Twin Primes and the Pentium Chip

It is an old maxim of mine that when you have excluded the impossible, whatever remains, however improbable, must be the truth.

– Sherlock Holmes

If p is a prime such that $p+2$ is also a prime then $p, p+2$ are known as *twin primes.* One of the outstanding unsolved problems in number theory is to prove (or disprove) that there are infinitely many twin primes. Euler had proved the infinitude of primes by showing that the series of reciprocals of primes diverges (see *Resonance*, Vol.1(3), pp.78-95, 1996). Guided by this some mathematicians considered the series of reciprocals of twin primes. If this series had been divergent then we could have concluded that there are infinitely many twin primes. But to make matters interesting, in 1919, V Brun proved that it converged to a value that has been calculated to be approximately 1.90216.

So the series of reciprocals of twin primes is of interest. Thomas Nicely, a number theorist, was compiling and extending the list of twin primes and computing the sum of their reciprocals using computers; this sort of exercise is referred to as *number crunching.* In 1994 when he was checking his calculations he discovered that there were errors:

I encountered erroneous results which were related to this bug as long ago as June, 1994, but it was not until 19 October 1994 that I felt I had eliminated all other likely sources of error (software logic, compiler, chipset, etc.). . . . .

Through trial and error and finally a binary search, the discrepancy was isolated to the pair of twin primes 824633702441 and 824633702443, which were producing incorrect floating point reciprocals (the ultra-precision reciprocals were also in error, by a lesser amount, evidently due to a minor dependency on floating point arithmetic in Lenstra's original integer arithmetic code).

Finally the source of the error was traced to the division algorithm implemented on the Pentium chip. The bug relates to operations that convert floating point numbers into integer numbers. Intel withdrew the defective chips from the market and re-released corrected pentiums. This instance should be enough to convince sceptics that number crunching has its uses! Apparently, the Pentium III family has a flaw that slows down the boot process in a small number of chips! I suppose 'eternal vigilance is the price of computing power!'

(In a different context, it seems, a launch failure of the Ariane 5 rocket, which happened less than a minute into the launch, was traced to behavior around an overflow condition in one of the softwares used in it! One of the computers on board had a floating point to integer conversion that overflowed, but because the overflow was not handled by the software the computer did a dump of its memory. Unfortunately, this memory dump was interpreted by the rocket as instructions to its rocket nozzles. Apparently, even a failure of an ISRO rocket was traced to one such programming error.)

Moral: If you are interested in number crunching just go ahead without worrying about its utility. The world may be grateful to you some day!

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