A brief review of the work of the three Nobel Laureates – Alexei A Abrikosov, Vitaly L Ginzburg and Anthony J Leggett is presented. Their work forms the basis for understanding the phenomena of superfluidity and superconductivity.

The Nobel Prize in Physics for 2003 was awarded jointly to Alexei A Abrikosov, Vitaly L Ginzburg and Anthony J Leggett for ‘pioneering contributions to the theory of superconductors and superfluids. The prize rewards the three scientists who have explained remarkable quantum-physical effects in matter close to absolute zero temperature’.

Superconductivity refers to the resistance-less flow of electrons in a material and superfluidity refers to the flow of a liquid without viscosity. Both these are low temperature phenomena.

Since the development of the kinetic theory of heat, temperature is associated with random motions of the elementary building blocks of a body. In addition to this random motion, the building blocks also experience forces due to various interactions, which lead to ordering. A low temperature ordered state arising from a particular interaction, will be destroyed on increasing the temperature, because it increases the energy of the random thermal motions. A liquid-gas phase transition is one such example. So every interaction (fundamental or otherwise) can be assigned a temperature range according to its strength, where it is effective in producing ordering. Thus, we can look at superconductivity and superfluidity as low temperature ordered states where
the interaction responsible for ordering has very small energies.

The transition temperature, for transition from the gaseous state of an element to its liquid state depends on the strength of Van der Waals forces which decide the atom-atom interaction. The strength of Van der Waals forces between $^4$He atoms is very small. Therefore $^4$He has a correspondingly low transition temperature to liquid – only 4.2 K above absolute zero. The liquification of helium by the Dutch physicist Kammerlingh Onnes in 1908 (Nobel Prize 1913), opened a new area of probing the physical forces that dominate at temperatures close to absolute zero. This directly led to the discovery of 'vanishing of electrical resistance' of mercury at this temperature (shown in Figure 2). Onnes himself stated that 'mercury has passed into a new state, which, on

![Figure 1.](image1)

![Figure 2.](image2)

The left panel shows the superconductivity of Mercury (adapted from H K Onnes, Comm. Leiden 120b(1911)). The right panel shows a superconductor (V-Ba-Cu-O) expelling the magnetic flux below the transition temperature. This causes levitation of a permanent magnetic disk.
account of its extraordinary electrical properties may be called the superconductive state'.

Unmeasurable low resistance to the flow of electrons and the expulsion of an already existing magnetic flux (Meissner effect) are the two characteristic features of a superconductor. A superconductor is also a perfect diamagnet i.e., it does not allow penetration of small magnetic fields. This is illustrated in Figure 2. However, when the applied field is increased, the superconductors show one of the two following types of behaviour. In Type I superconductors flux exclusion is perfect upto a critical field \( H_c \). When the field exceeds this value the superconductor becomes normal which then allows complete flux penetration (shown in Figure 3).

In Type II superconductors, on the other hand, perfect flux exclusion exists upto an applied value \( H_{c1} \) called the lower critical field. When the field exceeds \( H_{c1} \), some flux penetrates the superconductor, till at a value \( H_{c2} \) of the magnetic field, full flux penetration takes place. The electrical resistivity of the material, even during the partial flux penetration regime (between \( H_{c1} \) and \( H_{c2} \)), is zero and only at \( H_{c2} \) does the material loose its superconducting property i.e., it becomes normal (shown in Figure 3).
The theoretical framework describing the behaviour of superconductors in an external magnetic field was developed by a group of Soviet physicists in the late 1940s. The crucial element in this theory is a parameter called the ‘order parameter’ which is zero in the disordered state and takes a finite value in the ordered state. The order parameter concept was introduced by Lev Landau, a famous Russian physicist, in 1937, to describe a class of phase transitions called the second order phase transitions. For example, in the theory of ferromagnetism the order parameter is the spontaneous magnetisation. Since the order parameter describes a thermodynamic phase transition, the relevant thermodynamic function namely the free energy is written as a function of the order parameter. This kind of description was able to give the temperature dependence of the order parameter across a phase transition quite accurately. Vitaly Ginzburg, working along with Landau adapted this theory to describe the behaviour of a superconductor in the presence of a magnetic field $H$ where the order parameter may vary in space. So their free energy function also contained terms which are gradients of the order parameter. They chose the order parameter to be a complex quantity $\Psi$. The equilibrium thermodynamic state is got by minimising the free energy density with respect to $\Psi, \Psi^*$ and $A$. Here $A$ represents the vector potential describing the magnetic field. These equations are called the Ginzburg-Landau (GL) equations. For the sake of completeness we include them here.

$$\frac{1}{2m^*}\left(-i\hbar \nabla - \frac{e^*}{c} A\right)^2 \Psi + \alpha \Psi + \beta |\Psi|^2 \Psi = 0.$$  

$$I = \nabla \times \mathbf{H} = \frac{e^* \hbar}{2m^* c}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) + \frac{e^{*2}}{m^* c^2} |\Psi|^2 \mathbf{A}.$$  

Here $e^*$ and $m^*$ represent the effective charge and mass of the superconducting element. (Later, following the microscopic description of superconductors by Bardeen, Cooper and Schrieffer (BCS theory) in 1950, Ginzburg
and Landau realised that $e^* = 2e$ and $m^* = 2m$, where $e$ and $m$ are the charge and mass of an electron.)

The order parameter $\Psi$, was itself described as the effective wavefunction of the superconducting elements, and $|\Psi|^2$ gives their number density. It is clear from this description that all the elements in the superconducting state are described by a single wavefunction $\Psi$. The exemplary properties of the superconductors can be traced directly to the existence of such a macroscopic wavefunction. The GL equations gave rise to two characteristic length scales, $\lambda$ and $\xi$, which describe the properties of superconductors in the presence of magnetic fields. $\lambda$, the penetration depth, describes the length over which the magnetic field decays inside a superconductor and $\xi$, the coherence length, describes the distance over which any deviation of the order parameter from its equilibrium value decays. The stability of the superconducting phase in the presence of an external magnetic field, boils down to energy considerations of the surface separating the region over which the flux has penetrated (normal regions) and regions of negligible change in the order parameter (superconducting regions). It turns out that if the ratio $\kappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}}$, then the net surface energy is positive. This implies that it costs energy to make such a surface and these class of materials form the Type I superconductors. On the other hand if $\kappa > \frac{1}{\sqrt{2}}$ then the surface energy is negative implying that it is energetically favourable to have surfaces. These class of materials form the Type II superconductors. Thus Ginzburg and Landau could make a number of predictions for the critical magnetic field and critical current density for thin superconducting films which were borne out to be true by later experiments. They did not dwell much on the characteristics of Type II materials because the then superconducting materials had $\kappa \ll \frac{1}{\sqrt{2}}$.

The phenemenological Ginzburg-Landau theory was developed seven years before the microscopic BCS theory

3 e.g. a Cooper-pair (see p.57).
and succeeded in explaining many of the observed properties of superconductors. It is most used to describe the nature of superconductors in practical applications—like superconductors in the presence of strong magnetic fields and time dependent superconducting order. It was shown later that GL equations can be derived from BCS theory as well and it has applications in many areas of physics.

After the GL theory, a spate of experiments followed confirming its predictions. The number of materials showing superconductivity also had increased and some of these fell under the Type II class of superconductors. It was experimentally found that these superconductors retained their superconducting property upto a much higher critical field than that predicted by GL theory. Alexei A Abrikosov, a student of Landau, found explicit solutions to the GL equations for Type II superconductors and showed that the critical field \( H_{c2} \), where the superconducting order completely vanishes, can indeed be considerably higher than that \( H_c \) for Type I materials (Figure 3). Abrikosov also showed that in the intermediary regime between \( H_{c1} \) and \( H_{c2} \), magnetic field enters the superconductor partially in the form of flux tubes carrying a quantum of flux \( \frac{hc}{2e} \) each. He found that a periodic distribution of the flux tubes minimised the total energy. So these flux tubes arrange themselves in the form of a lattice. The core of the flux tube contains normal material where the superconducting order parameter is zero and complete field penetration takes place. Surrounding the flux tube, supercurrents flow, shielding the rest of the superconductor from the field. It is common to refer to these flux tubes as vortex tubes and the array of flux tubes as a vortex lattice. At \( H_{c2} \) the vortex cores begin to overlap and the system returns to its normal state.

It may be noted here that Abrikosov's prediction of the vortex state was remarkable in that it preceded any con-
crete experimental proof of its existence. It is said that Abrikosov discovered these solutions in 1953 and did not publish them till 1957. The suggestion by R P Feynman in 1955 that vortex filaments are formed in superfluid $^4$He prompted Abrikosov’s publication. Vortex lattices are now commonly observed in Type II materials. A regular hexagonal arrangement of the vortex tubes in a Type II superconductor is shown in Figure 4.

After the discovery of ‘high temperature superconductors’ which are extreme Type II superconductors by Gerd Bednorz and Alex Muller in 1986 (Nobel Prize 1987), research to understand and use these new materials has become very active. The vortex lines discovered by Abrikosov are very important for the properties of these materials. Type II materials are commercially used to wind the superconducting magnets for Magnetic Resonance Imaging (MRI) and in high energy charged particle accelerators.

We had mentioned earlier that superconductivity is an ordered state. The extraordinary property of resistanceless flow, implies that in the superconducting temperature range, there is no longer scattering of electrons by the underlying lattice of positive ions. This remarkable effect happens due to the following reasons as explained by the BCS theory.
Electrons with opposing momenta and spin pair. These pairs are called the Cooper pairs. The interaction between electrons in a Cooper pair is over macroscopic distances ($\approx 1000$ nm). This is mediated through exchange of acoustic waves (phonons) with the lattice.

The Cooper pairs are strongly correlated with each other. We can deduce from various experiments that there is macroscopic occupation of a single quantum state of cooper pairs. To state it simply, it means that all cooper pairs are in the same quantum state.

Now, electrons are particles with $1/2$ integer spin (fermions) obeying Pauli's exclusion principle which states that no two electrons can occupy the same quantum state. But Cooper pairs have zero total spin (bosons) and hence favour the occupation of the same state as given by Bose–Einstein statistics. At appropriate densities and temperatures, bosonic particles show a phase transition called the Bose–Einstein condensation where a macroscopic fraction of the total number of particles occupy the lowest energy state. It is this state which is referred to as $\Psi$ in the GL theory. Other bosonic particles also show this condensation phenomenon. For example, $^4$He is a boson (integral total spin) whose superfluid nature was discovered by Pyotr Kapitza in 1938 (Nobel Prize 1978). Some aspects of the superfluid nature of $^4$He can be attributed to this condensation, though here, the strong interaction between Helium atoms alter this naive picture. After the BCS theory, it was conjectured that another isotope of Helium, namely $^3$He, which is a fermion, should also show superfluidity through pairing (to form a composite boson) just as the fermionic electrons in a Cooper pair do. It was experimentally shown by Lee, Richardson and Osheroff in 1972 (Nobel Prize 1996) that by cooling to a low enough tempera-
Figure 5. Transition from a paramagnetic to a ferromagnetic state.

Temperature (2 milliKelvin), helium-3 atoms do indeed pair up. They also showed using NMR studies that there are two different superfluid phases of $^3\text{He}$ – namely the A phase and the B phase. However, the nature of superfluidity in $^3\text{He}$ is very different from that of either $^4\text{He}$ or Cooper pairs. To understand this we should understand the concept of spontaneously broken symmetry.

To illustrate this concept, we take the familiar example of transition of a material to a ferromagnetic state. The magnetically disordered high temperature paramagnetic state has spins randomly oriented in all directions. In the low temperature ferromagnetic state the spins line up along a preferred direction. This is shown in Figure 5. Clearly the existence of a preferred direction of spin implies that the symmetry of the ferromagnet under spin rotation is reduced (broken). This is the phenomenon of spontaneously broken symmetry (i.e., not caused by an external field). It describes the property of a macroscopic system, in an ordered state, lacking the full symmetry of the underlying microscopic dynamics. In BCS theory for superconductivity, and in the theory of superfluidity in $^4\text{He}$, the order parameter is a complex quantity with two components, an amplitude and a phase. The high temperature states in these systems can have any value for this phase (‘gauge’). However, the low temperature ‘super’ states have a particular value
for this phase. This is referred to as spontaneously broken gauge symmetry. Some systems can have order parameters with not just two components, but as many as 18 components, as in the case of $^3\text{He}$. This arises because, the paired $^3\text{He}$ atoms strongly repel each other. As a result the paired particles will be kept at some distance from each other. This implies a non-zero relative orbital angular momentum. Therefore not only the phase and amplitude of $\Psi$ is in question, but the relative orbital angular momentum of the paired $^3\text{He}$ atoms and their relative spin orientations also play a part. It turns out that, in addition to gauge symmetry being broken in superfluid phases in $^3\text{He}$, the rotational isotropy of the relative orbital angular momentum state and that of the spin states are simultaneously broken. Therefore altogether three symmetries are broken. This makes $^3\text{He}$ a highly anisotropic superfluid. In 1972, Anthony Legget made the theoretical prediction that several simultaneously broken symmetries can appear in condensed matter systems. He applied it to the case of $^3\text{He}$ and showed that the condensed pair of $^3\text{He}$ atoms are in a relative orbital momentum p-state ($L=1$) and the spins are in a relative triplet state ($S=1$).

We can think of these as two vector quantities. Leggett showed that if both the vectors end up pointing in particular directions (the case where the rotation and spin symmetries are separately broken) then this resulted in the A superfluid phase of $^3\text{He}$. Instead, if only the relative orientations of these two vectors is fixed (combined broken symmetries) then this resulted in the B phase. This is schematically shown in Figure 6. Leggett showed that the long range orientational ordering of (as in the case of liquid crystals) the spin and orbital angular momentum vectors in the A phase gave rise to the high frequency NMR signal as reported in experiments. The A and B phase were further identified with a particular quantum state namely the ABM state and the BW

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**Figure 6. Superfluid phases of He-3. Solid arrows indicate the orbital angular momentum and dashed arrows indicate spin angular momentum.**

A phase

B phase
The work of Anthony Leggett was crucial in understanding the order parameter structure in the superfluid phases of $^3$He. However, his discovery that several symmetries can be broken simultaneously during a transition to an ordered state, is of more general importance in understanding complex phase transitions in other fields like liquid crystals, particle physics and cosmology.

The theoretical understanding of the phenomena of superconductivity and superfluidity we have today, is the result of the seminal work of these three people who were clearly fascinated by these low temperature effects. The lure of low temperatures is partly because it represents a world without disorder. However, more fundamentally, it is a realm where our classical intuitions consistently fail and the quantum takes over – A world where the subtle dominates.

Suggested Reading

A Letter to a Student

As a young and enthusiastic undergraduate student of physics in the College for Women, Trivandrum, I was interested in reading about the history of physics and the biographies of eminent physicists. I used to collect details about their work and their lives. As an amateur artist, I also collected their photographs, from which I would make sketches. In 1974, I wrote to Professor A A Abrikosov, who was then the Head of the Solid State Theory Department in the Landau Institute, requesting him for some details regarding the Landau and Lifshitz series of monographs, and for a photograph of L D Landau and other well-known Soviet physicists.

A little while later, I was delighted to receive a very gracious letter from Professor Abrikosov answering all my queries. He also enclosed a photograph, and took the trouble to annotate the picture in his letter. I have preserved his letter and the photograph carefully in my album. Looking back after nearly thirty years, I am once again struck by the personal interest he showed and the pains he took to reply in detail to a letter from an undergraduate student from a far-off place. I will always cherish this gesture on the part of one of the greatest physicists of the day.

G Ambika
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Dear Miss Ambika,

I am sorry being so late with my answer. Various duties and a long stay out of Moscow stopped for some time my correspondence. Unfortunately, I can satisfy your wishes only to a rather small extent. The original Landau’s works are in Russian and therefore cannot be very useful to you. But there exists now an English version of his selected works (probably Pergamon Press, England). The publishing house has printed also English translations of all the books of the Landau-Lifshitz series. I don’t think you will have difficulties in getting them in India. Anyway, it is easier to get the English edition here in Russia, since it is sold out in a short time and no additional printing is done.

Now about Landau’s photo. There are many of them, but unfortunately, I have found only one which I send to you.
It represents the Theoretical Department of the Institute for Theoretical Problems. It is the Director of the Institute and Landau until his death was the head of the Theoretical Department. The people are (left to right):

Khalatnikov is now the Director of the Landau Institute and I am there as the head of the Solid State Theory Department. Dzyaloshinski is also with us as the head of the Sector of Magnetism, Lifshitz, Lifshitz, Vainshtein, stay at the Institute for Theoretical Problems. Prozorova also at the photograph is an experimenter. Arkipov is the head of the Theoretical Group at the Institute for High Pressures and Gershtein, the same at the Serpukhov Institute for Elementary Particle Studies. Of course now everybody has become 20 years older.

About the correspondence Please write if you have any questions although I must warn you that I am not extremely accurate with my answers.

Please accept my best wishes.

Yours
A.G. Khrenov