In this article we present briefly some preliminary ideas of map projection, followed by a detailed account of the Mercator projection.

Introduction

During our school days we get acquainted with maps and atlases because without them the learning of geography remains incomplete. Any attentive student would have noticed that at least one world map in the atlas bears under it the words Mercator’s Projection. But most of us do not know what ‘Projection’ means, or who Mercator was. Perhaps we do not know also the deep relation between mathematics and geography, or how a map depicted on a spherical globe can be converted into a map on a flat sheet of paper.

What is a Map?

A map is a depiction (total or partial) of the structure of the Earth (or sky) on a plane, such that each point on the map corresponds to an actual point on the Earth (or in the sky), according to a particular scale. Here we shall confine our discussions to maps of the Earth. How much information will be provided by the map depends on its scale, the nature of the projection, conventional signs used, efficiency of the map-maker and the method of map preparation.

Meaning of Map Projection

Map projection means drawing of a part or totality of the Earth on a plane, using latitudes and longitudes, in such a manner that every point of the Earth is represented in the diagram. The globe itself is a miniature of the Earth divided into different regions by latitudes and
longitudes. This network of latitudes and longitudes is known as a *graticule*. Thus we can say that map projection is a way of representing a graticule upon a plane.

**Perspective Projection**

In a *perspective* or *geometric projection*, the positions of latitudes and longitudes are determined according to the view from some point, or with the help of a shadow cast by a light source on a plane. The globe used to prepare the projection is called the *generating globe*, and the plane on which projection is constructed is called the *plane of projection*. In Figure 1, $P'$ is the perspective projection of the point $P$ from $O$, the center of the generating globe $N E S W$, to the plane of projection $N L$.

A projection which is not based on the view from some point, but made with the intention of preserving some property of the map on the generating globe, is called a *non-perspective projection*. In Figure 1, the projected distance of $P$ from $N$ is $NP'$; this exceeds the actual distance $NP$. If one wants to keep the distance intact, then one must project $P$ to $P''$ where $NP'' = \text{length (arc } NP)$. In this case, $P''$ is a non-perspective projection of $P$ on the $NL$ plane.

**Basic Problem of Map Projection**

If we cut a right circular cylinder along its length and lay it flat upon a plane (i.e., we *develop* the surface), it
The curved surfaces of a circular cylinder and a circular cone are developable — they can be 'developed' into a plane surface.

On the other hand, common experience tells us that the peel of an orange cannot be laid flat upon a plane without distortion, no matter how carefully it is done. Similarly, if one wants to cover the entire surface of a globe with paper, then creasing and crumpling of the paper are inevitable. Distortion occurs in these cases because the curved surface of a sphere is nondevelopable. (This fact was first shown by the great mathematician Leonard Euler in the mid-18th century.) For this reason, in projecting a map from the spherical globe to a plane, some distortion must inevitably occur. (If the shape of the Earth were cylindrical or conical instead of spherical, then the task of map projection would have been much easier!)

Therefore, during map projection suitable methods are used so as to minimize the distortion. The nature of the projection depends on the purpose of preparation of the map. Depending on the purpose, various types of map projections have been developed.

**Zenithal and Conical Projections**

If latitudes and longitudes are projected on a plane that is tangent to the generating globe, the projection obtained is a zenithal or azimuthal projection (Figure 2). The plane of projection touches the generating globe at only one point. Generally the point of contact is taken to be the North or South Pole, in which case the merid-
Gnomonic, stereographic and orthographic projections are all zenithal projections.

If a cone is placed over the globe, touching it at its mid-latitudes, and a projection is made from the globe to the cone, we get a conical projection (Figure 3). The projection is obtained by cutting the cone and laying it flat.

**Cylindrical Projection**

If a cylinder is placed over the globe, touching it along the equator, the projection obtained by radial projection from the center of the globe is called a cylindrical projection. We get the map by cutting the cylinder along a generator and laying it out flat. Let $R$ be the common radius of the generating globe and the cylinder (Figure 4). A point $P$ on the generating globe will be projected on a point $P'$ on the cylinder from the center $O$ of the globe; $P'$ is the image of $P$. After obtaining the image of each point on the globe, the curved surface of the cylinder is opened to obtain the map of almost the entire world.

Note that circles of latitude are projected into horizontal straight lines while meridians (longitudes) are projected into vertical straight lines. Since the north and south poles lie along the axis of the cylinder, the projections of those two points will be situated at infinity. On a globe, meridians meet at the poles, but they appear as
In cylindrical projection only portions in the vicinity of the equator can be depicted with some degree of accuracy. Parallel straight lines in this projection. This has the following consequence: the distance between two meridians appears to remain constant as one travels north or south from the equator. Similarly, though parallels of latitude are equally spaced on a globe, in this projection their spacing gradually increases as one moves north or south. Also, in this projection all latitudes look the same as the equator in terms of length. If $\phi$ and $\lambda$ are the latitude and longitude respectively of a point $P$ on the globe, and $(x, y)$ the coordinates of its image $P'$, then

$$x = R\lambda, \quad y = R \tan \phi.$$  \hspace{1cm} (1)

A serious drawback of this projection is the large degree of distortion in the $NS$ direction at high latitudes. This arises because of the presence of $\tan \phi$ in equation (1). We know that $\tan \phi$ is an increasing function of $\phi$. For this reason distortion increases in the $NS$ direction. So, in cylindrical projection, only portions in the vicinity of the equator (where the cylinder touches the generating globe) can be depicted with some degree of accuracy.

**History of Map Making**

The early maps were hand drawn; in general, positions, shapes and scales for different places were not properly shown. About three thousand years ago, the Egyptians for the first time constructed a map which was somewhat acceptable. In that map, prepared for revenue collection, dividing lines of land masses were shown.

In about 600 BC, Thales of Miletus presented the idea of a projection now known as a *Gnomonic projection*. Around 540 BC, Pythagoras of Samos (580–500 BC) said that the Earth is round. Gradually due to the works of Aristotle (384–322 BC), Eratosthenes (273–192 BC), Ptolemy (85–165 AD) and others, the ideas of the poles of the Earth, the equator, various climatic regions and drawing of maps using projections came into being.
Box 1.

Gerhard Mercator (1512–1594) was born on 5 March, 1512 at Rupelmonde in Flanders (earlier in Holland, now in Belgium). In accordance with the custom of educated people of that period, he adopted the Latin form ‘Mercator’ of his original surname ‘Kremer’ (this being a Dutch word which means trader), and since then he has been known by that name. Twenty years before Mercator was born, Columbus had made his historical voyage, and the new geographical discoveries inspired young Mercator. He entered the University of Louvain in 1530, and after graduating he became a map maker and machine designer. In 1544 he was arrested for his belief in the Protestant cult. Somehow he escaped and in 1552 he fled to the neighbouring town Duisberg, now in Germany, and spent the rest of his life there. After enjoying an illustrious and luxurious life he passed away on 2 December, 1594.

Anaxymander (610–540 BC) for the first time prepared a map of the entire world, relying on stories told by travellers, sailors, etc. Strabo stated for the first time that the characteristics of spherical Earth cannot all be represented correctly in a plane diagram, and it is he who suggested the need for corrections in latitudes and longitudes. The Geographia of Ptolemy contained a world map and twenty six other maps. However the book soon disappeared into oblivion, resulting in a deterioration in the art of map making. With its rediscovery in the 15th century, and the subsequent discovery of printing and engraving techniques, there was a revival in the art of map making. In the 16th century, publication of maps became a lucrative business. However, as regards distortion in shape and distance, these maps were of the same standard as that of Ptolemy’s map. The person who liberated map making from the influence of Ptolemy was Gerhard Mercator (Box 1).

Mercator’s Map

In 1568 Mercator took up the task of preparing a map which would satisfy navigators’ needs and also promote water transport as an exact science. From the start he based his work on two principles:

1. The map should be constructed on a rectangular grid where latitudes are represented by straight
lines parallel to the equator, and meridians are represented by vertical lines perpendicular to the equator.

2. At each point, the degree of distortion in the $NS$ direction must be equal to the degree of distortion in the $EW$ direction, so that shapes remain the same, locally. In other words, shapes and compass directions must remain “true”.

Now, the circumference of a parallel of latitude decreases with increasing latitude and becomes 0 at the poles. But in Mercator’s map these circles are represented by equal horizontal straight lines. Therefore, the degree of distortion in the $EW$ direction depends on the latitude. In Figure 5, a parallel of latitude $\phi$ is shown. The circumference of the circle is $2\pi R \cos \phi$ (where $R$ is the radius of the globe), and it is shown in the projection as being of the same length as the equator, whose length is $2\pi R$. Therefore, the circle is stretched by a factor of

$$\frac{2\pi R}{2\pi R \cos \phi} = \sec \phi.$$

So the degree of stretching depends on $\phi$, and $\sec \phi$ being an increasing function $\phi$, the distortion increases with increasing $\phi$ (see Table 1).

In order to keep the $NS$ distortion and the $EW$ distortion the same, the same amount of stretching of meridians must be effected in the $NS$ direction as in the $EW$ direction. This means that the distance between latitudes must be shown as increasing steadily as one travels towards the poles. One consequence of this is that

\[ \begin{array}{cccccccccc}
\phi & 0^\circ & 20^\circ & 40^\circ & 60^\circ & 80^\circ & 85^\circ & 90^\circ \\
\hline
\sec \phi & 1.00 & 1.06 & 1.31 & 2.00 & 5.76 & 11.47 & \infty
\end{array} \]
if a small region of the map is considered, no great distortion occurs with regard to scale, whereas if the region under consideration is large, there will be a significant amount of distortion (in scale). This fact is best illustrated by the observation that Greenland appears larger than South America in a Mercator projection, though in reality South America is nine times as large as Greenland. The factor by which area gets distorted is 4 at 60° latitude, 15 at 75° latitude, and 30 at 80° latitude. And as the poles are situated at infinity in Mercator’s projection, it is useless to show latitudes beyond 80°. For this reason, such maps are shown only till the 80° parallel.

In order to accomplish his purpose, Mercator had to work out the exact spacing of the latitudes. Exactly how he did this is unknown as he did not leave any document explaining his method. However, some insight into his thinking may be obtained from this extract: “In making this representation of the world, we had to spread on a plane the surface of the sphere in such a way that the positions of places shall correspond on all sides with each other both in true direction and in distance. With this intention we had to employ a new proportion and a new arrangement of the meridians with reference to the parallels. For these reasons we have progressively increased the degrees of latitude towards each pole in proportion to the lengthening of parallels with reference to the equator.” It is clear from this brief description that Mercator was quite aware of the mathematical principles of construction of his map.

In 1569, Mercator published his map under the title *New and Improved Descriptions of the Lands of the World, amended and intended for the Use of Navigators*. The map was large (54” × 83”), and it was divided into twenty-one parts. It is regarded as highly valuable; only three original copies survive. Mercator called his map *Atlas*, after the mythological character who held the Earth in
his hand. The book was published in three volumes; the third volume was published in 1595, a year after his demise.

One of the early pioneers in map publishing was Abraham Ortelius (1527–1598), whose *Theatrum orbis terrarum* was published in Antwerp in 1570, one year after Mercator’s *Atlas*.

**Coordinates in Mercator’s Map**

Consider a spherical rectangle determined on the globe by the meridians \( \lambda, \lambda + \Delta \lambda \) and the latitudes \( \phi, \phi + \Delta \phi \) (Figure 6); the lengths of its sides are \( R \cos \phi \Delta \lambda \) and \( R \Delta \phi \). Let the point \((\lambda, \phi)\) on the globe be projected to the point \((x, y)\) in the map. Then the spherical rectangle is projected into a plane rectangle whose sides are determined by straight lines \( x, x + \Delta x \) and \( y, y + \Delta y \), where \( x = R \Delta \lambda \).

Now, if the projection be such that the NS and EW distortions are equal, then the spherical rectangle and the plane rectangle should be similar to each other. Therefore,

\[
\frac{\Delta y}{\Delta x} = \frac{R \Delta \phi}{R \cos \phi \Delta \lambda}.
\]

This in combination with \( \Delta x = R \Delta \lambda \) results in:

\[
\Delta x = R \Delta \lambda, \quad \Delta y = R \sec \phi \Delta \phi. \quad (2)
\]
Edward Wright's Explanation

Mercator's projection is valuable for navigators because compass directions can be represented by straight lines. Unfortunately, it did not immediately gain popularity among navigators. Probably sailors failed to understand either the basis of the map or its virtues. It was left to Edward Wright (1560–1615), a British mathematician and machine designer, to find a proper explanation of the basic principles of Mercator's map.

In modern notation, the solution of the equations $\Delta x = R\Delta \lambda$ and $\Delta y = R \sec \phi \Delta \phi$ may be given as $x = R\lambda$ and $y = \ln \tan (\pi/4 + \phi/2)$, because the value of $\int_0^\phi \sec \phi d\phi$ is $\ln \tan (\pi/4 + \phi/2)$. This allows the coordinates of different points in Mercator’s map to be determined for different values of $\lambda$ and $\phi$. Wright evaluated $\int_0^\phi \sec \phi d\phi$ using tedious numerical methods. In his book *Certain Errors in Navigation*, published in 1599, he gave an explanation of Mercator’s map. It should be remembered that at the time of publication of Wright’s book, calculus was unknown; so he had no alternative but to use numerical integration. After the publication of the book, the value of Mercator’s projection was better appreciated by navigators.

The Following Years

In 1614, John Napier (1550–1617) invented logarithms. Shortly after, Edmond Gunter (1581–1626), a British mathematician, published a chart of logarithmic tangents. In 1645, Henry Bond, a mathematics teacher and high rank officer, discovered while comparing Gunter’s figures with Wright’s that if the angles in Gunter’s table are replaced by $\pi/4 + \phi/2$, then the two sets of figures become identical. From this correlation he was able to guess that

$$\int_0^\phi \sec t \, dt = \ln \tan \left( \frac{\pi}{4} + \frac{\phi}{2} \right)$$
Suggested Reading


But he was unable to prove this. After futile attempts of John Collins, Nicholas Mercator, Edmund Halley and others to prove the relationship, James Gregory found a rather complicated proof in 1668. Shortly after, Isaac Barrow, the predecessor of Isaac Newton in the Lucas Chair at Cambridge, found an easy proof. Barrow used the idea of partial fractions in his proof — the first known use of this technique, which today is so widely used for evaluating indefinite integrals.

Loxodromes

A speciality of Mercator’s projection is that if a straight line is drawn anywhere on the map, it makes equal angles with all the meridians. This results in a line of constant bearing which is known as a Loxodrome or Rhumb line. This line always represents a true direction. This fact is of great use to navigators at sea, for if the line is followed then there is no need for always changing direction. It results from the following two facts: (a) parallels of latitude and meridians intersect at right angles, and (b) the vertical and horizontal scales are equally distorted at each point.

Epilogue

Over the two centuries following the publication of Mercator’s map, rapid progress was made in the art of map making, and maps began to be used widely for trade, exploration and military purposes. Mercator can be regarded as a pioneer in developing the art of map making into an applied science. For this reason, in spite of its few shortcomings, Mercator’s projection is still regarded as a milestone in the history of map making.

Address for Correspondence
Utpal Mukhopadhyay
Barasat Satyabharati Vidyapith
P.O. Nabapally 700126
Dist. North 24-Parganas
West Bengal, India.