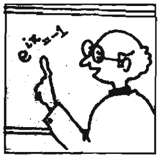
Classroom



In this section of* Resonance, *we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

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Tennis Ball Flight under Strong Wind

Predicting the path of flying objects is the most exciting aspect of sports. Tennis ball cricket offers plenty of excitement when played under *windy* conditions. This force of wind on flying objects called *drag* affects the flight path significantly. In high school physics and college mechanics, we solved many interesting problems without considering wind effects.

Here, we discuss how winds bring in many mathematical complications, and how physics helps us to simplify matters for developing the tennis ball theory under strong winds, or TBTSW for short.

Suppose a batsman hits a tennis ball. Let the ball velocity be $\mathbf{v}^{bg}$, where superscripts denote the velocity of the ball with respect to the ground. Assuming a steady wind velocity $\mathbf{v}^{wg}$, the relative velocity of the ball with respect to the wind is $\mathbf{v}^{bw} = \mathbf{v}^{bg} - \mathbf{v}^{wg}$. In general, $\mathbf{v}^{bg}$ and $\mathbf{v}^{bw}$ have all three components while $\mathbf{v}^{wg}$ has only two components parallel to the cricket ground.

The drag force varies as the square of the relative velocity magnitude and acts in the direction opposite to the vector $\mathbf{v}^{bw}$. In vector notation, the drag expressed per unit mass is:

$$\frac{\mathbf{D}}{m} = -k\left(v^{\text{bw}}\right)^2 \frac{\mathbf{v}^{\text{bw}}}{v^{\text{bw}}} = -kv^{\text{bw}}\mathbf{v}^{\text{bw}}$$

In the above equation k is a drag coefficient, which depends on the shape, size, orientation, speed and texture (fur, moisture, etc) of the flying object. It is convenient to introduce a standard wind speed c_0, which produces a drag equal to the ball weight. For a standard 60g tennis ball c_0 is about 25 m/s. Thus,

$$\mathbf{D} = -mg\frac{v^{\text{bw}}}{c_0^2}\mathbf{v}^{\text{bw}}$$

We are now armed to attack the vector equation of motion:

$$m\mathbf{a}^{\text{bg}} = -(\mathbf{D} + m\mathbf{g}),$$

where $\mathbf{a}^{\text{bg}}$ is the acceleration vector.

In order to fix the directions of the unit vectors, we take the batsman as the origin of co-ordinates (X,Y,Z). The X-axis is along the pitch, Y-axis is pointing in the direction of the leg umpire, and Z-axis pointing to the sky above the batsman. The wind vector is $\mathbf{v}^{\text{wg}} = -(\mathbf{i}\, v_x^{\text{wg}} + \mathbf{j}\, v_y^{\text{wg}})$ and the acceleration due to gravity is $-\mathbf{k}g$.

The vector equation of motion looks deceptively short and simple, but wait until you read about all the complications caused by the **D** term! Expanding this equation into its scalar components.

$$a_x^{\text{bg}} = -g\frac{v^{\text{bw}}v_x^{\text{bw}}}{c_0^2}$$

$$a_y^{\text{bg}} = -g\frac{v^{\text{bw}}v_y^{\text{bw}}}{c_0^2}$$

$$a_z^{\text{bg}} = -g\left[1 + \frac{v^{\text{bw}}v_z^{\text{bw}}}{c_0^2}\right].$$

Clearly, there is no hope for an easy solution considering that hardly anything is known about the variation of $\mathbf{a}^{bg}$ and $\mathbf{D}$ with time in the three dimensional wind blown space. It seems like it is not good cricket! But, wait! For strong winds $\mathbf{v}^{bw} \cong -\mathbf{v}^{wg}$! Remember, we are developing TBTSW. Tennis ball cricket does not stop because of a gale or two! Now, under this gale force, the formidable nonlinear coupled equations bow down to simple decoupled linear equations:

$$a_x^{bg} = -g\frac{v^{wg}v_x^{wg}}{c_o^2}$$

$$a_y^{bg} = -g\frac{v^{wg}v_y^{wg}}{c_o^2}$$

$$a_z^{bg} = -g$$

It is indeed remarkable that strong winds make the going smooth by way of decoupling the maze of coupled non-linear differential equations into a docile set of three linear equations! To keep things even more simple, we assume that the wind is blowing along the direction of the pitch into the batsman along the negative X axis. Further, we assume the ball is hit either high over the bowler or pulled low over the leg umpire.

In the first case the ball balloons up over the bowler and the wind drags it back towards the batsman. Curious? Let us derive a formula to drive home this idea. The docile set of three equations becomes a sweet set of two:

$$a_x^{bg} = -g\left(\frac{v_x^{wg}}{c_o}\right)^2$$

$$a_z^{bg} = -g\,.$$

Omitting superscripts and integrating the equations

$$v_x = v_{x0} - \alpha g t$$

$$v_z = v_{z0} - g t,$$

where $\alpha = \left(\frac{v_x^{wg}}{c_0}\right)^2$; v_{x0}, v_{z0} – initial velocity components.

Integrating again and eliminating the time variable t gives the flight path. You may prove this to be given by the equation

$$\frac{(x-\alpha z)^2}{zv_{x0}-xv_{z0}} = \frac{2(\alpha v_{z0}-v_{x0})}{g}.$$

This equation represents a *tilted* parabola. The angle of tilt is given by $\arctan(1/\alpha)$ with respect to the ground. The wind effect is like playing on a mountain slope. Thus, strong winds and level playing grounds do not go together!

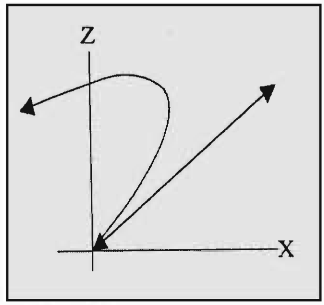


Figure 1.

Some typical trajectories are shown in *Figure* 1. Observe the strange case of the ball moving up and down a straight line. In this extraordinary situation the wind returns the ball back to the bat. This is not as odd as it seems when there is no wind. A ball thrown vertically up comes down to the same point eventually. Under light breeze conditions, this special angle is about 80 degrees, and under strong winds angles as low as 45 to 60 degrees are possible.

We shall conclude this class on TBTSW with a second example of a tennis ball hit low over the leg umpire. We are now interested in the way the ball drifts in the direction of the wind when viewed from the top in the XY plane. Therefore, we need only two equations.

$$a_x^{bg} = -g\left(\frac{v_x^{wg}}{c_0}\right)^2 = -g\alpha$$

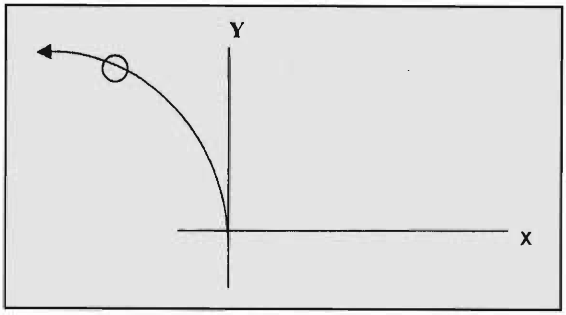
$$a_y^{bg} = 0.$$

Integrating after dropping superscripts

$$v_x = \alpha g t$$
$$v_y = v_{y0} = \text{const.}$$

In this case, the wind carries the ball behind the position of the leg umpire before the ball hits the ground. Looking from above,

Figure 2.



the ball sweeps a parabolic arc in the XY plane (*Figure* 2).

$$y^2 + \left(\frac{2\, v_{y0}^2}{ag}\right) x = 0.$$

So, there you are! Tennis ball cricket is a great spectacle for learning mechanics and mathematics. Of course, the same principles apply to shuttle badminton, soccer, frisbees and even diwali *parachutes* shot into the sky. For plate like objects like frisbee, aerobie and boomerang, however, we have to include the *lift* force. Lift forces are also generated for round shapes when they are spinning. Recall the famous soccer banana kick! Finally, do not forget all those wonderful diwali missiles with their own ingeneous devices for propulsion and guidance. These are *active* objects as opposed to *passive* balls and boomerangs.

In this class we learnt how winds complicate matters mathematically, and how simple sports sense and physics can peel off the complexity layer by layer to reach the juicy core of the problem. In fact, this was the way Timoshenko's teacher Prandtl unveiled his famous boundary layer theory to explain drag on flying objects in 1904 by simplifying the formidable nonlinear Navier–Stokes equations[1]. This class is just for starters. The real fun starts when strong winds subside into weak ones. Note that even when there is no wind there is always a drag on moving objects. You may try the exercises to learn more about air drag on a ball hit vertically up. These exercises also introduce you to a frame resembling the capital letter **A** to study projectile path. It will be nice to hear about your ground experiences while

Timoshenko: Father of Engineering Mechanics, *Resonance,* Vol.7, No.10, pp.2-3, 2002.
Ludwig Prandtl and Boundary Layers in Fluid Flow, *Resonance,* Vol.5, No.12, pp.48-63, 2000.

playing tennis ball cricket or soccer. So go out, play, learn and kick up some dust. But, make sure it does not boomerang in your face! Hurry up before television and third umpires corrupt this great Indian pastime.

Dedication: This article is dedicated to the memory of Indian cricket legend M L Jaisimha (1939-1999).

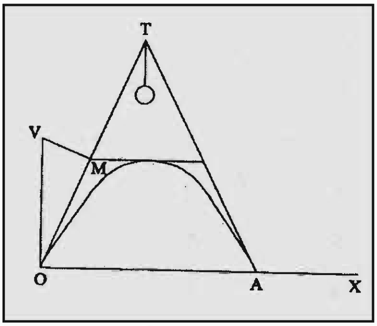


Figure 3.

Exercises

1) We can neglect wind and air drag for a cricket or a golf ball, and construct a frame using the letter **A** (*Figure* 3). In this figure, OV is the height attained when the ball is projected vertically. OM is the initial direction of flight. VM is the perpendicular bisector to OT. Notice the ball pendulum hanging vertically from the top when there is no wind. This pendulum represents the axis of symmetry for the **A** frame as well as for the parabolic flight. From basic physics and geometry, prove that this idea of the **A** frame really works!

2) Again neglecting air and wind drag, prove that a ball traces its *longest* path when launched at an angle of about 56.5 degrees. Note that 45 degrees gives the maximum range.

3) Here is an exercise to demonstrate air drag on a ball hit vertically up. Suppose a ball is launched vertically up with 20 m/s. It goes up to some height and falls back to the ground with a speed of 10 m/s. With this information, calculate the value of the drag coefficient k introduced in the first equation of this classroom. Suppose the ball keeps bouncing up and down. Assuming that there is no loss in energy during the bouncing process, prove the following:

(a) Speed of the ball after the *fifth* bounce is 5 m/s; and, after the *eighth* bounce is 4 m/s.

(b) Total distance traveled by the ball in going up and down just prior to the n^{th} bounce is $ln(1+3n)^{1/k}$.

(c) Time taken for the *first* ascent and *second* ascent (after

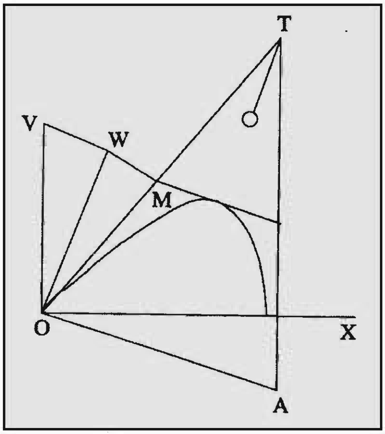


Figure 4.

bouncing once) are 1.2338 s and 0.8419 s, respectively; and, time taken for the *first* and *second* descents are 1.5493 s and 0.9241 s, respectively.

4) Develop the **A** frame idea to comprehend the effect of strong winds (*Figure* 4). Notice the pendulum is now tilted by the wind force. Otherwise, the **A** frame is similar to the previous case. Here, the angle VOW represents the angle of tilt. OW is the projection of OV on the new pendulum direction. Now WM is the perpendicular bisector of OT, the initial direction of flight. Prove all this!

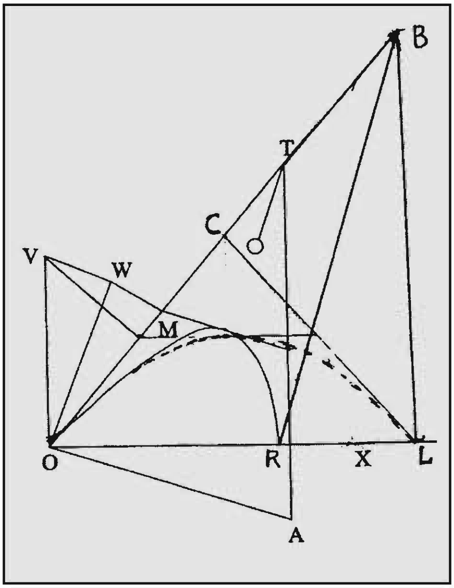
5) Under strong wind conditions, show that the horizontal range achieved by a ball hit at an angle θ with initial velocity v_a is:

$$R = \frac{v_a^2 \sin 2\theta}{g} (1 - \alpha \tan \theta) \cdot$$

Note: The range can be positive or negative due to wind!

6) Extend the **A** frame idea in Exercise 4 to obtain the range under windy conditions (*Figure* 5). In this figure, OL is the range neglecting wind effect. BL is perpendicular to the ground OX and BR is parallel to the ball pendulum. Prove that OR is geometrically equivalent to the range formula in Exercise 5.

Figure 5.



7) Show that the maximum range under strong wind is achieved when the sum of the angles of take-off and landing is 90 degrees, and further, that these two angles are the roots of cot $2\theta = \alpha$.

Note: When there is no wind effect(α = 0), the famous angle of 45 degrees is recovered.