Structural Origami

A Geodesic Dome from Five Postcards

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We have taken the modular construction of surfaces to the complexity of an eicosahedron, which describes a surface crafted from congruence of five equilateral triangles resulting in the inscription of a pentagon motif. Taking any pairs of opposite poles, each pole will have a pentagon, with the equatorial girdle harboring ten equilateral triangles. The surface of eicosahedron, a platonic solid with twenty equilateral triangles, can be viewed as a layering of interdigitating regular pentagons.

Compare this in complexity with a geodesic dome, a surface as close to sphere as you can get. Here one perceives the congruence of twelve regular pentagons, but each constituted by 10 triangles, which are irregular ($a \neq b \neq c$), with the neighboring module aligned in a mirror image configuration!

The word geodesy has esoteric origins! This branch of mathematics developed as a result of determining the properties of a spherical object like Earth, in terms of integration of largest number of plane surfaces. The two dimensional projection of the three dimensional Earth is done using such data. From mathematics, geodesic surfaces and the study of their properties have advanced to navigation, aerospace, electronic networks, molecular clusters and even to covered areas, with no support, termed now as geodesic domes.

We will construct a geodesic dome, from five postcards! (*Figure* 1)!

Keywords
Origami, construction, geodesicdomes, eicosahedron.

A postcard, that costs 50 paise, is approximately 13.5×9 cm in size. This has to be trimmed precisely to size 13.5×9, where the ratio is 3:2. Each postcard will provide 4 modules harboring six right angled triangles with unequal sides, arranged contigu-

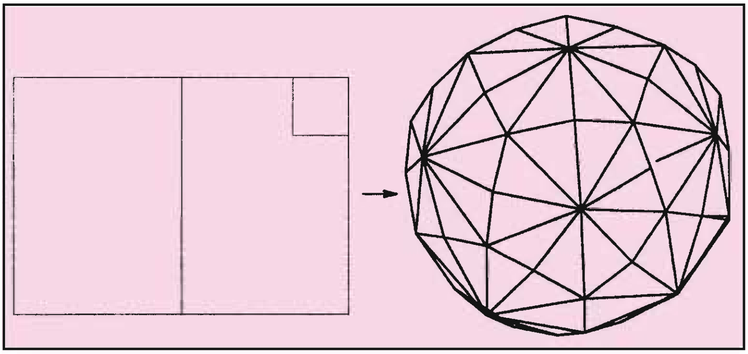

Figure 1.

ously in a mirror image fashion with the non-hypotenuse sides in the precise ratio of 3:2. The procedure is illustrated in *Figure* 2.

Use a black marker pen and the front side of the postcard (1). Draw the two diagonals, A→E′, A′→E to create 4 triangles with mid-point X. Now divide the card into 16 rectangles by equally dividing along both the axes. Note that each rectangle will be of the size, 3 .4 cm × 2.25 cm and their ratio would be ~3:2 (2). Draw diagonals, G→ C′, C′→ G′, G′→ C and C→ G to complete the first motif (3). Cut out the four triangles, AaG, GbA′, E′dG′ and G′CE (4). Using a sharp scissors, neatly cut out the 12 triangles numbered 1-12 (5). Notice that each of these is duplex of two triangles which are disposed in a mirror image fashion and whose non-hypotenuse sides are in the ratio 3:2.

The remaining two angles of the triangle element can be easily computed as shown in (6). These triangles, a total of 120 needed for the geodesic dome, come from 60 duplexes, each card giving rise to 12.

Arrange 3 duplexes in a contiguous mirror image pattern (7). Best results are obtained when the duplexes are joined from the *inside* using strips from '3M Scotch magic tape'. The real skill comes in the joining of these duplexes to modules, modules to pentagons and pentagons to the dome. Tape each duplex to the neighbor from inside, with perfect alignment. Check each operation to ensure that the pasting has lead to a perfect mirror

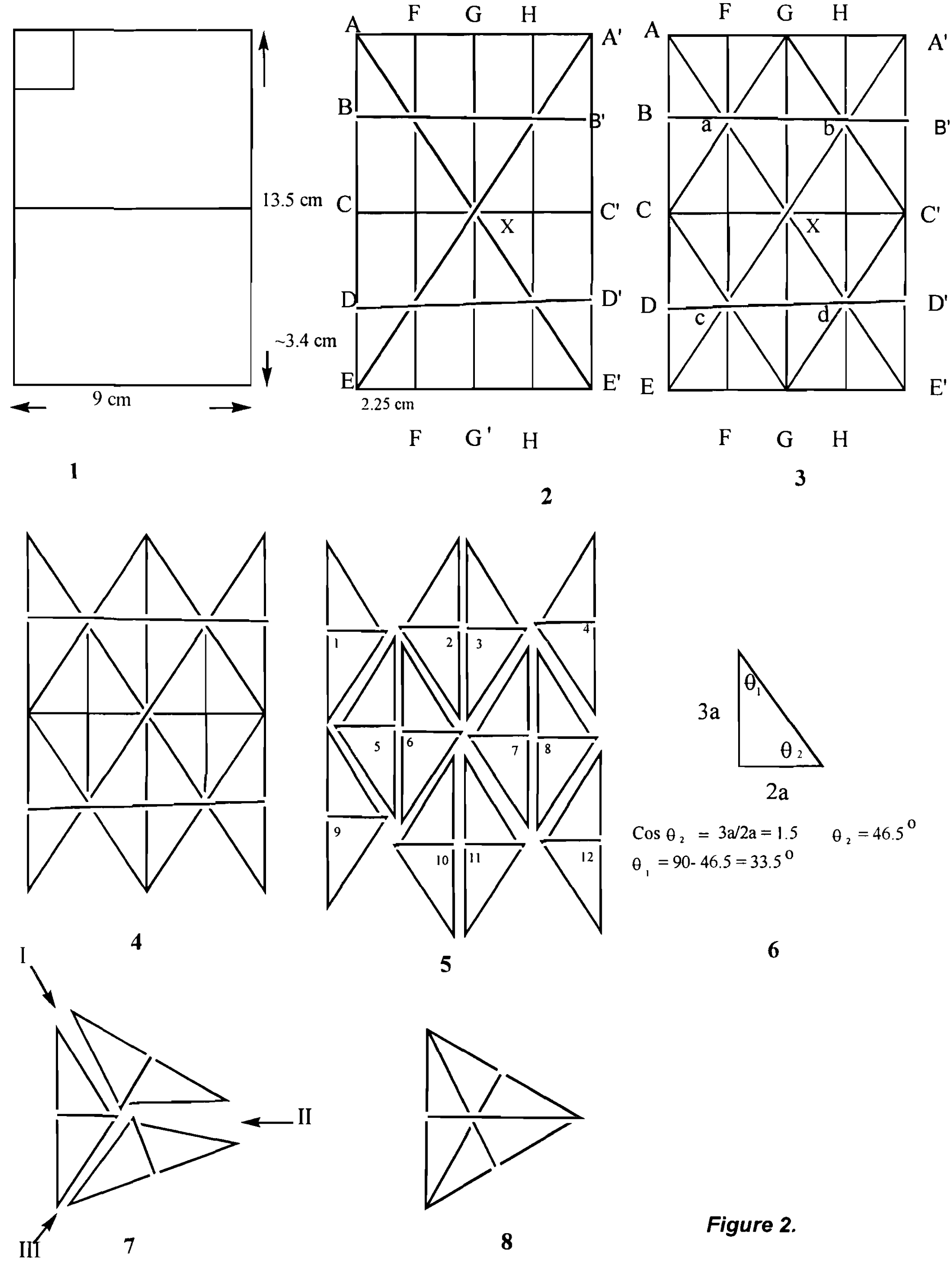
13.5 cm
~3.4 cm
9 cm
1
A F G H A'
B B'
C X C'
D D'
E E'
2.25 cm
F G' H
2
A F G H A'
B a b B'
C X C'
D c d D'
E E'
F G H
3
4
1 2 3 4 5 6 7 8 9 10 11 12
5
3a
θ1
θ2
2a
Cos θ2 = 3a/2a = 1.5 θ2 = 46.5°
θ1 = 90- 46.5 = 33.5°
6
I
II
III
7
8

Figure 2.

image overlap, by folding along the seam. The joining to the third duplex will, as shown in **7**, leave a gap of $180 - (5 \times 33.5) = 12.5^{\circ}$. This is the elegant part of the design, since when the last pair of edges are pasted the center would be lifted up, thus providing the necessary curvature. A planar projection of the module would look like **8** (*Figure* 2).

The basic pentagon unit of the geodesic dome (*Figure* 3, 10) would result from joining of five modules as shown in *Figure* 3, **9** Note that of the 30 triangles in the 5 modules, ten are used for making the pentagon and 20 to provide attachment for other modules, as shown by arrows in **10**. In both **9** and **10** the vertices of the module are marked by dark circles, which, as could be seen, align to form a pentagon.

The remaining 15 modules are to be pasted individually .The procedure for adding a second pentagon is shown in **11** (*Figure* 3). Two of the modules are perfectly aligned with the right side arrows of **10**, to generate bonds, 1→ 2 and 8→ 7 of the daughter.

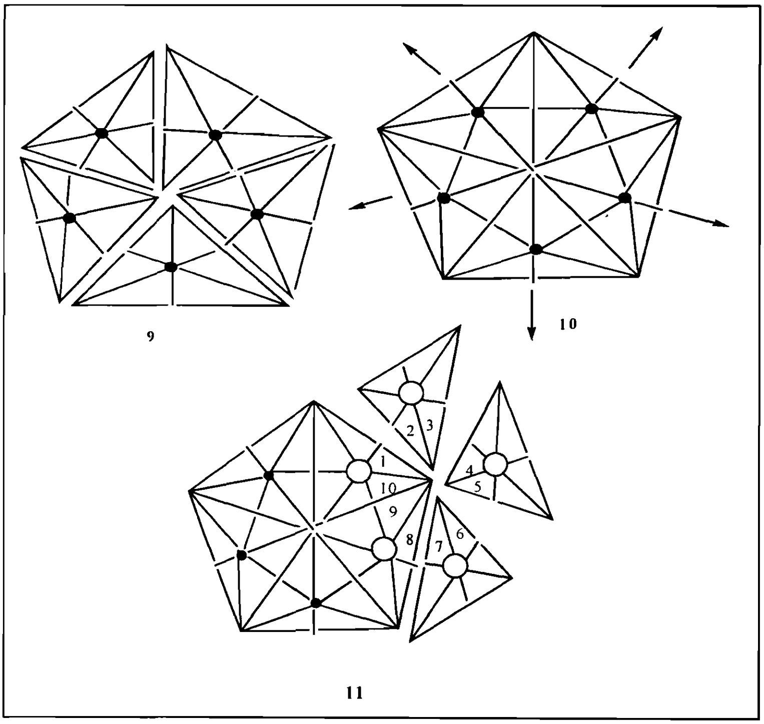

Figure 3.

Figure 4.

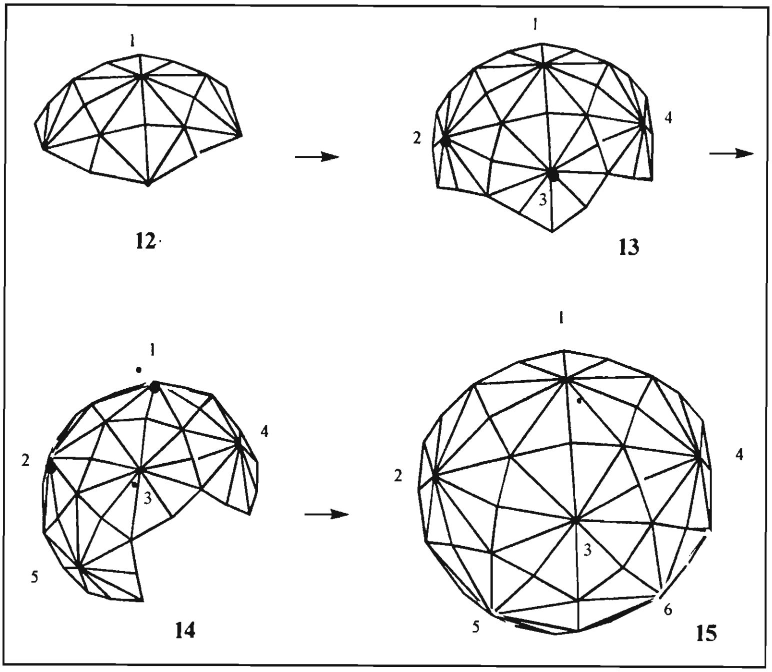


Figure 5.

The three modules needed here are placed in a staggered arrangement. The vertices of the daughter pentagon are indicated by open circles and the 10 triangles that are involved are marked 1→ 10 (*Figure* 3). From **11**, it is clear that the vertices of the 20 modules develop to that of the pentagons. Each module has 3 corners, 1 vertex and three open lines, which would be half the length of a pentagon arm. Based on these principles, modules can be added as shown in **12**→ **15** (*Figure* 4). Model **15** gives a good idea of the modular assembly and the principles involved. If all goes well, you will have, from five post cards, the dome, *Figure* 5.

Suggested Reading

[1] Structural Origami I: Let us make bucky balls, Structural Origami II: Let us make DNA double helix, Structural Origami III: Let us make platonic solids, available, free of cost, at http://crsi.org.in.education.html, or, http:www.orgchem.iisc.ernet.in, and click on 'Chemical Links'.

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