S S Pillai: 5 April 1901 – 31 August 1950

S Sivasankaranarayana Pillai was an outstanding mathematician who gained fame for his contribution to the solution of Waring's problem. He was born on April 5, 1901, at Vallam near Cutralam town (famous for its waterfall) in Tamil Nadu. His mother died within a year after his birth. Until the age of nine or so, he was educated at home after which he studied at the middle school in Senkottai. During his matriculation, his father too passed away suddenly. Fortunately, a former teacher came with monetary support enabling him to complete his school education and study further. Pillai got a scholarship to do his Intermediate course in the Scott Christian College at Nagercoil and his BA at Maharaja's College, Trivandrum. In 1927, Pillai undertook a research studentship at the University of Madras. He became greatly influenced by Anand Rao and Vaidyanathaswamy, as evidenced already in his early work. One of his most famous contributions is to the solution of Waring's problem. K Chandrasekharan refers to this in his obituary on Pillai as "almost certainly his best piece of work and one of the very best achievements in Indian Mathematics since Ramanujan". Most of his work was done during the period 1929-1941 when he was a lecturer in Annamalai University.

For any positive integer $k > 1$, let $g(k)$ be the least value such that every positive integer is the sum of $g(k)$ non-negative integers, each of which is a perfect k-th power. For instance, that every positive integer is a sum of 4 squares was proved by Lagrange in 1770, 130 years after it was stated by Fermat. Note that 7 is not a sum of three squares. That is, $g(2) = 4$. In the same year (1770), Waring, at Cambridge University, conjectured in his *Meditationes Algebraicae*, that every number is the sum of 9 cubes and of 19 fourth powers and also that the number $g(k)$ is finite for every k. Let alone verify the given values 9, 19 and so on, even the very existence of $g(k)$, for every k, was an open question for a long time. It was finally proved by Hilbert in 1909. During the same year, Wieferich also proved that $g(3) = 9$. For $k > 1$, let $[(3/2)^k]$ denote the greatest integer less than or equal to the k-th power of 3/2. Pillai conjectured that $g(k) = 2^k + [(3/2)^k] - 2$ for all k and proved this in 1935 for all $k > 6$, assuming the truth of the inequality $\{(3/2)^k\} + \frac{[(3/2)^k]+3}{2^k} \leq 1$, where $\{(3/2)^k\}$ is the fractional part of $(3/2)^k$. The number $2^k + [(3/2)^k] - 2$ comes up naturally because this is the number of summands needed to express the number $2^k[(3/2)^k]-1$ as a sum of k-th powers; it is interesting to note that this was proved in 1772 by a son of the famous Leonhard Euler. In 1940, Pillai obtained $g(6) = 73 = 2^6 + [(3/2)^6] - 2$ unconditionally. It has been proved now (Jing-run Chen in 1964) that $g(5) = 37 = 2^5 + [(3/2)^5] - 2$ and that $g(4) = 19 = 2^4 + [(3/2)^4] - 2$ (R Balasubramanian. J-M Deshouillers, F Dress in 1986).

Another deep result of Pillai was on Diophantine approximations. He formulated the conjecture (still open) that in the equation $a^x - b^y = c$, for a given c, there are only finitely many a, b, x, y solving it. This is vast generalization of the Ramanujan - Nagell equation and Catalan's equation. Recently, T N Shorey has proved that the generalized abc-conjecture implies Pillai's conjecture. This connection brings forth the depth of Pillai's conjecture also since the abc-conjecture is a powerful conjecture

of which various things like Fermat's last thorem are also consequences. Another noteworthy contribution by Pillai is his extension of Ramanujan's work on highly composite numbers. Pillai was awarded a DSc degree from Madras University and was the first to receive such a recognition in mathematics from that university.

Apart from number theory Pillai was also interested in tackling famous tough problems in other areas. In 1942, when he was a lecturer at Calcutta University, he was interested in the problem of whether a continuous, periodic function necessarily has a point of convergence.

He was at the peak of his work in 1950, and was invited to be a visiting member for a year at the Institute for Advanced Study, Princeton, and also to participate in the International Congress of Mathematicians at Harvard University. But, tragically, he died when the aircraft carrying him crashed near Cairo on August 31.

Pillai was shy of any publicity about himself. Once when a Calcutta editor of a newspaper had sent a photographer to take a picture of Pillai, the latter agreed after the cameraman had kidded him saying he had been asked by Princeton University itself to get Pillai's photograph! Pillai declined to wear either a coat or a tie mimicking Oliver Cromwell when he said, "Just as I am, with warts and all."

It is amusing to know that Pillai's dinner guests – including foreign mathematicians invited to his house – were occasionally served traditional Tamil food on plantain leaves and requested to squat on the floor and eat using bare hands! Pillai had strong beliefs with regard to public issues: he was highly critical of the first use of the atomic bomb and espoused quitting the Commonwealth while retaining the English language. He is said to have possessed the very rare quality of intellectual honesty due to which he had to forego many material advantages.

It seems incredible that he was never a Fellow of any of the Academies or Institutes in India although there is a letter from Sir C V Raman indicating that he would like to propose Pillai's name for Fellowship to the Indian Academy of Sciences. This, and some other interesting tidbits like photographs and copies of letters can be seen at www.geocities.com/ananthirt; I am indebted to R Thangadurai from HRI, Allahabad for this reference.

Littlewood said in 1934 (before Pillai had gained fame for his work on Waring's problem), "Dr.Pillai's work is fresh and original. I consider him as one of the very best of Indian mathematicians."

T Vijayaraghavan said in 1937, "Since the time Ramanujan died, the work of no other Indian mathematician has brought to Indian mathematics greater credit than Dr. Pillai's on Waring's problem."

It is high time that the Indian Academy of Sciences recognises Pillai's magnificent contributions to mathematics.

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