Quantization of the Radiation Field

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I briefly review the seminal 1927 paper of Dirac which emphasized the need for the quantization of the electromagnetic field and then showed how to carry it out. After mentioning few historical developments, I then point out some of the outstanding successes of quantum electrodynamics – the kind of agreement which exists between theory and experiment is unprecedented in the history of science.

Light has wave-like properties in interference and diffraction experiments and particle-like properties when it is emitted or absorbed by atoms. Dirac by quantizing electromagnetic field, was able to bring about the first synthesis of these two dual aspects of radiation. In 1927 Dirac wrote (one of his many) seminal papers entitled 'The Quantum Theory of the Emission and Absorption of Radiation' [1] in which he argued why electromagnetic field also needs to be quantized and then showed how this can be done. This led to the formulation of quantum electrodynamics (QED) in which there is an unprecedented agreement between theory and experiment which has not been bettered in any branch of science. Not surprisingly, QED has become the prototype for a comprehensive theory of strong and electro-weak interactions as well as for theories beyond like grand unified theories. Further, the same QED also provides foundation for the quantum mechanics of atoms and molecules as well as condensed matter physics, not to mention their numerous applications in chemistry, biology, etc.

Keywords
Quantum electrodynamics, Maxwell's equations, radiation field, quantization, Lamb shift.

In this article, I shall first give some arguments which led Dirac to think about quantizing electromagnetic radiation and then briefly discuss his 1927 paper [1]. Further, I indicate some of the future developments which led to what is known today as QED. I shall also point out some of its predictions and compare them with experiments.

By 1926 the basic formulation of non-relativistic quantum mechanics by Schrödinger, Heisenberg and Dirac was already well established and the concept of wave-particle duality for matter was well accepted. Dirac was intrigued by the fact that while dual nature of matter was recognized which eventually led to non-relativistic quantum mechanics, nothing had been done about the dual nature of electromagnetic waves. In fact, historically, it is in the electromagnetic theory that departure from classical theory became necessary. In particular, in the problem of radiation in thermal equilibrium with a black body, the classical theory leads to the well-known ultra-violet difficulty. To avoid this difficulty, Planck (and later Einstein in photoelectric effect) assumed that the energy of a monochromatic wave with frequency ν could only assume values which are integral multiples of a certain unit, i.e. $E = nh\nu$, where n is an integer and h is the Planck constant. Further, it is this quantization of monochromatic waves together with the law of conservation of energy which led to Bohr's well-known frequency condition for atomic transitions. Thus Dirac felt that this dual nature of light necessitates the quantization of light waves for its description.

As Dirac started thinking more about this problem, he was struck by the profound difference between matter and radiation (i.e. electromagnetic waves). In fact this is what Dirac said in his 1927 paper: "In the new quantum theory, one can treat mathematically the problem of any dynamical system composed of a number of particles with instantaneous forces acting between them, provided it is describable by a Hamiltonian function, and one can interpret the mathematics physically by a quite definite general method. On the other hand, hardly any-

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thing has been done up to the present on quantum electrodynamics. The questions of the correct treatment of a system in which the forces are propagated with velocity of light instead of instantaneously, of the production of an electromagnetic field by a moving electron, and of the reaction of this field on the electron have not yet been touched."

Thus Dirac correctly recognized the fact that as soon as one wants to quantize electromagnetic field, one will have a relativistic quantum field theory. He also realized that unlike any non-relativistic theory, here there is a possibility of particle production. Thirdly, since electromagnetic field can be viewed as discrete infinite number of particles, quantizing electromagnetic field amounts to quantizing a system with discrete infinity of light quanta (i.e. harmonic oscillators).

At classical level, the electromagnetic fields in vacuum satisfy the Maxwell equations

$$\nabla\ \ \mathbf{E}=\rho,\quad \nabla\ \ \mathbf{B}=0,\quad \nabla\times\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=0,$$
$$\nabla\times\mathbf{B}-\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}=\mathbf{j} \tag{1}$$

which are invariant under Lorentz transformations. Now instead of two vectors **E** and **B** satisfying four equations, one can reduce them to simpler equations between a vector and a scalar function (called potentials) defined by

$$\mathbf{B}=\nabla\times\mathbf{A},\quad \mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}-\nabla\phi \tag{2}$$

Note however that the vector and the scalar potentials **A** and ϕ are not completely determined by the magnetic and the electric fields **B** and **E**. The different possible choices one can make for **A** and ϕ, leaving **E** and **B** unchanged, are called gauges, and the invariance of **E** and **B** under these transformations is called gauge invariance. The concept of gauge invariance has played a profound role in QED as well as in the development of modern theories of basic interactions in nature.

Dirac considered the problem of interaction of an atom with radiation and to simplify the issue, he considered the problem in the Coulomb gauge in which there is instantaneous interaction between charges while the electromagnetic field is purely transverse. By including the interaction of atoms with radiation as given by classical theory, he was able to build a fairly satisfactory theory of emission and absorption of radiation by atoms and obtained correct values for the Einstein A and B coefficients.

This seminal work of Dirac opened a flood of activities. Jordan and Wigner [2] considered the quantization of electron (and positron) fields. It is worth pointing out here that unlike photons which satisfy Bose–Einstein statistics, electrons and positrons satisfy Fermi–Dirac statistics and hence instead of canonical commutation relations, they in fact satisfy anti-commutation relations. Subsequently, Heisenberg and Pauli [3] considered the full QED, i.e. electrons, photons and their interactions. One of the drawbacks of Dirac's quantization of electromagnetic field was that his treatment was not Lorentz covariant. Fermi [4] rectified that defect by including longitudinal as well as scalar photons. The profound analysis by Landau and Peierls and specially by Bohr and Rosenfeld made it clear that at least the QED of vacuum must be correct.

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I might add here that initially grave difficulties appeared in the full theory of QED; there was the old problem of diverging self-energy of a point charged particle, now aggravated by an additional transverse self-energy. It turned out that the answer to almost every physical question was a divergent integral. Subsequently, it was realized that all infinities that appear in the theory originate from a few diverging quantities which are unobserv-

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able; namely a contribution to the mass and the charge of a particle due to its interaction with the electromagnetic field. It was found that if one argued that the bare mass and charge of the particle are also divergent such that the physically observed charge and mass of the particle are finite, then there is no obstacle in the way of giving an unambiguous finite answer to every legitimate physical question and the answer is independent of the gauge in which such a calculation is carried out. Further, such a calculation can be systematically carried out order by order in perturbation theory

This is how the final picture of QED emerged in the hands of Feynman, Tomonaga and Schwinger who shared the 1965 Nobel Prize for their work. The outstanding success of the theory is in quantitative explanation of the interaction of electrons with photons [5]. I might add here that electron, being a lepton, does not experience any strong force and electromagnetic interaction is the dominant force experienced by it and so electron-photon interaction is the right place to apply QED. One of the outstanding successes of the theory is in its prediction of anomalous magnetic moment of the electron. According to QED, electron is a point particle which is surrounded by the photon cloud. Normally a charged, spin $\hbar/2$, point particle possesses normal magnetic moment of one Bohr magneton. However, quantum electrodynamics predicted that due to the electron-photon interaction, this magnetic moment and hence Dirac's g-factor gets a tiny correction, i.e. $g = 2(1 + a_e)$. It was found that the theoretical prediction for this tiny value agrees with experiment to seven decimal places, an agreement unprecedented in the history of science! In particular, the latest values are

$$a_e^{\mathrm{th}} = 1159652133(29) \times 10^{-12} ,$$

$$a_e^{\mathrm{exp}} = 1159652188.4(4.3) \times 10^{-12} \qquad (3)$$

where the numbers in parentheses indicate the error.

The other outstanding success of QED is in the quantitative explanation of the radiative displacement of the atomic energy levels. For example, whereas the Dirac theory predicts that the $2^2S_{1/2}$ and $2^2P_{1/2}$ levels of hydrogen atom are degenerate, QED says that due to the interaction of the zero-point field fluctuations with electron of hydrogen atom, there will be a tiny splitting between the two levels, popularly known as Lamb shift. The latest theoretical and experimental values for Lamb-shift $\Delta E \equiv E_{2^2S_{1/2}} - E_{2^2P_{1/2}}$ are

$$(\Delta E)_{\text{th}} = 1057.866(11) \text{ MHz},$$

$$(\Delta E)_{\text{exp}} = 1057.845(9) \text{ MHz}. \qquad (4)$$

I might add that there are several other consequences of the zero-point fluctuations of the electromagnetic field which have been verified experimentally.

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More than these successes, what I regard as the outstanding success of QED is that it created a *new successful gauge dogma* in elementary particle physics. Inspired by the outstanding successes of QED, gauge theory of electro-weak interactions as well as quantum chromodynamics (strong interactions) has been written down and has proved immensely successful. Recent attempts of constructing grand unified theories are also based on this gauge principle. It is worth adding here that even the General Theory of Relativity by Einstein is a gauge theory being based on the invariance under general coordinate transformations.

There are numerous other successes of QED. Few such examples are, radiative transitions between discrete atomic levels, photoelectric effect, natural width of spectral lines, etc. [6]. The other success story is in Cavity QED [7] where conducting dielectric material is intentionally placed nearby, thereby perturbing the electromagnetic field in such a way that the radiative properties of an atom are substantially modified. By a suitable choice of the cavity geometry, one is able to change the spectrum and spatial distribution of electromagnetic quantum noise in a well defined way and hence to control things like Lamb shift.

The other place where QED has made a major impact is in the area of quantum optics [8]. Here a more appropriate basis is not the Fock states (i.e. the number states) but coherent states which have indefinite number of photons but a more precisely defined phase. It may be noted that for the Fock states the phase is completely random. The coherent states are meaningful states in that the field generated by a highly stabilized laser operating well above threshold is a coherent state.

Suggested Reading

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"... it is more important to have beauty in one's equations than to have them fit experiment. ... . It seems that if one is working from the point of view of getting beauty in one's equations, and if one has really a sound insight, one is on a sure line of progress".

– Dirac