Think It Over

This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

A Maze Problem

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There are many stories about people getting lost inside complicated mazes. One way to avoid getting lost is to always touch the left wall (or always touch the right wall) as you grope your way. Then you are sure to find your way back to the entrance. This is obvious since you will always move in a loop which comes back to the starting point. But the trouble is that this will not ensure that you have visited the *entire* maze, as in *Figure* 1.

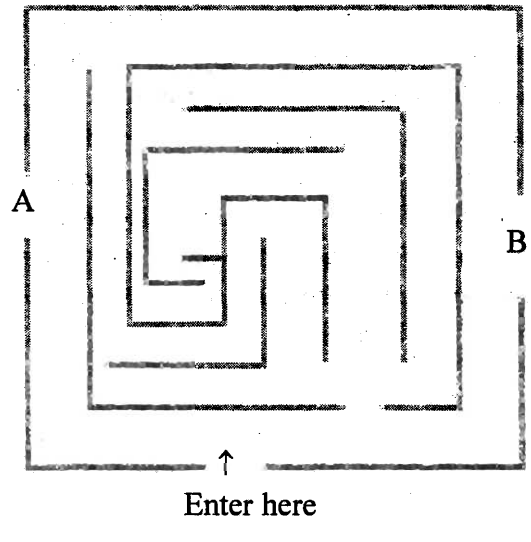

Figure 1. If you always follow the lefthand side wall, you will come out through exit A, and never get to see the inside of the maze. Similarly, following the right-hand side wall will send you out through B.

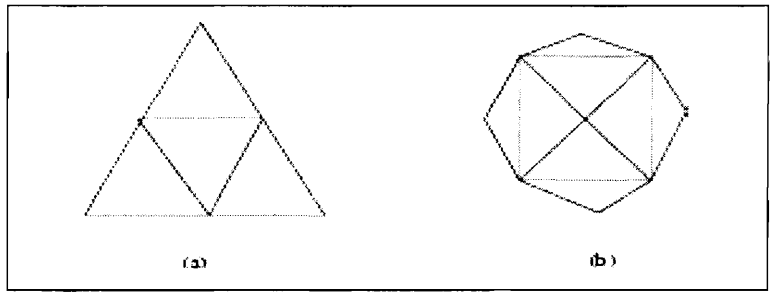

Figure 2. Try to draw these without lifting your pen and without going over any line twice.

To visit the entire maze some brave souls in storybooks equip themselves with lots of stone pieces, and go on marking their route with them, so that they can retrace their steps. Now assume that you have a huge supply of stones with you when you enter the maze. The stones have no particular shape, and hence cannot be used to denote directions. Thus if you come to a stone that you dropped earlier, it will only tell you that you had been to that place earlier, but not the direction along which you were walking during that first visit. How will you use your stones? The answer "Drop stones everywhere you go" won't work, since you cannot retrace your step through a crossing.

Here's a hint. Can you draw the diagram, *Figure* 2a without lifting your pen off the paper and without going over any line more than once, and finally come back to the starting point? What about *Figure* 2b? Any such figure where only an even number of lines meet at each point can be drawn in this way. No other figures can be drawn thus. If, however, I ask you to go over each line *exactly twice*, it becomes *very* easy, doesn't it? Once you figure this point out, try to device a way by which you will traverse every lane in the maze exactly twice. Do not use the stones to merely mark your footsteps, use them intelligently to count how many times you have crossed a lane. The method should work for *any* maze, even if it has tunnels and bridges.