The underlying recursion is the same as given earlier:

\[ (a_{n+2}, b_{n+2}, c_{n+2}) = 4 \ (a_{n+1}, b_{n+1}, c_{n+1}) - (a_n, b_n, c_n), \]

with \((a_1, b_1, c_1) = (15, 1, 24)\) and \((a_2, b_2, c_2) = (56, 4, 90)\).

We have accomplished the task we set out to do. It is interesting to ask whether there are any solutions other than the ones given above.

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**A Geometric Dissection Problem**

*Find a way of cutting up a regular octagon into twelve congruent isosceles triangles and one square.*

**Solution**

We shall show, more generally, how to cut up a regular \(2n\)-gon \(P\) into \(n(n-1)\) congruent isosceles triangles and a regular polygonal figure \(Q\) with \(n(n-3)\) sides; here \(n \geq 3\). For \(n > 4\) the inner polygon \(Q\) is non-convex.

The given problem corresponds to the case \(n = 4\); the inner polygon is a square in this case.

Let the vertices of the polygon be labelled in sequence 1, 2, 3, \ldots, \(2n\); let \(s\) be its side. Construct \(n(n-1)\) congruent copies of an isosceles triangle whose equal sides have length \(s\) and whose apex angle is \(\theta = \pi/n\) radians.

Each interior angle of the polygon is \((n-1)\theta\), so \((n-1)\) of these triangles can be placed inside \(P\) without overlap with their apexes at vertex 1. This can be repeated for vertices 3, 5, \ldots, \(2n-1\) (alternate vertices only). In this way all the \(n(n-1)\) triangles get ‘used up’, and there is no overlap; the central region left uncovered is a regular polygonal figure \(Q\) having \(n(n-3)\) sides.

Sketches for the cases \(n = 4\) and \(n = 5\) are displayed in *Figures* 1 and 2 respectively.