The Mutable Galaxies
How Galaxies Enrich with Heavy Elements

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Soon after the discovery of the expansion of the universe, astronomers hoped to determine the parameters of the universe by comparing distant galaxies with nearby ones. Observing distant galaxies, however, meant observing them as they were long ago. A crucial assumption in these studies was that galaxies hardly change, in the time scale of the expansion of the universe. Beatrice Tinsley was one of the first astronomers to point out that galaxies do change. Here we discuss a simplified model of galactic evolution, and show that the comparison of its prediction with observations uncovers clues to how galaxies evolve with time.

A cursory look at the starry sky at night is likely to make us think that stars, galaxies and the universe are immutable and changeless. Stars and galaxies do however change with time. The beginning of the last century was marked by the discovery of the expansion of the universe and also the development of our knowledge of the evolution of stars. Yet, the overall evolution of galaxies, as a result of stellar evolution, was considered too messy, and perhaps even negligible.

The study of stars told us that stars shine as a result of thermonuclear reactions inside their cores. These reactions synthesize heavy elements like carbon, oxygen, nitrogen and so forth, beginning with hydrogen and helium (which were created in the first few minutes of the birth of the universe). Stars which are more massive than a certain amount spew out their accumulated wealth of these heavy elements in space as a result
of spectacular explosions, called supernovae. The next generation stars which grow out of this ‘enriched’ material, produce yet another batch of heavy elements, a portion of which is again ‘recycled’ through the supernova explosion of massive stars.

Galaxies then must go through this ‘chemical’ evolution, slowly changing the abundance of heavy elements in its gas and in stars. A spectacular confirmation of these ideas came recently from the observation of gamma rays from radioactive elements in Milky Way, like $^{26}$Al. This element has a half life of less than a million years ($7 \times 10^5$ yr, to be precise), and it emits gamma ray photons of energy 1.8 MeV. Detection of these gamma rays showed that this element was produced recently, less than a million years ago.

When Beatrice Tinsley began working as a graduate student on the issue of using distant galaxies to determine the parameters of the universe we live in, she realised that the evolution of galaxies themselves made them unsuitable for such studies. How would one disentangle the effects of cosmology (how the universe expands and how this expansion has changed with time) from the effects of internal changes in galaxies? If the measuring instruments begin to change, how does one use them to measure the change in something else?

Unlike many stalwarts at that time who dismissed such worries, Tinsley began to think in earnest about tackling the evolution of galaxies. We will discuss a particular aspect of these early studies by her and others.

Evolution of stars is a messy topic. As mentioned earlier, stars use their supply of hydrogen and helium to synthesize heavy elements to shine. Their ‘lifetime’ is therefore constrained by the amount of hydrogen/helium fuel. In general, massive stars use their fuel vigorously and consequently ‘live’ for a short time. These stars are hot and bright. Low mass stars continue to shine (faintly)
The distribution of stellar masses in our Galaxy is found to be dominated by low mass stars.

for a long time. Stars more massive than, say about 8 times the mass of sun (8 solar mass, or $8M_{\odot}$), explode at the end of their lifetime. These supernova explosions disperse a fraction of the processed material in the interstellar space. The amount (both absolute and relative) of each heavy element thus recycled depends on the mass of the star in a complicated way.

There is the additional problem that the relative proportion of stars of different masses is not a constant but a function of the mass itself. This function known as the Initial Mass Function plays a crucial role in determining the course of chemical evolution of a galaxy. Matters could be further complicated if this function, (IMF) depended on time, or the 'enrichment' of the gas and if it varied from one galaxy to the other. Then there are other problems that the rate of formation of stars could depend on time, and some gas may flow out of the galaxy or may fall into it from the ambient medium.

The distribution of stellar masses in our Galaxy is found to be dominated by low mass stars. In other words, there are more low mass stars than massive stars (see Figure 1). Typically, the fraction of mass that is contributed by stars less than one solar mass ($1 M_{\odot}$) is approximately 70%-80%.

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Figure 1. The distribution of stellar masses. Low mass stars (shown as small circles) are more abundant than massive (large circles) stars.
Let us make a few simplifying assumptions to make the problem tractable. We divide the stellar population into two parts – one which ultimately goes supernova and recycles its wealth of heavy elements, and another which keeps its wealth. In other words, instead of the circles of a variety of sizes in Figure 1, we divide the ensemble into two parts, of big and small circles (see Figure 2a). Since massive stars go supernovae in less than about 10 million years, which is a very short time compared to other relevant time scales, let us then assume that the first type of stars become supernovae instantly, and that the processed material is mixed with the rest of the gas efficiently. Figure 2b shows the disappearance of the massive stars, and also that the gas has been enriched somewhat (darker shade of the empty space around circles in Figure 2b, compared to that in Figure 2a). The first category stars then continually return their processed material to the surrounding gas, whereas the second category stars keep on adding to the total stellar mass, generation after generation. Figure 2c (and 2d) shows the result after another generation of star formation, with increased number of low mass stars and increased abundance of heavy elements in gas. The amount of gas has certainly decreased.

Figure 2. A schematic diagram for the simplified version of chemical evolution of galaxies. (a) Stars are divided into two categories, – massive stars are assumed to recycle their processed material almost instantaneously. Panel (b) shows their disappearance, along with a slight enrichment of the gas (darker shade of empty space around stars) – massive stars will not be shown in the later panels. Panel (c) shows how after the massive stars created in the last time interval have enriched the gas further and created another set of low mass (long lived) stars of higher abundance of heavy elements. The trend continues in panel (d).
If one assumes that there is no outflow (or inflow) of matter from (or into) the galaxy, then one can easily show that the abundance of heavy elements in gas at any instant is given by,

\[ Z = \rho \ln \frac{1}{\mu}, (Z \ll 1). \]

This startlingly simple result for a complicated problem of chemical evolution of galaxies remains one of the most widely used results in galactic evolution.

In this simplified version, one can then look for a relation between the residual amount of gas and the abundance of heavy elements in it, at any instant of time. One could perhaps also find the relative number of (second category) stars with different amount of heavy element abundance – the relative number of stars with different shading in Figure 2c or 2d. Perhaps one could then compare this with what is observed in galaxies. Let us find out how this can be quantified in order to be able to compare with observations.

The second type of stars tend to 'lock up' a fraction of mass since they do not recycle their processed material. Let us call this fraction the 'lock-up fraction', \( \alpha \). Let us denote the fraction of mass in the form of heavy elements (heavier than hydrogen and helium) by \( Z \). For example, the value of \( Z \) inside Sun (denoted by \( Z_\odot \)) is approximately 0.01. We will denote this fraction inside stars by \( Z_s \) and for gas by simply \( Z \). Another important parameter is the 'yield', \( \rho \) which is defined as the mass of heavy elements ejected per unit mass that is locked in (low mass) stars. This might seem like a bizarre parameter to define, but it turns out to be an important parameter in the chemical evolution of galaxies.

Let us also denote the fraction of the galactic mass that is in the form of gas (and not in stars) as \( \mu \). If one assumes that there is no outflow (or inflow) of matter from (or into) the galaxy, that is, if one assumes galaxies to be separate 'closed-boxes', then one can easily show that (see Box 1), the abundance of heavy elements in gas at any instant is given by,

\[ Z = \rho \ln \frac{1}{\mu}, (Z \ll 1) \]  \hspace{1cm} (1)

This is a startlingly simple result for a complicated problem of chemical evolution of galaxies. As expected, it remains one of the most widely used (perhaps used too much!) results in galactic evolution.
Box 1.

Let us denote by \( g \) and \( s \) the mass of gas and stars at any instant, respectively. Suppose \( \psi(t) \) denotes the rate of formation of stars. One can then write, for closed-box galaxies and assuming ‘instantaneous recycling’ (that is, there are only two categories of stars as described in the text),

\[
\frac{ds}{dt} = -\frac{dg}{dt} = \alpha \psi,
\]

where \( \alpha \) is the lock-up fraction, the fraction of mass in low mass stars.

The total mass of stars (of both categories) produced is certainly \( s/\alpha \). If \( p \) is defined as the mass of heavy elements produced by unit mass in the lock-up category of stars, then the total mass of heavy elements per unit mass in both category stars is \( \alpha p \). The total mass of heavy elements ejected by stars until a given time, will be \((\text{total mass of stars produced until then}) \times (\text{mass of heavy elements per unit stellar mass}) = (s/\alpha) \times (\alpha p) = ps\). This amount of heavy elements is shared by gas and stars. In other words,

\[
ps = sZ_s + gZ,
\]

which can be written (after defining the gas fraction \( \mu = g/(g + s) \)) as,

\[
Z_s = p - \frac{\mu}{1 - \mu} Z.
\]

This relation shows that as gas is completely depleted \((\mu \to 0)\), \( Z_s \to p \). In other words \( p \) can be estimated by the mean abundance of heavy elements in stars, if \( \mu \) is very very small, which is the case in our galaxy. One finds that \( p \) is of the order of heavy element abundance in the Sun, or \( p \sim Z_\odot \).

One can also write the evolution of heavy elements in gas as,

\[
\frac{d}{dt}(gZ) = -\alpha Z \psi + p \alpha \psi,
\]

where the first term on the RHS describes the continual disappearance of a certain amount of heavy elements in low mass stars (which ‘lock up’ the heavy elements), and the second term denotes the production of new heavy elements. Since \( gdZ/dt = d(gZ)/dt - Z dg/dt \) one can also write this as, \( g \frac{dZ}{dg} = \alpha p \psi \). Using \( dg/dt = -\alpha \psi \), we can write,

\[
\frac{dZ}{dg} = p
\]

which is readily integrated to give (with the initial condition that \( Z = 0 \) when \( g = g(0) \)).

\[
Z(t) = p \ln \frac{g(0)}{g(t)} = p \ln \frac{1}{\mu(t)},
\]

continued ...
where \( \mu(t) \) is the gas fraction at time \( t \). We should point out that equation (d) is true only when \( Z \ll 1 \) so that the above solution is valid only in this regime.

Consider the cumulative distribution of heavy element abundance in stars ever formed. Suppose the current stellar mass and gas fraction are given by \( s_0 \) and \( \mu_0 \). The fraction of stellar mass formed until the instant when the gas fraction was \( \mu \) is,

\[
\frac{s}{s_0} = \frac{1 - \mu}{1 - \mu_0}.
\]  

(h)

These stars formed when the abundance of heavy elements was less than or comparable to \( p \ln(1/\mu) \). So, the fraction of stars with heavy element abundance less than \( Z \), is,

\[
s(Z) = \frac{1 - \exp(-Z/p)}{1 - \mu_0} = \frac{1 - \mu_0 Z/Z_0}{1 - \mu_0}.
\]  

This is the distribution of stars of different heavy element abundance.

It is also a very reasonable result. The abundance of heavy elements in gas is zero at the beginning, when \( \mu = 1 \), that is, there are no stars and everything is in the form of gas. As gaseous material is depleted to form stars, as \( \mu \) decreases, the abundance of heavy elements in it increases.

It is then a straightforward exercise to determine the relative number of stars of an identical type with different abundances of heavy elements. Suppose the current value of \( \mu = \mu_0 \) and \( Z = Z_0 \). Mathematically, one can write the fraction of stars which have heavy element abundances less than a certain amount \( Z \), as,

\[
s(Z) = \frac{1 - \mu_0 Z/Z_0}{1 - \mu_0}.
\]  

(2)

This is a distribution that can be readily compared with observations. To do this one has to identify a stellar type which can be sampled over timescales comparable to the lifetime of the Milky Way galaxy, so that even the first generation of such stars is available to us today for making observations. Since massive stars from a previous, less enriched generation are not observable.
at the present time, one has to look at stars that are of lower mass. A G dwarf, a star with a calculated lifetime of about 10 billion years, provides such a sample, for even the first-generation G dwarfs in our Galaxy should still be living and of course, their later generations too. When a selected sample of G dwarfs was thus studied, it was found that there are too few of them with small values of $Z$, compared to the above prediction. The same exercise could have been done with stars of type K but since it was done with a sample of G dwarfs first, the problem has come to be known as the ‘G dwarf problem’. Our Sun is also a G dwarf.

Clearly some (or all) of the assumptions that we made must be modified. Perhaps the IMF varied with time such that there were relatively a larger number of high-mass stars in the past than there are now – thereby

Figure 3. The solid line shows the prediction of the simple model of galactic evolution, in terms of the distribution of stars with different heavy element abundance, and the dashed line shows the observed distribution. The deficit of stars with low abundance of heavy elements on the left is acute.
making the lock-up fraction small in the past, and making the ancient generation of stars (with small amount of heavy elements) a rare commodity today. Or, perhaps the assumption of galaxies as ‘closed-boxes’ is wrong, and material falling into galaxies enriches it beyond the above prediction. At any rate, the G dwarf problem points towards some yet unknown but important clues to evolution of galaxies.

Astrophysicists have continued in the last few decades to refine the pioneering calculations done by Tinsley and others in order to explain the G dwarf problem. It is indeed a tribute to the perseverance of Tinsley who did not want to avoid the problem of galactic evolution (in the hurry to use galaxies for cosmological studies) but decided to study its effects.

Beginning with the pioneering work of Tinsley, astronomers have developed models of how a ‘population’ of stars should change their look with time – to be precise, how the composite spectrum, and brightness should change with time, as a result of the evolution of stars and their continual recycling of processed material into the gas and formation of new generations of stars. In the last few decades, astronomers have improved upon their models, which are now being used to compare with observations of ancient galaxies that have become possible with the advent of large telescopes.

**Suggested Reading**


The evolution of galaxies is a subject that involves almost every field of astronomy and astrophysics to some extent, since everything within galaxies was made and distributed by an evolutionary process.

*Beatrice Tinsley*