Think It Over

This section of Resonance presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', Resonance, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

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Problem posed by S N Maitra in February 2003 issue:

A particle 1 is projected from a point of position vector \( \vec{r}_0 \) with velocity \( \vec{u}_0 \). After a lapse of time \( T \), another projectile, say, particle 2 is projected from a point of position vector \( \vec{R}_0 \) with velocity \( \vec{v}_0 \) to pursue the former projectile. Find the condition required to be fulfilled for a collision between the two projectiles. Also, find where and when they will collide. Neglect the air-resistance.

Solution by the Proposer:

Let \( t \) be the time taken by the projectile 2 to collide with projectile 1. Let \( \vec{R} \) be the position vector of the point reached by projectiles 1 and 2 in times \( (T + t) \) and \( t \). We begin with the equation \( \frac{d\vec{r}}{dt} = -\vec{g} \) and integrate twice applying the limited conditions to get

\[
\vec{R} = \vec{r}_0 + \vec{u}_0(T + t) + \frac{1}{2} \vec{g}(T + t)^2 = \\
\vec{R}_0 + \vec{u}_0t + \frac{1}{2} \vec{g}t^2 
\]

or, \( (\vec{r}_0 - \vec{R}_0) + (\vec{u}_0 - \vec{V}_0)t + \vec{u}_0T + \frac{1}{2} \vec{g}(T + 2t)T = 0. \)
Taking cross product of (2) with $\vec{g}$ and simplifying,

$$(\vec{r_0} - \vec{R_0}) \times \vec{g} + (\vec{u_0} - \vec{V_0}) \times \vec{g} + \vec{r_0} \times \vec{g} = 0$$

or,

$$(\vec{u_0} - \vec{V_0}) \times \vec{g} t = -(\vec{r_0} - \vec{R_0} + \vec{u_0} T) \times \vec{g} t$$

Taking modulus of both sides

$$or, t = \frac{|(\vec{r_0} - \vec{R_0} + \vec{u_0} T) \times \vec{g}|}{|\vec{u_0} - \vec{V_0}| \times \vec{g}|} \quad (3)$$

Employing (3) and (1) we get

$$\vec{R} = \vec{R_0} + \left\{ \vec{V_0} + \frac{1}{2} \vec{g} \frac{|(\vec{r_0} - \vec{R_0} + \vec{u_0} T) \times \vec{g}|}{|\vec{u_0} - \vec{V_0} \times \vec{g}|} \right\}$$

$$\frac{|(\vec{r_0} - \vec{R_0} + \vec{u_0} T) \times \vec{g}|}{|\vec{u_0} - \vec{V_0} \times \vec{g}|} \quad (4)$$

and thereafter taking cross product of (2) with $(\vec{u_0} - \vec{V_0})$ and scalar product of $\vec{g}$ with the resulting vector we get

$$\{(\vec{r_0} - \vec{R_0}) \times (\vec{u_0} - \vec{V_0})\} \vec{g} - (\vec{u_0} \times \vec{V_0}) \vec{g} T = 0$$

because $[\vec{g} \times (\vec{r_0} - \vec{V_0})] \vec{g} = 0, \vec{V_0} \times \vec{V_0} = 0, \vec{g} \times \vec{g} = 0$.

Hence

$$T = \frac{[(\vec{r_0} - \vec{R_0}), (\vec{u_0} - \vec{V_0}), \vec{g}]}{[\vec{u_0}, \vec{V_0}, \vec{g}]} \quad (5)$$

which gives the required condition to have a collision between the projectiles 1 and 2.