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Probabilities in the Card Game of Three Cards

In the game of Three Cards (also known as Flush or Poker), certain combinations of playing cards are considered valuable, and conventionally ranked according to their value in the order (a) Trail, (b) Flush, (c) Run, (d) Colour, and (e) Pair. In this note we obtain the probabilities of these combinations and check whether the conventional wisdom is right. In this game, the valuable combinations are defined as follows:

Trail: All the three cards of the same denomination;

Flush: Three consecutive cards of the same suit, such as QJ10, 543, etc. The ace fits in at both the ends, such as AKQ or 32A, but the combination 2AK is not allowed, which means that this combination is considered to be of a lower value;

Run: Three consecutive cards of any suit, with the same proviso as in flush being applicable; it must exclude Flush;

Colour: Any three cards of the same unit; it must exclude Flush;

Pair: Two cards of the same denomination, the third being any other card; must exclude Trail.

We shall only calculate the probability for Trail and Flush. The exercise is simple and very rewarding. We note that there are altogether $52 \times 51 \times 50/6 = 22100$ distinct combinations of three cards. We pick up three cards from the pack, without replacement.

Trail: First note that the number of combinations resulting in 3 aces (AAA) is $4 \times 3 \times 2/3! = 4$. Similarly for KKK, ... , 222. So the number of combinations resulting in a 'Trail' is $13 \times 4 = 52$. Thus the probability of a Trail is $52/22100 = 1/425 = 0.002353$.

Flush: In this case again it is convenient to proceed by finding the favourable number of ways and dividing it by the total number of combinations 22100 obtained above. We want three consecutive cards of the same unit, barring the combination 2AK. Let us write the following series of 12 cards: AKQ109876 543.

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We note that one may start from any of the above 12 cards and complete the Flush by taking two more on its right, such as QJ10, 765, etc. In the case we start with 4 or 3, we may complete the Flush with 432 or 32A, respectively. But one cannot start from 2 and proceed because the combination 2AK is not allowed. Hence 2 is not included in the above series. Thus one can have any of the 12 cards as the starting point of the Flush, and there are 4 suits. This gives us a combination of three cards which produce a Flush. Thus the number of favourable combinations for Flush is $12 \times 4 = 48$. The probability of dealing a Flush is therefore

$$48/22100 \approx 0.002172 \approx 1/460.4. \qquad (1)$$

The probabilities for Run, Colour and Pair are straightforward to calculate, and the values are as follows:

$$\text{Run:} \quad \frac{720}{22100} \approx 0.03258 \approx \frac{1}{30.7}; \qquad (2)$$

$$\text{Colour:} \quad \frac{1144}{22100} \approx 0.05176 \approx \frac{1}{19.3}; \qquad (3)$$

$$\text{Pair:} \quad \frac{3744}{22100} \approx 0.1694 \approx \frac{1}{5.9}. \qquad (4)$$

Thus we see that the probabilities are ranked in the order Flush, Trail, Run, Colour and Pair, as shown below:

	Flush	Trail	Run	Colour	Pair
	0.002172	0.002353	0.03258	0.04959	0.1694
1 in (~)	460	425	31	20	6

In each of the above, we have given the integer nearest to the reciprocal of the probability. We conclude that conventional wisdom is wrong in assigning the top place to Trail which should rightly have gone to Flush. Indeed in some parts of the world, the game is called Flush, though the top rank is given to Trail. Perhaps it is a nice feeling to see three Aces in one's hand.