table determination are examples. Non-invasive procedures such as ultrasound and MRI medical scans make similar demands on us: given the sound or spin echoes, determine the underlying pathology. In fact a large number of research experiments, photoemission and Auger spectroscopy to name just two, are simply sophisticated versions of the black box experiment.

One Final Observation: One should be permitted the luxury of guess–estimating the unknown variables in any black box situation. Let us do so for the mass m of the ball. We have been given the radius of the ball, viz. 1.1 cm and also that kinetic friction is small. The examination was conducted with around 200 students at a given time. This required mass production of the set up and it is reasonable to assume that the balls were cast from a metal. The density of metals is around 10 g-cm^{-3}. From this data the mass is easily estimated to be 56 g which is not too far from the experimental value of 62 g!

Suggested Reading

[1] Vijay A Singh and Manish Kapoor, The Magnetohydrodynamic Generator: A Physics Olympiad Problem 2001, *Resonance*, Vol.7, No.7, pp. 68 - 75, 2002.

[2] H C Pradhan, Report on the First National Physics Olympiad and the 29th International Physics Olympiad, *Resonance*, Vol .4, No.2, p. 104, 1999.

[3] Vijay A Singh and R M Dharkar, The International Physics Olympiad - 1999, *Physics News*, Vol.30, pp. 60-64, 1999.

[4] Vijay A Singh, Ampere versus Biot-Savart, *Resonance*, Vol.5, No.8, pp.84-91, 2000.