Origami, Modular Packing and the Soma Puzzle

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Can one fit a square peg into a round hole? Never! Can you create high symmetry by assembling modules that lack them? Yes!

After three decades of teaching and research at the Indian Institute of Technology, Kanpur, S Ranganathan has continued these activities, earlier at RRL Trivandrum and now at IICT, Hyderabad, where he is a distinguished scientist. His current fascinations are, molecular biology and mathematical topology.

Consider this. Take any module – molecule or material – having a particular symmetry property, destroy this by assembling the same module in irregular ways and then put them together to create a macroscopic system that enjoys the precise symmetry of the original module! This is the crux of the 'Soma puzzle', discovered in the early part of the last century by the mathematician, Piet Hein. The potential applications of the principles here are enormous. The irregular composites can be used, in storage of information through endless networks, in materials to generate packing designs, in the molecular and structural arena ranging from, combinatorial science, supra-molecular chemistry, packing mechanisms and molecular biology.

Fortunately, to an extent; the feeling for three dimensions (3D) is a born instinct. A baby, three months old, when laid on a table with black checker design, above a floor having precisely the same design, will instinctively keep away from the edge! Pineapples are irregular objects and the way they are packed by unlettered vendors takes your breath away! Having said all this, the finer nuances of 3D have to be acquired and the history of science attests to what the mastering of this can lead to. From Einstein in physics, to Pauling and Woodward in chemistry, Archimedes, Euler, Kepler, Hawkings in mathematics, Michaelangelo, Leonardo DaVinci, Picasso, Monet, Escher and Klee in arts, Ramachandran, Watson and Crick in molecular biology, they indeed form a brilliant galaxy!

Keywords
Modular construction of Soma patterns.

In this article we will take a small step towards enjoying the exciting area of 3D, by amalgamating origami with mathematical principles; we will do this by transforming two dimensional

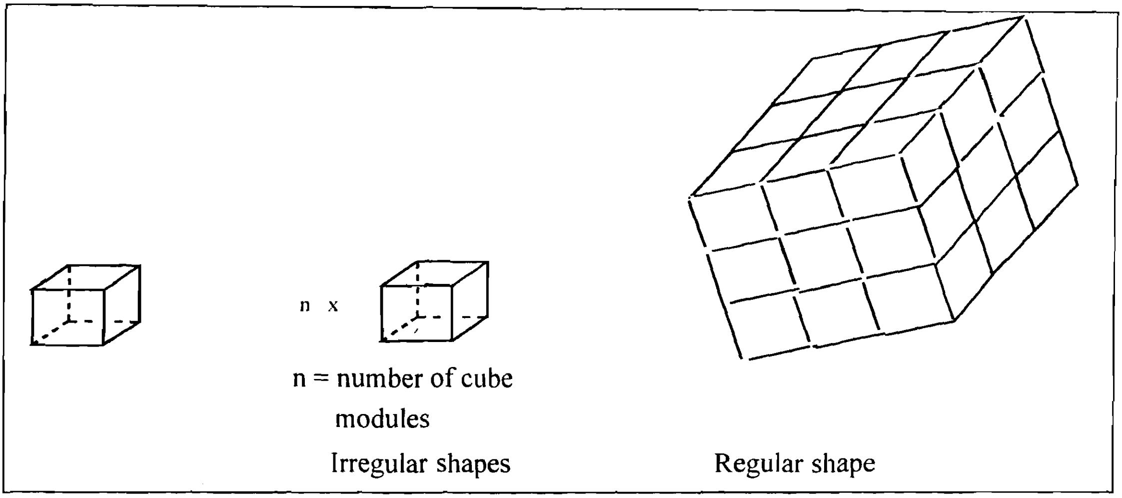


Figure 1.

sheets of paper into irregular objects having a basic cube module and then combining them to form a cube! We will illustrate that the irregular assemblies can be used to create endless 3D objects. The genesis of the Soma puzzle is steeped in high science. The Danish writer, poet and mathematician Piet Hein conceived the Soma cube during a lecture on quantum physics by Werner Heisenberg. Whilst Heisenberg was waxing eloquent on the slicing of 3D space into geometric entities, shuffling them and mixing them up, Hein's supple imagination thought of using a regular cube to illustrate this principle.

Let us start constructing irregular shapes from a regular cube (a basic cube module), with one of the objectives being to create a cube from these irregular shapes (*Figure* 1).

These can be constructed by a planned addition of single modules to the first irregular shape **1**, along the X, Y, Z coordinates (*Figure* 2). Whilst the paths here are crooked, the final mathematics is neat, according to the equation, $27 \times a^3 = (3a)^3$, where a is the edge length of the single module.

The core irregular shape **1** arises by a three-module combination (*Figure* 2), which has a cleft, or corner or bent. Six, and only six, unique options are open for the addition of another unit, such that the original irregularity is increased and not destroyed!

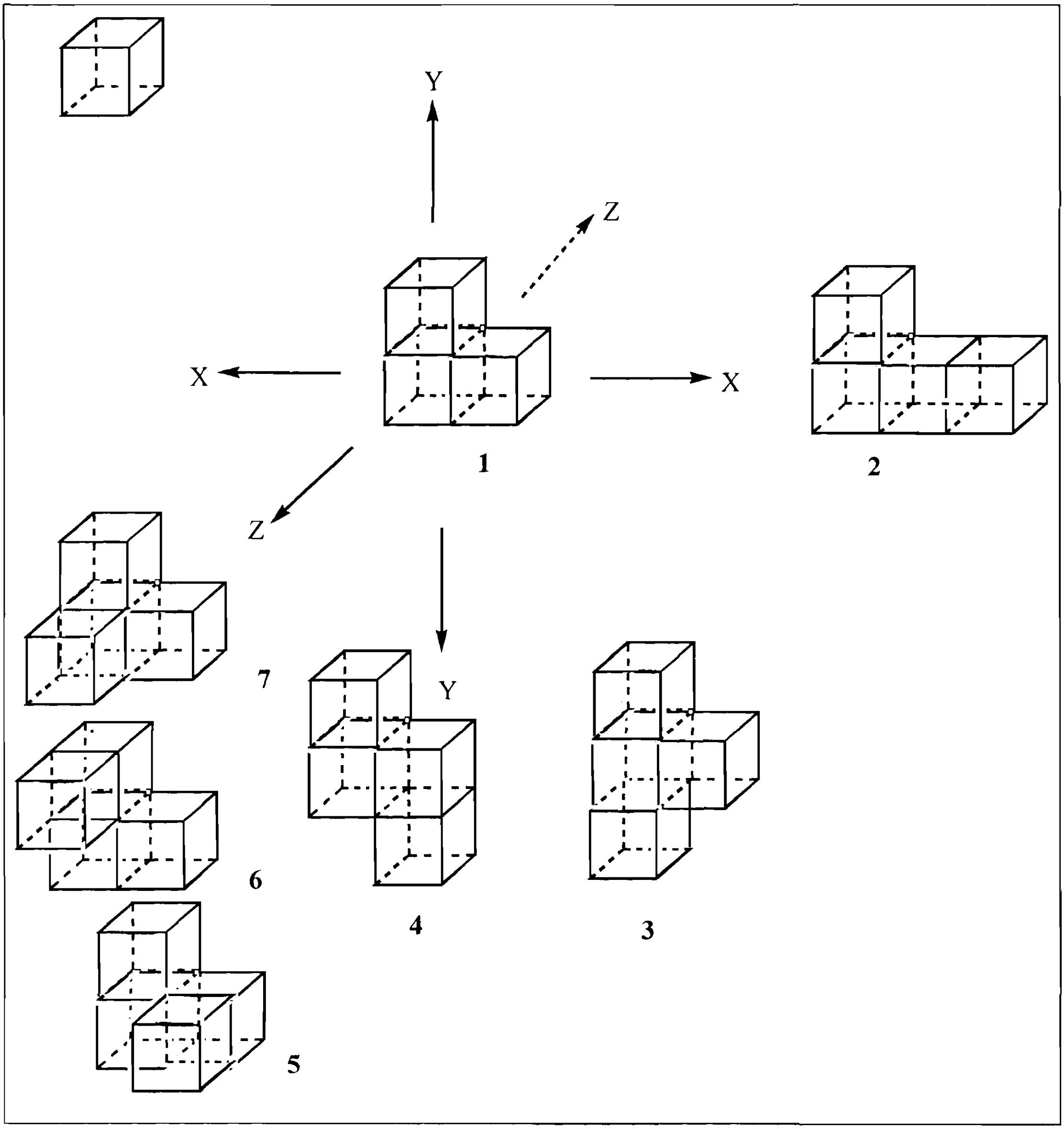


Figure 2.

These are presented in *Figure* 2. To rationalize this, the planar composite **1**, is placed at the origin of a 3D coordinate system. The construction of irregular objects by addition of a single module to **1** is shown in *Figure* 2. This procedure leads to **2-7**, each having four modules. Note that **5** and **6** are mirror images and **7** uniquely asymmetric. None of the six composites is similar to the other and all lack the symmetry properties of the cube

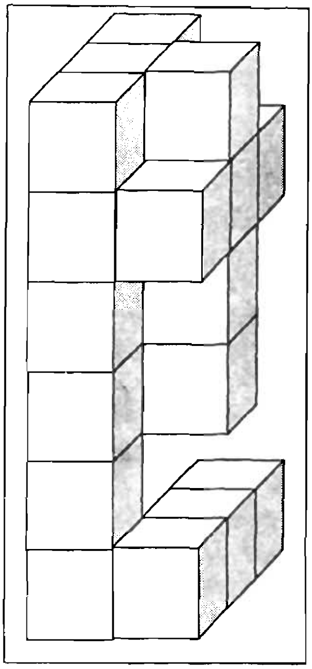


Figure 3.

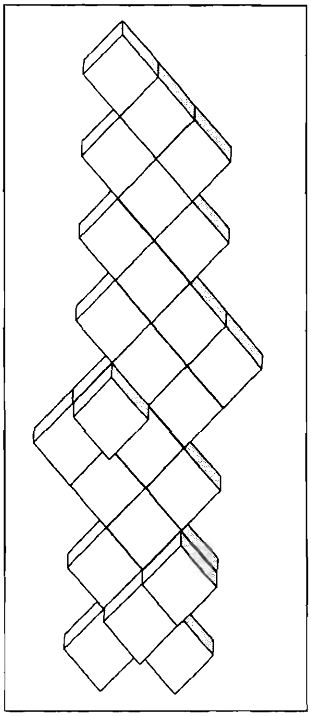


Figure 4.

module, though they may possess other forms of symmetry. Composites 1-7 are the Soma pieces and infinite forms can be created from these, much as that from a tetrahedral carbon. The real challenge is to arrange the composites to reach a predetermined objective, like in retro analysis protocol. This will involve a mental slicing of the end object, much as Piet Hein did with the cube! Indeed, more than 230 essentially different solutions, not counting rotations and reflections, are available for assembling **1-7** into a cube (*Figure* 1). With a good 3D sense, this can be done in minutes; if not so endowed, several hours! Look at the aesthetically pleasing cross motif in *Figure* 3. Even with a good 3D sense this may take some time. The real fun with Soma pieces is in this kind of exercise, which can really sharpen the 3D senses, a skill worth acquiring. One can also create abstract forms (*Figure* 4). This kind of creative activity can be introduced in classrooms.. The handicap is the difficulty in making the seven Soma pieces without much effort. It is this problem that is remedied here, by an origami approach.

Construction of Soma Pieces

First let us make the six cube modules to be added to **1** (*Figure* 2)

Start with a thick marble board or thick hand made sheets and draw on it a rectangle of size 4 inches × 9 inches (*Figure* 5). Cut this into three 3" × 4" rectangles. Each of these can give two 'T' shaped modules that can be folded into a perfect 1"× 1" × 1" cube (*Figures* 1 and 5). Crease all the lines in the 'T' module and make the hollow cube by bringing edges ff' to gg' together and taping them. Folding and sealing the flaps (abcg and a'b'c'g') will generate the cube module.

Construct **1** is the core for building the Soma pieces **2-7**. We need seven pieces of this and the procedure to build **1** is shown in *Figure* 6.

Let us start with a 4" × 4" grid and cut it into the 'T' shirt motif and two constructs which are 3" × 1". The big piece can be folded into a cube duplex, and the cube precursor (the two 3" × 1"

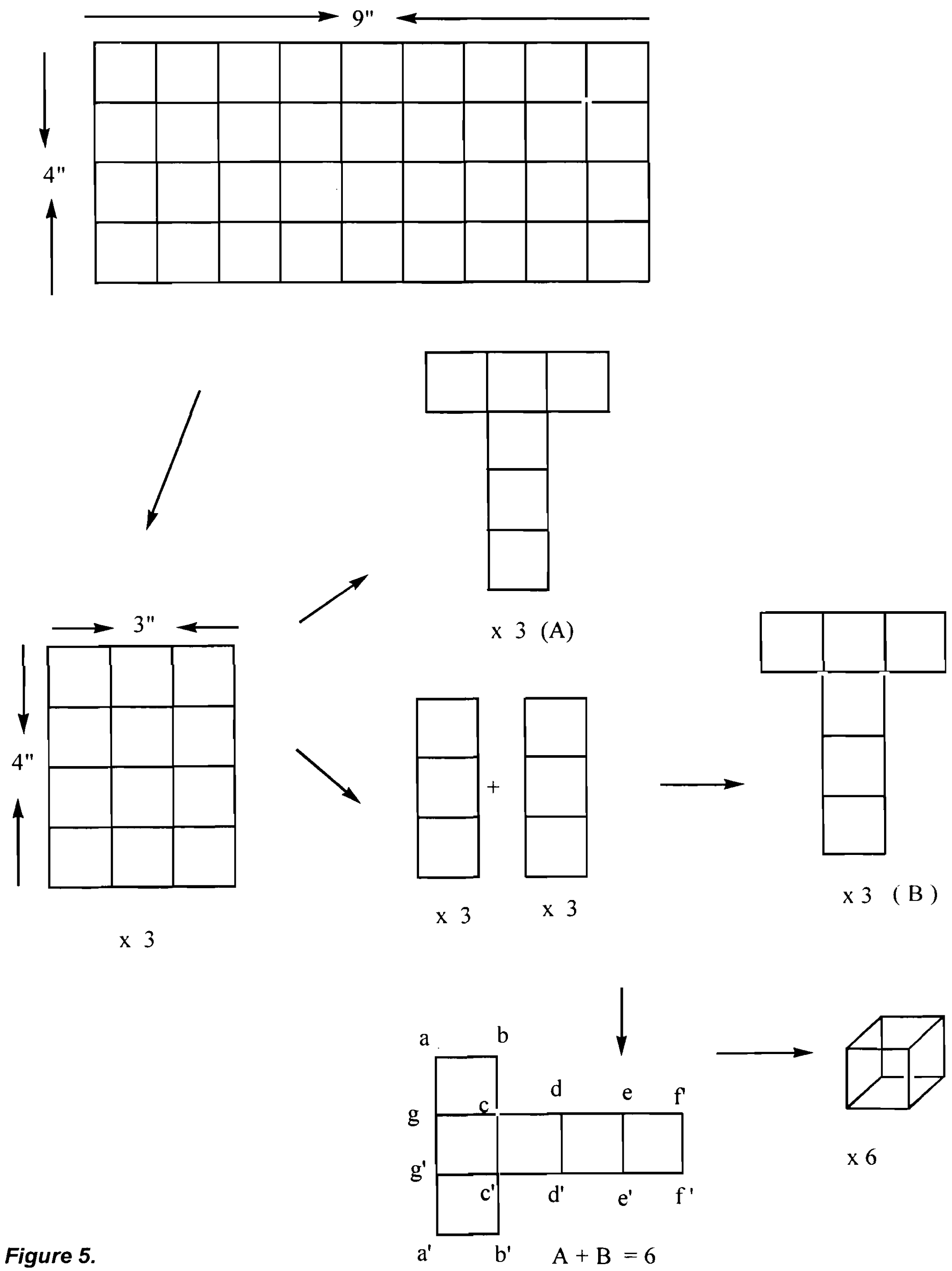


Figure 5.

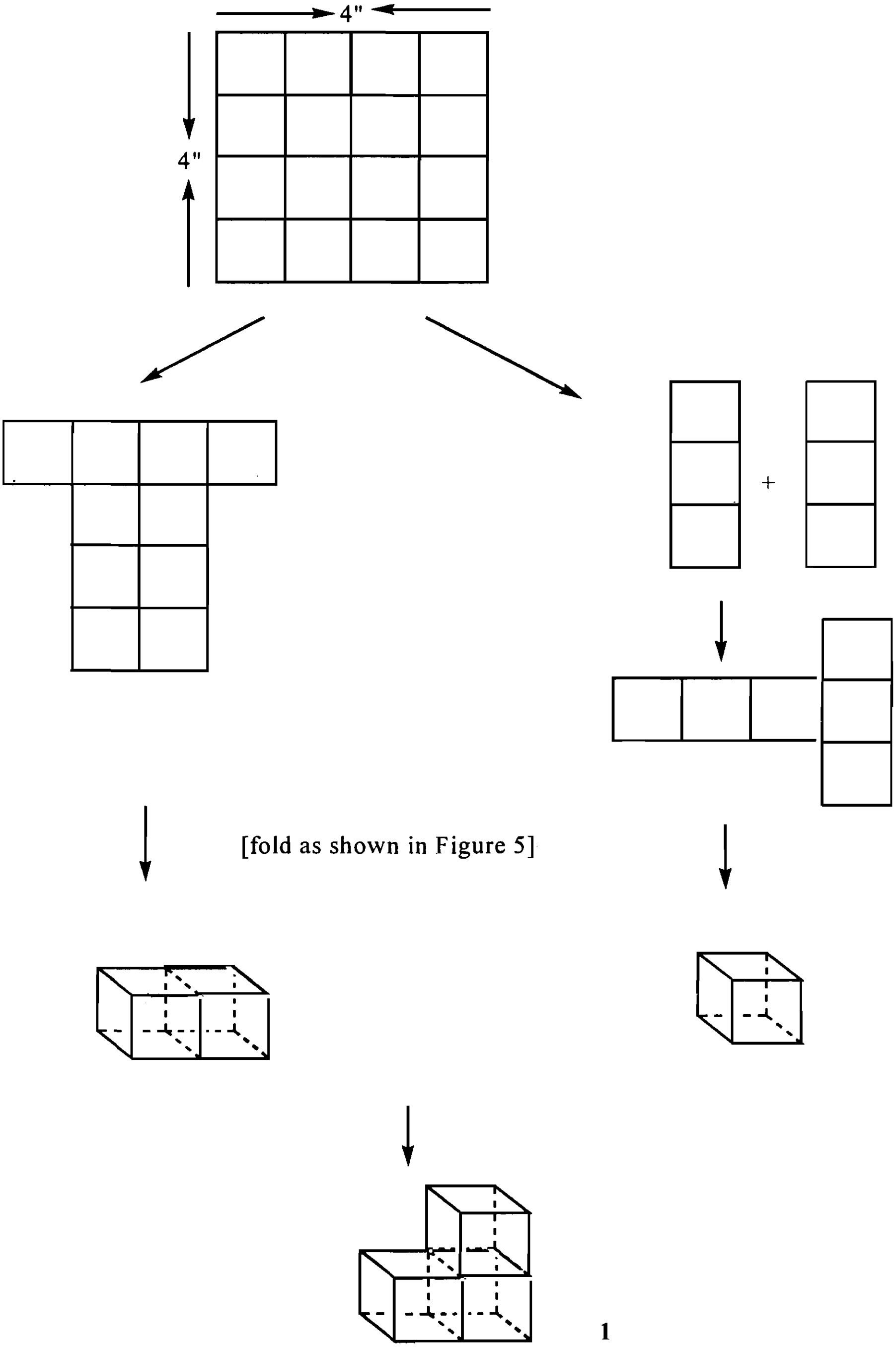


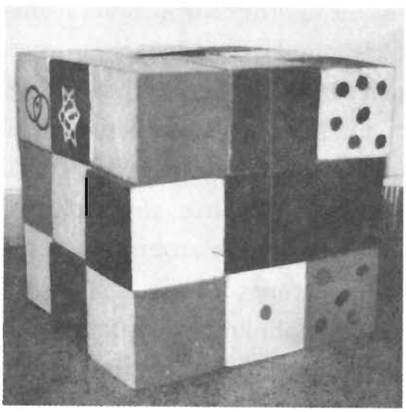
Figure 6.

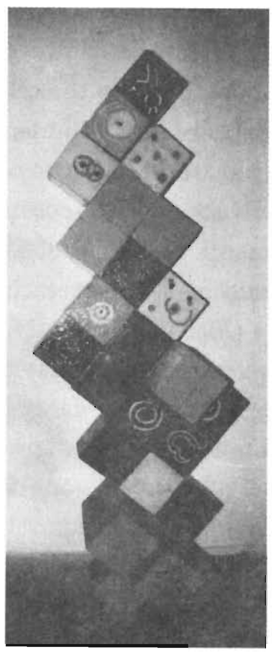
constructs) is transformed into the cube. Glue the single cube over the duplex as shown to generate **1**. Repeat the operation six more times. We will then have seven of Construct **1**.

To construct Soma pieces **2-7**, join the single cube to **1** following precisely the pathways shown in *Figure* 2. In all we will have seven Soma pieces.

We urge you to make these models, experiment with various designs and let us know of any exciting findings. In case you are not able to make the cross, let us know; we have the solution!

If you become a Soma enthusiast you can go to the next step of making a $4 \times 4 \times 4$ composite with 64 cubes. I have made it from 16 pieces and after two days remade the cube. Once disassembled, I have not been able to make it again. So, the complexity of the Soma problem can be enhanced, the limit being your intuition and patience. If you are a mathematician, I would like you to derive a general theorem about the Soma cubes and let me know.





Patterns from painted Soma modules made from 9″× 9″ × 9″ wooden cubes, by the author.

Suggested Reading

[1] **Subramania Ranganathan, Molecular origami: Modular construction of platonic solids as models for reversible assemblies, *Resonance*, Vol.5, No.9, pp.83-91, 2000.**

[2] **Martin Gardner, *More Mathematical Puzzles and Diversions*, Penguin, 1961.**

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