The underlying recursion is the same as given earlier:

$$(a_{n+2}, b_{b+2}, c_{n+2}) = 4 \quad (a_{n+1}, b_{n+1}, c_{n+1}) - (a_n, b_n, c_n)\,,$$

with $(a_1, b_1, c_1) = (15, 1, 24)$ and $(a_2, b_2, c_2) = (56, 4, 90)$.

We have accomplished the task we set out to do. It is interesting to ask whether there are any solutions other than the ones given above.

A Geometric Dissection Problem

Find a way of cutting up a regular octagon into twelve congruent isosceles triangles and one square.

Solution

We shall show, more generally, how to cut up a regular $2n$-gon $\mathcal{P}$ into $n(n-1)$ congruent isosceles triangles and a regular polygonal figure $\mathcal{Q}$ with $n(n-3)$ sides; here $n \geq 3$. For $n > 4$ the inner polygon $\mathcal{Q}$ is non-convex. The given problem corresponds to the case $n = 4$; the inner polygon is a square in this case.

Let the vertices of the polygon be labelled in sequence 1, 2, 3, , $2n$; let s be its side. Construct $n(n-1)$ congruent copies of an isosceles triangle whose equal sides have length s and whose apex angle is $\theta = \pi/n$ radians.

Each interior angle of the polygon is $(n-1)\theta$, so $(n-1)$ of these triangles can be placed inside $\mathcal{P}$ without overlap with their apexes at vertex 1. This can be repeated for vertices 3, 5, .., $2n-1$ (alternate vertices only). In this way all the $n(n-1)$ triangles get 'used up', and there is no overlap; the central region left uncovered is a regular polygonal figure $\mathcal{Q}$ having $n(n-3)$ sides.

Sketches for the cases $n = 4$ and $n = 5$ are displayed in *Figures* 1 and 2 respectively.

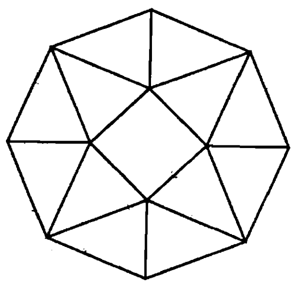

Figure 1. n=4.

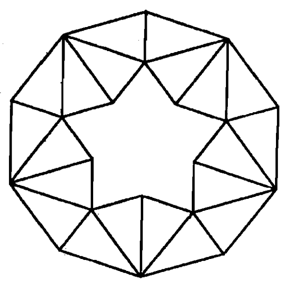

Figure 2. n=4.