# Think It Over

**This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.**

Soubhik Chakraborty
Marwari College
Bhagalpur University
Bhagalpur 812 007, India.

## A Curious Set of Numbers

**Problem.** Find the largest possible set $P \subset \mathbf{N}$ ('largest' in the set-theoretic sense) with the property that a number is in $P$ if and only if it is a sum of two distinct integers in $P$ *or* a sum of two distinct integers not in $P$?

**Solution.** Such sets exist, trivially, and there are infinitely many of them; any singleton set $P = \{n\}$ with $n \geq 3$ satisfies the given requirement. We want, however, a set-theoretically maximal set. We construct it inductively as follows; we assume at the outset that it exists and construct it; the mode of construction will then make it clear that the set $P$ has the required property, and is maximal.

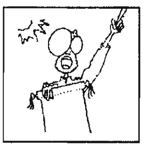
We must clearly have $1 \notin P$ and $2 \notin P$, as 1 and 2 cannot be written as sums of two distinct positive integers. Now we must have $3 \in P$, because $3 = 1 + 2$ (remember that $P$ must be 'maximal'). As $1 \notin P$ and $3 \in P$ we must have $4 \notin P$. Next, we have $5 \in P$ (as $5 = 1 + 4$) and $6 \in P$ (as $6 = 2 + 4$). Following this, we have $7 \notin P$ (as 7 can only be written as $1 + 6$, $2 + 5$ and $3 + 4$, but none of these will do; in each sum, one number is in $P$ and the other number is not in $P$).

Next, $8 \in P$ (for $8 = 1 + 7$), $9 \in P$ (for $9 = 2 + 7$) and $10 \notin P$. After this we find that all the integers 'fall' into $P$; i.e., all $n \geq 11$ lie in $P$. We only need to check that $11(= 1 + 10)$, $12(= 2 + 10)$ and $13(= 5 + 8)$ lie in $P$. After this we can write $n = 3 + (n - 3)$ for any $n \geq 14$.

So the required set is $P = \mathbf{N} \setminus \{1, 2, 4, 7, 10\} = \{3, 5, 6, 8, 9, 11, 12, 13, 14, 15, 16, \ldots\}$.

If we replace the 'two' in the problem statement by 'three' we get another interesting problem; now we want a subset $P_3$ of $\mathbf{N}$ such that a number is in $P_3$ if and only if it is a sum of three distinct integers in $P_3$, *or* a sum of three distinct integers not in $P_3$. Once again, a maximal such set may be found; we get, writing $\overline{P_3}$ for the set $\mathbf{N} \setminus P_3$,

$$\overline{P_3} = \{1, 2, 3, 4, 5, 13, 14, 15\}.$$



# Information and Announcements

## Nobel Prize 2003

**Physics** – *"for pioneering contributions to the theory of superconductors and superfluids"* to

**Alexei A Abrikosov**, Argonne National Laboratory, Argonne, Illinois, USA,
**Vitaly L Ginzburg**, P N Lebedev Physical Institute, Moscow, Russia, and
**Anthony J Leggett**, University of Illinois, Urbana, Illinois, USA.

**Chemistry** – *"for discoveries concerning channels in cell membranes"* to

**PeterAgre**, Johns Hopkins University School of Medicine, Baltimore, USA – *"for the discovery of water channels"* and
**Roderick MacKinnon**, Howard Hughes Medical Institute, The Rockefeller University, New York, USA – *"for structural and mechanistic studies of ion channels"*.

**Physiology or Medicine** – *"for magnetic resonance imaging"* to
**Paul C Lauterbur**, University of Illinois Urbana, IL, USA, and
**Peter Mansfield**, University of Nottingham, School of Physics and Astronomy Nottingham, United Kingdom.