The Coming of a Classical World

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Quantum theory's unusual predictions stem from its basic formalism which involves concepts like the wavefunction or probability *amplitudes* instead of probabilities. Many serious doubts have been raised about quantum theory's connection with perceived classical dynamics. How does quantum mechanics, with all its strange ideas, unfold to give us the 'reality' of the familiar physical world? What is the connection between the *classical* and the *quantum*? If quantum mechanics is, indeed, the fundamental theory of nature, as is widely accepted, then how does it explain *classicality*? In the following, some of the fascinating conceptual problems of quantum mechanics are highlighted. The 'environment-induced-decoherence' approach is then discussed as one practical attempt at explaining the emergence of a classical world from an underlying quantum substrate.

1. Introduction

The emergence of quantum mechanics as a fundamental theory of nature symbolizes one of the greatest revolutions in physics. Since its inception a century ago, the theory has introduced stunning new ideas and made precise predictions about a wide range of physical phenomena. Throughout its development, quantum mechanics has been subjected to the most intense scrutiny by the scientific establishment. Yet, experiment after experiment has confirmed the predictions of quantum theory, no matter how preposterous they might have sounded. Quantum mechanics has the unique distinction of being the most successful working theory of nature and

Keywords
Quantum superpositions, quantum measurement, entangled states, quantum coherence, decoherence.

there is no known example of any conflict between its predictions and experimentally observed results. However, in spite of its impressive successes and the fact that it is currently accepted as one of the best tools for understanding the physics of the micro world, quantum physics remains an enigma. The predictions of quantum mechanics are often at odds with what people regard as 'common sense'. The concepts of quantum mechanics seem absurd when related to the world of our 'experience' – the familiar physical world, and the philosophical implications of the theory remain highly controversial.

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Quantum mechanics, it seems, fails to provide a natural framework to accommodate our 'classical' perceptions of the physical world. Our normal perceptions of the world around us are classical in the sense that they are based on classical ideas like Newton's laws, and well-understood concepts like precise positions, momenta and trajectories for moving objects. A central concept of classical dynamics is predictability. In the classical way of looking at things, the world is considered to be made up of *observable objects*, like particles, fluids, fields, etc., and they obey definite laws of force. Classical systems evolve deterministically and are expected to possess definite properties. Our classical perceptions help us form simple, direct, mental pictures in space and time of the way the world works. Quantum mechanics, however, calls for a radically different vision of the world. Triggered by Max Planck's revolutionary concept of 'quanta' in 1900, the theory was born and grew in the years that followed. In its current form, quantum mechanics is stunningly powerful. However it has compelled us to completely reshape and revise our ideas of reality and notions like position, momentum, cause and effect. The quantum view appears abstract and counterintuitive to our classically coloured minds. Though the practitioners of quantum theory have no doubt that it is one of the most elegant and satisfying

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descriptions of phenomena at the atomic scale, its clash with the more comfortable classical scheme continues to bother physicists.

At the heart of quantum mechanics is the wave function, ψ, a mathematical entity that contains all possible information about the system to which it is attributed. The wave function evolves in time according to the Schrödinger equation, which is linear and deterministic:

$$i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle. \tag{1}$$

Here H is called the Hamiltonian of the system and $\hbar$ is $h/2\pi$, h being Planck's constant.The wave function, however, does not have a physical counterpart, since it itself is not an observable. According to the Born interpretation, the wave function is understood in terms of probabilities. For example, if one measures the position of an electron, the probability of finding it in a given region depends on the 'intensity' of the wave function there. This means that in spite of the apparent determinism manifested in the Schrödinger equation, knowledge of ψ, the wave function, does not ensure a precise knowledge of the observable properties of the system – the kind we are familiar with in the classical world, e.g. position, momentum, etc. There seems to be a fundamental randomness built into the laws of quantum mechanics. This challenge to our cherished common-sense perceptions of, and preference for, a deterministic universe perhaps made Einstein utter his oft quoted remark, "I can't believe that God plays dice"

An intriguing consequence of the linearity of Schrödinger's equation is that wave functions could describe combinations of different states or 'superpositions'. For example, an electron could be described by a wave function, which describes a superposition of several different locations. When this linear superposition principle of quantum mechanics is extrapolated to macroscopic systems, which are conventionally described by classical mechanics, we are faced with the famous counterintuitive example of 'Schrödinger's cat'. Schrödinger's cat is the unfortunate victim of a nasty contraption where the decay of a radioactive atom triggers a device which kills the cat. The quantum mechanical description of this scenario demands that a superposition state of 'decayed' and 'not decayed' for the atoms leads to the cat being both dead and alive in superposition! This bizarre state of suspended animation of Schrödinger's cat is a classic illustration of the clash between the predictions of quantum theory and familiar classical insights. A closely related problem is that of quantum measurement. If quantum theory is accepted to be the fundamental theory of nature, then the process of measurement and all its ingredients must be described quantum mechanically. In a quantum measurement, the coupling between a microscopic system and a macroscopic measuring apparatus results in an *entangled* state where quantum mechanics seems to allow the apparatus (meter) to exist in a coherent superposition of macroscopically distinct states – a Schrödinger's cat like situation, which is classically unimaginable. Such concepts raise many problems about quantum theory's connection with the emergence of classicality and the elusive boundary between quantum and classical worlds.

An apparent resolution of the quantum measurement paradox is often forced by the notion of a sudden *collapse* of the state vector into one of the eigenstates of the dynamical operator.

Over the years, there have been several attempts to understand and explain the strange and novel concepts of quantum mechanics, the quantum measurement problem, and the nature of classicality as an emergent property of an underlying quantum system. von Neumann postulated that an irreversible *reduction* process takes the quantum superposition to a *statistical* mixture which is classically interpretable and meaningful. Thus, an apparent resolution of the quantum measurement paradox is often forced by the notion of a sudden *collapse* of the state vector into one of the eigenstates of the dynami-

The quantum and the classical undoubtedly share an intimate bond.

cal operator. However, the *non-unitary* nature of this reduction is at odds with the inherent unitary nature of the Schrödinger equation (see *Box* 1). This seems to imply that the mechanism lies outside the realm of quantum mechanics, thus questioning the validity of the theory itself. What, then, is the connection betwen the 'classical' and the 'quantum' worlds? Is there a definite relationship? Are classical mechanics and quantum mechanics two mutually exclusive incompatible theories or are they two aspects of the same underlying philosophy? Classical objects are eventually composed of elements of the microworld which can be described quantum mechanically. So, how and where can there be a boundary between the two worlds? The quantum and the classical undoubtedly share an intimate bond. Among the various explanations that seek a resolution to the conceptual problems of quantum mechanics are the Copenhagen interpretation, the Hidden Variable theory, the Many-Worlds interpretation, the de-Broglie Bohm theory, the environment-induced decoherence theory, etc. Though all these different interpretations seem mutually conflicting at the philosophical level, they all accurately explain the outcomes of known experiments and correctly predict the outcomes of new experiments! Each physicist's preference, therefore, for a particular interpretation of

Box 1. Quantum Measurement

The quantum mechanical description of a system is contained in its wave function $|\psi\rangle$, which lives in an abstract Hilbert space. The state $|\psi\rangle$, in general, can be represented as a linear combination of an *orthonormal set of basis vectors in Hilbert space.* The dynamics of the wave function are governed by the Schrödinger equation:

$$i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle.$$

Here H is called the *Hamiltonian.* The mathematical structure of the Schrödinger equation is such that it is *linear* and *deterministic.* A consequence of it being *deterministic* is that if $|\psi(0)\rangle$ is known, then the equation can specify $|\psi(t)\rangle$ for all times. Another property is that quantum evolution as governed by the Schrödinger equation is *unitary.*

Box 1. continued...

This means that if we have some arbitrary quantum system, U, that takes as input a state $|\phi\rangle$ which evolves to a different state $U|\phi\rangle$, then mathematically we can describe U as a *unitary linear tranformation.* This means that $U^+U = 1$, where U^+ is the complex transpose of U. Examples of unitary transformations are rotations and reflections. Unitary transformations preserve *inner products*, and, therefore, *norms*:

$$(\psi, \phi) = (U\psi, U\phi).$$

Inspite of the apparent determinism manifested in the Schrödinger equation, a precise knowledge of $|\psi\rangle$ does not ensure a precise knowledge of the *observable* properties of the system- the kind that we are familiar with in the classical world, e.g. position, momentum etc. Dynamical variables or *observables* are represented in quantum mechanics by *linear Hermitian operators* which act on the state vector. An operator $\hat{A}$ corresponding to a dynamical quantity A is associated with *eigenvalues* a_is and corresponding *eigenvectors* $\{|\alpha_i\rangle\}$, which form a *complete orthonormal set.* A consequence is that any arbitrary state vector $|\psi\rangle$ can be expanded as a linear superposition of these eigenvectors: $|\psi\rangle = \sum c_i|\alpha_i\rangle$. How does standard quantum mechanics describe *measurement*? A basic postulate of quantum mechanics is that a measurement of A can only yield one of the eigenvalues a_is, but the result is not definite in the sense that different measurements for the quantum state $|\psi\rangle$ can yield different eigenvalues. Quantum mechanics predicts the *probability* of obtaining the eigenvalue a_i to be $|c_i|^2$. In quantum mechanics one writes the *expectation* value of $\hat{A}$ in the state $|\psi\rangle$ as:

$$\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle = \sum a_i|c_i|^2.$$

An additional postulate of quantum mechanics is that the measurement of an observable A, which yields one of the eigenvalues a_i (with probability $|c_i|^2$) culminates with the *reduction* or *collapse* of the state vector $|\psi\rangle$ to the eigenstate $|\alpha_i\rangle$. This means that every other term in the linear superposition vanishes except one. Such a transition is obviously impossible to achieve via the norm preserving unitary evolution discussed above. This is where the measurement postulates clash with unitary Schrödinger evolution.

If a system has a state vector $|\psi\rangle$, then it is possible to define the *density operator* for the system by the outer product

$$\hat{\rho} = |\psi\rangle\langle\psi|.$$

Then the equivalent formula for the expectation value of an operaor $\hat{A}$ is

$$\langle\hat{A}\rangle = Trace\{A\rho\}.$$

The density operator, thus, is essentially an object which is equivalent to the state vector. It is a convenient formal tool to compare and contrast quantum and classical systems in terms of probabilities. The conceptual problems of quantum measurement become more transparent when analyzed in the language of density matrices, and hence this is often the preferred approach.

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quantum mechanics remains a matter of taste until the day some irrefutable experimental evidence chooses one interpretation over all others.

In the following we will highlight the 'environment induced decoherence theory'– one attempt to explain the emergence of classicality from quantum dynamics. This theory employs the methods developed by several authors to analyse the quantum mechanics of a system in interaction with its environment. The central idea of this approach is that classicality is an emergent property triggered in *open* systems by their environments. The theory explains why we do not routinely see quantum superposition in the familiar physical world around us. Though this theory might seem to some as less dramatic compared to its rival counterparts, it is being widely accepted as a successful and practical solution for explaining the emergence of a classical world. The strength of this approach is that it provides explanations within the realm of quantum mechanics and does not call for any modification of the existing framework of theory.

The achievements of the decoherence theory seem particularly relevant in the light of several recent proposals to exploit purely quantum mechanical features such as the linear superposition principle, and quantum entanglement to build high speed quantum computers.

Recently, interest in the understanding of decoherence has been heightened by stunning advances on the experimental front, where some of the predictions of the decoherence approach have been successfully demonstrated. The achievements of the decoherence theory seem particularly relevant in the light of several recent proposals to exploit purely quantum mechanical features such as the linear superposition principle, and quantum entanglement to build high speed quantum computers. There have also been several attempts to implement other interesting ideas from this novel field of quantum information like quantum cryptography and quantum teleportation in the laboratory. Since environmental influence is often unavoidable in a real life laboratory scenario, the ubiquitous decoherence mechanism can ruin the functioning of such futuristic systems which rely heavily on the maintenance of quantum coherence.

Thus, for the more practical physicist, an understanding of decoherence and its quantitative and qualitative predictions holds more significance when looking for realistic explanations for the emergence of classical features in a quantum world.

2. The Problem: Quantum Measurement, Entangled States and the Pointer Basis

Consider a measurement-like scenario, which is to be described quantum mechanically. A simple and illustrative example is when both the system whose properties are being measured and the apparatus, which measures these properties, are quantum-mechanical spin-1/2 particles. The measurement interaction between the two should be such that the property of the system gets correlated with some property of the apparatus. This way, by looking at the state of the apparatus (meter) one can infer what state the system is in. Let σ and L represent the Pauli spin operators for the system and the apparatus, respectively. Let $|\uparrow_z\rangle$ and $|\downarrow_z\rangle$ be the eigenstates of σ_z and $|\Uparrow_z\rangle$ and $|\Downarrow_z\rangle$ be the eigenstates of L_z. It is reasonable to assume that the system and the apparatus are initially uncoupled. Their initial combined wavefunction (state) can then be written as a direct product state of the form:

$$|\psi(t=0)\rangle = (a|\uparrow_z\rangle + b|\downarrow_z\rangle) \otimes (c|\Uparrow_z\rangle + d|\Downarrow_z\rangle), \quad (2)$$

where the system and apparatus are, in general, in the superposition states $(a|\uparrow_z\rangle + b|\downarrow_z\rangle)$, and $(c\Uparrow_z\rangle + d\Downarrow_z\rangle)$, respectively. a, b, c, and d are complex numbers. The mechanism of a measurement process demands that the system and the apparatus *interact*. Let us assume that this happens via a Hamiltonian of the form

$$H_{SA} = g\sigma_z L_y, \quad (3)$$

where g is the strength of the system-apparatus coupling. It can be easily shown that the Hamiltonian evolution takes an initial state of the kind (2) into an *entangled* state (see *Box* 2) of the form:

$$|\psi_{S+A}(t)\rangle = k_1|\uparrow_z\rangle|\Downarrow_z\rangle + k_2|\downarrow_z\rangle|\Uparrow_z\rangle, \qquad (4)$$

where k_1 and k_2, in general, depend on the coupling strength, g, the complex coefficients a, b, c, d and time, t, in a complicated way. For a certain choice of c, d and t, we can have the simplified form of (4), which we will

Box 2. Superpositions and Entangled States

One of the consequences of the linearity of the Schrödinger equation is that its solutions are subject to the *linear superposition principle.* This means that the equation allows for solutions which are linear superpositions of other solutions. For example, if ψ_1 and ψ_2 are solutions of the equation for a certain system, then

$$\psi = c_1\psi_1 + c_2\psi_2$$

is also a perfectly legitimate solution for the same system, where c_1 and c_2 are some constant complex numbers. Such states are specially intriguing and counterintuitive when one considers situations where ψ_1 and ψ_2 each represent *macroscopically distinct* states of the system. When the superposition principle is applied to more than one quantum particle, say, to a system of *two* interacting particles, then one has what are known as *entangled states.* This phenomenon is even more peculiar than a superposition state of a single particle as described above. Schödinger's famous cat-paradox deals with such an *entangled state* between the states of the atom (decayed and undecayed) and the cat (dead and alive):

$$\psi = c_1|\mathbf{decayed}\rangle|\mathbf{dead}\rangle + \mathbf{c_2}|\mathbf{undecayed}\rangle|\mathbf{alive}\rangle.$$

Like the cat and the atom in the state above, the members of an entangled state do not have their own individual pure quantum states. Only the combination as a whole has a well-defined state. In a measurement scenario, the entangled state exists between the system and the measuring apparatus. This is completely at odds with anything we know in classical physics. Entangled objects behave as if they were connected with one another no matter how far apart they are and always contain quantum correlations. Einstein Podolsky and Rosen were the first to draw attention to the possibility of what they called "spooky action-at-a distance" or nonlocal effects involving entangled particles. In the nineteen sixties, John Bell predicted that entangled quantum states allow crucial experimental tests that distinguish between classical and quantum physics. Subsequently, experimenters have confirmed the existence of these "nonlocal" correlations. More recently, efforts have gone into exploiting the peculiar nature of entangled particles in the exciting field of quantum information (see [2]).

use to illustrate the measurement problem. Equation (4) is no longer a direct product state of the system and apparatus but an *entangled* state where the system and apparatus develop *quantum correlations* which makes them *one inseparable quantum unit.* Such a situation, like the state of Einstein, Podolsky and Rosen (EPR) (see *Box* 2) is a uniquely quantum mechanical state of affairs, which is difficult to comprehend classically. Consider the density matrix corresponding to the entangled state (4). The density matrix corresponding to such a state is, in general, 'non-diagonal' and constructed as the projection operator of the form:

$$\begin{aligned} \rho_{pure} &= |\psi\rangle\langle\psi| = |k_1|^2|\uparrow_z\rangle\langle\uparrow_z||\Downarrow_z\rangle\langle\Downarrow_z| + |k_2|^2|\downarrow_z\rangle \\ &\langle\downarrow_z||\Uparrow_z\rangle\langle\Uparrow_z| + k_1^*k_2|\downarrow_z\rangle\langle\uparrow_z||\Uparrow_z\rangle \\ &\langle\Downarrow_z| + k_1k_2^*|\uparrow_z\rangle\langle\downarrow_z||\Downarrow_z\rangle\langle\Uparrow_z|. \end{aligned} \tag{5}$$

The system and apparatus develop *quantum correlations* which makes them *one inseparable quantum unit.*

While (4) and (5) represent mathematically legitimate solutions of the Schrödinger equation for the Hamiltonian evolution, there is a serious difficulty in their physical interpretation, when one appeals to their usual meanings in the language of probabilities. How do they explain the outcomes of a real-life measurement process? If one appeals to the usual definition of density matrices as probabilities, then the off diagonal elements (the third and fourth terms in (5)) are puzzling. They do not make sense to our classically tuned minds like the diagonal elements do. The diagonal terms conform with our understanding of classical probabilities. In the measurement scenario described here, they can be interpreted as the probabilities corresponding to the cases where the spin system is in the 'up' or 'down' states. The off-diagonal elements make no such sense and we would be happy with an expression where they did not exist. In fact, the off-diagonal terms are signatures of the *quantum correlations* contained in the entangled state. The only way, it seems, to reconcile this situation with a classical measurement-scenario is to accept that the undesirable 'off-diagonal' elements somehow disappear.

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$\rho_{pure} = |\psi\rangle\langle\psi|$ contains apparent system-apparatus correlations which 'suffer' from what is often referred to as the 'ambiguity of basis'.

Subsequently, the density matrix is *reduced* to a diagonal *statistical mixture* of the form:

$$\rho_{red} = |k_1|^2 |\uparrow_z\rangle\langle\uparrow_z|| \Downarrow_z\rangle\langle\Downarrow_z| + |k_2|^2 |\downarrow_z\rangle\langle\downarrow_z|| \Uparrow_z\rangle\langle\Uparrow_z|. \tag{6}$$

This is equivalent to von Neumann's non-unitary reduction process or the *collapse postulate.* ρ_{red} makes classical sense. Its coefficients may be interpreted as classical probabilities corresponding to the two potential outcomes, 'up' and 'down' of the spin-systems. Thus ρ_{red} represents classical correlations between the system and the apparatus in a familiar measurement situation.

It might be argued that both ρ_{pure} and ρ_{red} contain the potential outcomes of 'up' and 'down' spins. Why can't we, then, look at just the diagonal elements of ρ_{pure} and simply ignore its off-diagonal elements? Why is it necessary to have the non-unitary reduction process that completely gets rid of the off-diagonal elements?. In other words, doesn't ρ_{pure} through its diagonal elements already contain the description of a completed measurement process? The answer is, no! The fact is, $\rho_{pure} = |\psi\rangle\langle\psi|$ contains apparent system-apparatus correlations which 'suffer' from what is often referred to as the 'ambiguity of basis'. To illustrate this point, let us choose the coefficients k_1 and k_2 such that $k_1 = -k_2 = \frac{1}{\sqrt{2}}$. This makes the density operator ρ_{pure} a projection operator constructed from the following pure state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow_z\rangle|\Downarrow_z\rangle - |\downarrow_z\rangle|\Uparrow_z\rangle\Big]. \tag{7}$$

The state (7) is *rotationally invariant.* This means that if the state is re-written in a different basis, e.g. in the eigenstates of σ_x and L_x for the system and apparatus, then

$$|\uparrow_z\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow_x\rangle + |\downarrow_x\rangle\Big], |\downarrow_z\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow_x\rangle - |\downarrow_x\rangle\Big],$$

$$|\Uparrow_z\rangle = \frac{1}{\sqrt{2}}\Big[|\Uparrow_x\rangle + |\Downarrow_x\rangle\Big], |\Downarrow_z\rangle = \frac{1}{\sqrt{2}}\Big[|\Uparrow_x\rangle - |\Downarrow_x\rangle\Big]$$

gives

$$|\psi\rangle = \frac{-1}{\sqrt{2}}\Big[|\uparrow_x\rangle|\Downarrow_x\rangle - |\downarrow_x\rangle|\Uparrow_x\rangle\Big], \qquad (8)$$

and similarly, in the eigenstates of σ_y and $\mathbf{L}_y$,

$$|\psi\rangle = \frac{-i}{\sqrt{2}}\Big[|\uparrow_y\rangle|\Downarrow_y\rangle - |\downarrow_y\rangle|\Uparrow_y\rangle\Big]. \qquad (9)$$

$\rho_{pure} = |\psi\rangle\langle\psi|$ in each case would have the same form as (5). However, everytime we change the basis, $\rho_{pure} = |\psi\rangle\langle\psi|$ would end up with off diagonal terms which contain correlations of different states of the system and apparatus. The democratic world of quantum mechanics gives us the freedom to choose any basis since all bases are alike and equivalent. The price to pay for this freedom is that ρ_{pure} ends up being an ambiguous and confusing object when one appeals to it to describe a completed measurement process. By simply looking at the diagonal elements of ρ_{pure}, one cannot assert that the apparatus as well as the system is each separately in a *definite* state with one-to-one correlations. As we have seen, a mere change in the basis will transform the picture. This makes the interpretation of its diagonal terms as probabilities for the two potential outcomes quite meaningless. For a classically meaningful situation one must be able to talk of the system/apparatus existing in specific states with classical probabilities. This necessarily requires a description in terms of a density matrix that is *diagonal* in a uniquely specified *pointer basis* which is *preferred* over all others. The crux of the measurement problem and associated difficulties is to explain the transition from ρ_{pure} to ρ_{red} in a preferred *pointer basis*, which, in essence is a satisfactory explanation for the transition from 'quantum' to 'classical'. Is it possible to explain the mechanism of reduction within the framework of the theory of quantum mechanics?

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3. The Decoherence Approach

As stated earlier, the off-diagonal elements of the density

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matrix are the signatures of *quantum correlations.* (4) and (5), in fact represent the bizarre state of Schrödinger's cat when extrapolated to the macroscopic domain. In quantum mechanics, wave functions evolve via the Schrödinger equation which is linear and deterministic and this evolution is *unitary* in nature. Unitary evolutions ensure that eigenvalues are preserved. There is no way that some terms of the density matrix can vanish in the course of a unitary evolution. How, then, can 'classical behaviour' (as discussed above) ever emerge from this substrate of the quantum world where entanglements and coherences are ubiquitous and inevitable?

The answer lies in realizing the fact that the Schrödinger equation driving unitary evolutions is actually applicable to completely *isolated* systems. The real world, however, is far from this pristine pure situation. Macroscopic systems are almost never isolated from their surroundings. They are constantly interacting with a complex environment. In a measurement like situation, the apparatus is almost always a macroscopic object from which one reads out the measured property of the system. In fact, the apparatus is not only considered macroscopic, it is also regarded as 'classical' in its dynamics.

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The 'decoherence' explanation is that it is the influence of the environment that makes a quantum system appear 'classical'. The environment 'washes away' quantum coherence, leaving behind a system which looks and behaves like a classical object of our cherished commonsense world. The 'open system' is coupled to a large number degrees of freedom which constitute the environment. However, one is always monitoring only a few degrees of freedom, which are of relevance. In mathematical language, this amounts to *tracing* over all other degress of freedom. This tracing over has the effect of causing the much desired transition:

$$\rho_{pure} \longrightarrow \rho_{red} \tag{10}$$

An illuminating and popular paradigm for understanding this is the phenomenon of Brownian motion, named after the British botanist, Thomas Brown, who first discovered it in pollen grains suspended in water. We all know now that such a suspended particle, when examined, is seen to bounce around in a random, irregular, 'zig-zag' fashion. Einstein showed that this pattern of behaviour is exactly what should be expected if the pollen grain is being repeatedly 'kicked' by other unseen submicroscopic particles (water molecules). The random motion of the suspended particle can be statistically explained by taking into account its interaction with a large number of particles which constitute the reservoir of water molecules. This is the environment whose influence on the suspended particle causes its random zig zag motion. When we see Brownian motion, we are only focussing on the dynamics of the suspended particle and do not monitor each and every particle of the environment (the water bath). Mathematically, we *trace* over all the degrees of freedom of the environment and look only at the *reduced* system, the suspended particle. As a consequence, the tagged particle is found to show a dynamics that contains *dissipation* (a steady loss of energy or relaxation) and *diffusion* (the random zig-zag motion).

The phenomenon of Brownian motion, named after the British botanist, Thomas Brown, was first discovered in pollen grains suspended in water.

Quantum Brownian motion describes a similar situation at the quantum mechanical level. The quantum measurement situation can be described in the following manner. The microscopic system couples to a macroscopic apparatus, which in turn is interacting with a large number of degrees of freedom which constitutes the environment. Schrödinger's equation is applied to the entire *closed* universe of system-apparatus-environment. Hamiltonian evolution drives this closed system from an initial uncoupled state into a gigantic entangled state containing all the degrees of freedom. A *tracing over* all the environmental degrees of freedom salvages the *re-*

The random motion of the suspended particle can be statistically explained by taking into account its interaction with a large number of particles which constitute the reservoir of water molecules.

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duced system-apparatus combine from this mess. After a characteristic time, the apparatus, impacted by the environment, appears classical in its dynamics. Thus, the environment causes a general quantum state to decay into a statistical mixture of *pointer states* which can be understood and interpreted as classical probability distributions. This in essence is the approach of the decoherence theory to explain the emergence of classicality. A commonly used model to describe the environment is to consider it as a reservoir of quantum oscillators, each of which interacts with the quantum system in question. The Hamiltonian for this would look like the following:

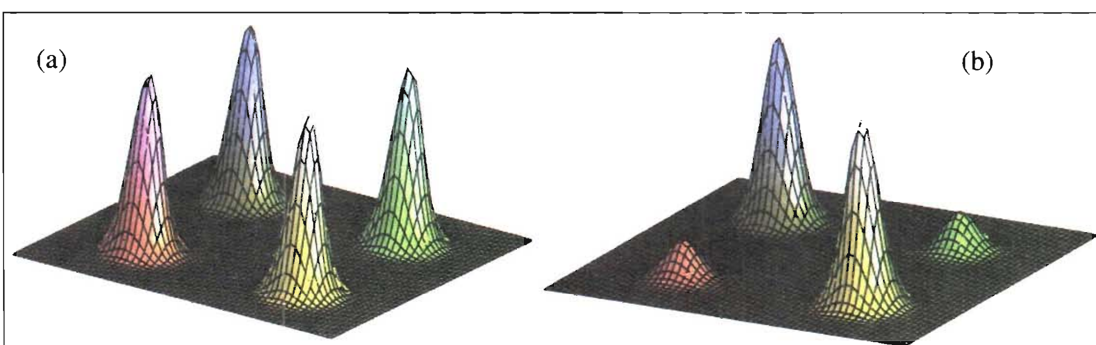
$$H = H_s + x\sum_k g_k Q_k + \sum_k \frac{1}{2M_k}\left(P_k^2 + M_k^2\omega_k^2 Q_k^2\right), \quad (11)$$

where H_s is the system Hamiltonian. The third term represents the Hamiltonian of the collection of oscillators which constitutes the environment. P_k and Q_k are the momentum and position coordinates of the k^{th} oscillator. The second term represents the *coordinate-coordinate coupling* between the system and the environment, x being the position coordinate of the quantum system. The dynamics for the closed universe of the system and environment is governed by the Hamiltonian evolution via (11) and the Schrödinger equation (1). Subsequently, the process of tracing over all the degrees of freedom of the environment results in an equation describing the dynamics of the *reduced* density matrix: $\rho(x, x', t) = Trace_k\left[\rho\left(x, (Q_k); x', (Q'_k), t\right)\right]$.

The reduced density matrix evolves according to a *master equation* which is obtained by solving the Schrödinger equation for the entire universe of the system and the environment and then tracing over the environment degrees of freedom. Several authors like Deiter Zeh and Woejeich Zurek, among others, have worked extensively on the decoherence approach using the *master equation*. Without going into the details of the *master equation*, it suffices to point out that the equation naturally separates into three disticnt terms, namely (i) a term describing the von Neumann equation which can be derived from the Schrödinger equation and thus represents the pure quantum evolution, (ii) a term that causes *dissipation*, which can be understood as a steady loss in energy or a *relaxation* process, and (iii) a term causing *diffusion*, which can be understood as the fluctuations or zig-zag movement seen in Brownian motion. The initial work of Zeh, Zurek and subsequent work where the decoherence approach has been applied to typical quantum-measurement-like situations like the Stern Gerlach experiment have shown that coherent quantum superpositions persist for a very short time. They are rapidly destroyed by the action of the environment. In fact, results show that the larger the quantum superposition, the faster the decoherence. For truly macroscopic superpositions such as Schrödinger's cat, decoherence occurs on such a short time scale that it is impossible to observe these quantum coherences. A simple example by Zurek illustrates this point (see *Figure* 1). Here an initial state constructed as a coherent superposition of two spatially separated gaussian wavepackets decoheres into a statistical mixture (diagonal density matrix) when its dynamics is governed by the *master equation* discussed above. Several authors have studied many more examples and decoherence calculations for these studies are seen to contain two main featues which can be seen as *signatures* of decoherence: (a) the decoherence time, τ_D,

Coherent quantum superpositions persist for a very short time.The larger the quantum superposition, the faster the decoherence.

Figure 1. Decoherence of the density matrix for an initial coherent superposition state of two spatially separated gaussian wavepackets: (a) The initial density matrix in the position representation showing the diagonal and off-diagonal terms of the quantum superposition. (b) The decohered density matrix where the off-diagonal peaks have decayed to leave the statistical mixture of diagonal terms which is classically interpretable.



These experiments have done no less than *create* Schrödinger-cat-like entangled states in the laboratory and seen them transform into classically recognizable objects under the influence of environmental coupling.

τ_D, over which the superpositions decay is much much shorter than any characteristic time scale of the system, e.g., the *relaxation time*, γ^{-1}. In the example chosen by Zurek, $\tau_D/\gamma^{-1} \sim 10^{-40}$, and (b) τ_D varies inversely as the square of a quantity that indicates the 'size' of the quantum superposition. In the example of Zurek, this is Δx, the spatial separation between the two gaussian wavepackets (see *Figure* 1).

4. Decoherence and Experiments

In recent years, the predictions of the decoherence theory have been tested in several spectacular experiments which put the theory on a firm footing. Of these, two experiments are particularly noteworthy and we will briefly mention them. Both these experiments have succeeded in monitoring the decoherence mechanism, i.e., the actual *transition* from a pure entangled state to a statistical mixture. Moreover, the experiments also give a quantitative estimate of the decoherence time, τ_D. These experiments have done no less than *create* Schrödinger-cat-like entangled states in the laboratory and seen them transform into classically recognizable objects under the influence of environmental coupling.

Among the first successful attempts is a beautiful experiment by Brune *et al.* at the École Normale Supérieure in Paris. Using Rubidium atoms and high technology superconducting microwave cavities, Brune *et al.* created a superposition of quantum states involving radiation fields. The superposition was the equivalent of a 'system + measuring apparatus' situation in which the 'meter' was pointing simultaneously towards two different directions. This is a Schrödinger-cat-like entangled state. Through a series of ingenious 'atom-interferometry' experiments, Brune *et al.* managed to not only 'read' this pure state but also monitored the decoherence phenomenon as it unfolded, transforming this superposition state to a statistical mixture. Besides providing a direct insight into the role of the environment in a quantum measurement process, their experiment also confirmed the basic tenets of the decoherence theory. The two main signatures of the decoherence theory were clearly observed in this classic experiment. The environment in this experiment are the 'modes' of the electromagnetic field in the cavity.

The decoherence explanation has certainly provided valuable insights into the actual mechanism of the loss of quantum coherences. Its qualitative and quantitative predictions are highly relevant to all experimental implementations of the novel ideas of quantum information.

At the National Institute of Standards and Technology, Boulder, Colorado, the group headed by Wineland chose a different experimental candidate to confirm the signatures of the decoherence mechanism. They created a Schrödinger-cat-like state using a series of laser pulses to *entangle* the internal (*electronic*) and the external (*motional*) states of a Beryllium ion in a 'Paul trap'. The motion of this trapped ion couples to an electric field which changes randomly, thus simulating an environment. Wineland *et al.* call this environment an *engineered reservoir* whose state and coupling can be controlled. Through their measurements, Wineland *et al.* have successfully demonstrated the two important signatures of the decoherence mechanism.

The above two experiments, along with several others have provided important insights into the role of the environment in bringing about *classicality*. The decoherence theory is strengthened by these spectacular observations.

5. Are the Problems Resolved?

At the end of the day can we say that the decoherence explanation makes quantum mechanics appear less enigmatic and mysterious? Have we explained away some of its conceptual problems? Physicists partial to the decoherence approach think so. The experiments discussed in the previous section clearly show how decoherence washes away quantum coherences. This is fairly convincing evidence for explaining the absence of Schödinger's

There are many who find the decoherence theory too prosaic a resolution to the complex and uncanny world of quantum mechanics.

cats in the real world. The decoherence explanation has certainly provided valuable insights into the actual mechanism of the loss of quantum coherences. Its qualitative and quantitative predictions are highly relevant to all experimental implementations of the novel ideas of quantum information. As for the realm of quantum measurements, to quote Tegmark and Wheeler, "decoherence produces an effect which looks and smells like a collapse..." This is an opinion shared by many physicists who see in the decoherence explanation a satisfactory settlement of the quantum measurement problem.

However, there are many who find the decoherence theory too prosaic a resolution to the complex and uncanny world of quantum mechanics. To them, many of the conceptual problems of quantum mechanics are still unresolved and deeply disturbing. This spectacularly powerful theory is still believed to conceal many secrets. Whatever one chooses to think, quantum mechanics remains a theory of great beauty and mystery, and a wonderful experience for anyone who has encountered it.

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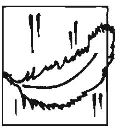
Suggested Reading

[1] Max Tegmark and John Archibald Wheeler, 100 years of Quantum Mysteries, *Scientific American*, p.68, February 2001.

[2] R Simon, From Shannon to Quantum Information Science, *Resonance* Vol.70, No.2, p.66, February 2002; Vol.70, No.5, p.66, May 2002.



The aim of science is not to open the door to infinite wisdom, but to set a limit to infinite error.

Berolt Brecht
From: *The Life of Galileo*