Stellar Masses

B S Shylaja

B S Shylaja is with the Bangalore Association for Science Education, which administers the Jawaharlal Nehru Planetarium as well as a Science Centre. After obtaining her PhD on Wolf-Rayet stars from the Indian Institute of Astrophysics, she continued research on novae, peculiar stars and comets. She is also actively engaged in teaching and writing essays on scientific topics to reach students.

Introduction

The twinkling diamonds in the night sky make us wonder at their variety – while some are bright, some are faint; some are blue and red. The attempt to understand this vast variety eventually led to the physics of the structure of the stars.

The brightness of a star is measured in magnitudes. Hipparchus, a Greek astronomer who lived a hundred and fifty years before Christ, devised the magnitude system that is still in use today for the measurement of the brightness of stars (and other celestial bodies). Since the response of the human eye is logarithmic rather than linear in nature, the system of magnitudes based on visual estimates is on a logarithmic scale (*Box* 1). This scale was put on a quantitative basis by N R Pogson (he was the Director of Madras Observatory) more than 150 years ago.

The apparent magnitude does not take into account the distance of the star. Therefore, it does not give any idea of the intrinsic brightness of the star. If we could keep all the stars at the same distance, the apparent magnitude itself can give a measure of the intrinsic brightness. This measurement by hypothetically keeping the stars at a distance of 10 parsec[1] is called the absolute magnitude. The Sun, if moved to that distance would have a magnitude of +5. Thus the determination of absolute magnitude demands the precise knowledge of distance.

[1] Parsec is a natural unit of distance used in astronomy. One parsec is the distance at which the radius of the Earth's orbit around the Sun subtends an angle of 1 arcsec.

The colour of stars may also be quantitatively measured by a similar method. Filters with established colour transmission characteristics are standardized and the starlight is measured through them. Depending on the colour of the star, the magnitudes measured will differ. For example, a blue star will register more flux in the blue (B) filter than in the green (V) filter; a red star – less flux in (B) than in (V). Generally, the magnitudes thus

Keywords
Stars, luminosities, masses, evolution.

Box 1. The Magnitude Scale

Let A and B be two stars, with a flux ratio 1:100. That implies that energy received on Earth per cm^2 per sec from the two stars, F_A and F_B are in the ratio $F_A/F_B = 100$. Now the magnitudes are defined as

m_A = magnitude of star A = $-2.5 \log F_A$
m_B = magnitude of star B = $-2.5 \log F_B$
$m_A - m_B = -2.5 \log F_A/F_B = -5.$

Therefore, if $m_A = 0$, $m_B = +5$ or if $m_A = 1$, $m_B = +6$.

If the numerical value of the magnitude is larger, the star is fainter.

The magnitude thus measured is called the apparent magnitude. Sirius, the brightest star has a magnitude of -1.4, Sun has a magnitude of about -26 and the full moon has -12. The magnitudes of planets vary from +2 to -4 depending on their distance. The faintest star detectable by the eye has a magnitude of about +6.

Published catalogues listing the magnitudes of over million stars are available.

measured in V are indicated as m_V or simply V. The difference of magnitude in the two filters is thus a measure of the colour i.e., $(B-V)$ will be negative for violet stars and positive for red stars. This is called the colour index.

For all these measurements α Lyrae or Vega (or Abhijit) is used as the primary standard. Its magnitude and colour index, are both set to zero.

The absolute magnitude and the colour index of a star can be used to derive the luminosity and the temperature of the star. Luminosity is the total energy radiated by a star in a second, which in the case of the Sun is 3.9×10^{26} Joules. This quantity is represented by $L_\odot$ and the luminosity of all other stars, L are expressed as a ratio to this. The fraction of this energy radiated from the Sun is received at the Earth in accordance with the inverse square law of distance. Thus the flux F received at Earth from a star whose luminosity is L is

$$F = L/4\pi d^2 = \text{flux received at Earth,}$$

where d is distance to the star.

A blue star is hotter than a yellow star, which, in turn, is hotter than a red star. The colour index changes from a negative value to a positive value in the same order.

This expression leads to $M-m = -5 \log d/10$, where M = absolute magnitude, m = apparent magnitude and d = distance in parsecs.

The colour index and the temperature are related. The hotter the star, greater the flux in the blue (short wavelength) region and the colour index is negative. A blue star is hotter than a yellow star, which, in turn, is hotter than a red star. The colour index changes from a negative value to a positive value in the same order.

One of the landmarks in understanding the stellar structure and evolution is the work carried out by Hertzsprung and Russell independently. They attempted to identify the distribution of stars in terms of the luminosity and temperature. This graph now finds a place in all textbooks of astronomy as HR diagram (*Figure* 1). The plot clearly demonstrated that majority of the stars occupy a region called the 'main sequence', which runs

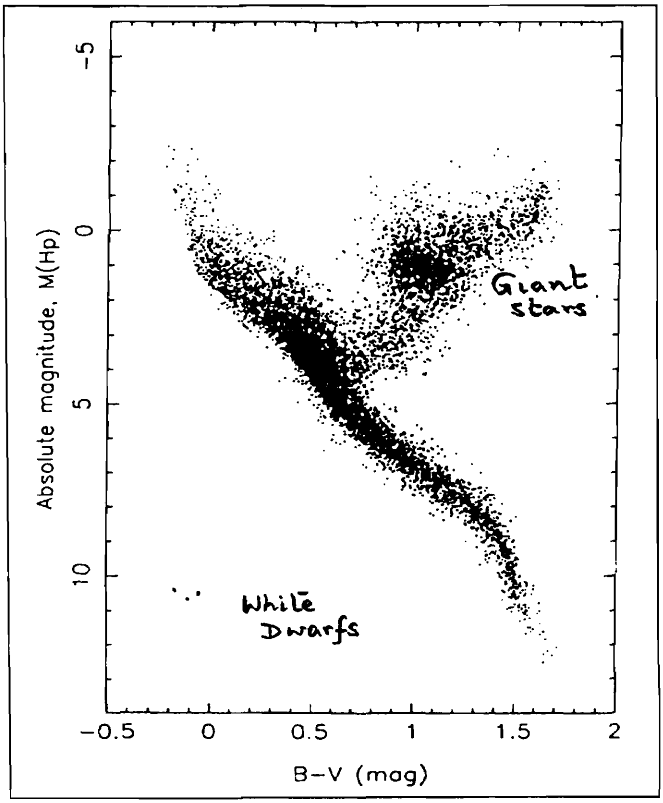


Figure 1. The HR diagram is the simple graphical representation of the variation of the absolute magnitude with the colour index. It also represents the variation of luminosity with temperature.
(Reproduced from James Binney and M Merrifield, Galactic Astronomy, Princeton University Press, New Jersey, 1988.)

almost diagonally on the graph. The fact that blue stars on the main sequence are more luminous than the red stars on it is clear from this graph.

Typically, a stellar spectrum shows a continuum, similar to a blackbody but crossed by hundreds of dark lines.

It is interesting to study how the temperature was linked to the observable parameters of the stars. Spectra of stars were regularly recorded since the 1880's. This was done by attaching spectrographs to the telescopes, the photographic plate acting as the detector. Typically, a stellar spectrum shows a continuum, similar to a blackbody but crossed by hundreds of dark lines. The spectra show an enormous variety with certain recurring patterns. This led to the classification of stellar spectra. The scheme of spectral classification was almost complete at the turn of the twentieth century. The lines in the spectrum had the clues on temperature hidden in them. We now know that the O, B, A stars, referred to as the early type stars, show ionized helium, helium and hydrogen lines in their spectra in that order; the later type stars namely F, G, K show hydrogen and metal lines. The classification into OBAFGKM is a sequence representing colour.

It was Meghnad Saha's ionisation formula which unfolded the secret of the spectral sequence. The formula was successfully applied to the astrophysical context by H N Russell. Cecilia Payne Gaposchkin extended the application to the large collection of spectra of Harvard. Thus the spectral classification which was earlier only a qualitative deduction was quantified as well.

It was Meghnad Saha's ionisation formula which unfolded the secret of the spectral sequence.

In the meanwhile, techniques had been evolved to measure the distribution of energy of the star at different wavelengths. A rough estimate of the temperature could be obtained with Wien's law, which relates the wavelength at the peak energy with the temperature of the star. Precise values were estimated by comparing with the theoretical curve of Planck. For this comparison, the theoretical models of the stellar spectrum accommodated different values of surface gravity to match with the observations. It was found that the temperature of the star as well as the surface gravity determined the spectrum. The spectrum also gave clues to what the stars are made of. Stars with peculiar chemical composition showed very different spectra. Thus, gradually, one was trying to look into the interiors of the star.

The Determination of Mass

A theoretical interpretation of the main sequence requires the knowledge of the basic parameter – the mass. To verify or to extend the theories of evolution to other stars we need to know their masses more precisely.

The stars may be regarded as isolated entities in the sky, keeping in mind the vast distances between them. The determination of mass therefore becomes a challenge. The mass of the Sun is the only precise estimate available. If we try to extend the same method as we used for the Sun, we need to look for the planets of these stars, a technique, which is still in its infancy.

As we extend our search beyond the solar system, we encounter a very interesting situation. The nearest star Alpha Centauri is a triple star system with components A, B and C. While A and B form a binary, the component C goes round the pair; during its routine journey it comes marginally close to the solar system. Hence it is called Proxima Centauri. Although it is so 'near', we cannot see it with naked eye. The other nearest star called Barnard's star is suspected to be having a companion. Sirius, at a distance of 8.6 light years also has a companion, the first white dwarf to be discovered observationally. At 8.9 light years there is another pair called L 726-8. Thus within a distance of 10 light years we encounter about 9 binary stars out of 14. Therefore, one can be optimistic about the estimation of the mass of these stars.

Only when the two components are gravitationally bound to each other, can they be called binaries.

However, all the doubles are not to be confused with optical pairs, which appear with a small angular separation by virtue of our line of sight. Only when the two components are gravitationally bound to each other, can they be called binaries. The two components are resolved in a telescope and one can measure the angular separation at different time intervals. The classic example is the optical pair ξ Ursa Majoris, (Mizar and Alcor, or Vasishta and Arundhati). Mizar itself is a binary with an orbital period of about 60 years. (*Figure* 2). The components A and B are, in turn, binaries. Among the examples mentioned above L 726 – 8 is also an optical pair.

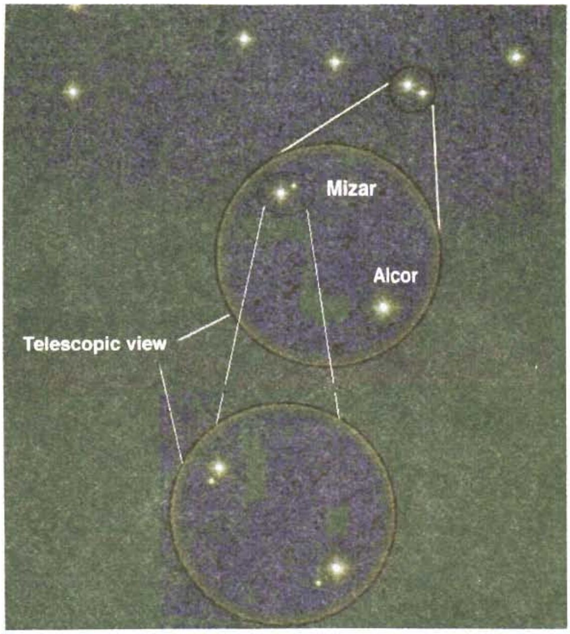


Figure 2. The best example of a double star is the Alcor–Mizar pair in the constellation of Ursa Major. The angular separation can be measured with a small telescope. Interestingly, Mizar is a complex system because each member of this binary itself is a binary.

The simplest case is that of a visual binary in which the components are close enough but get resolved through a telescope distinctly. It is possible to measure the inclination of the orbital plane with reference to the line of sight. One can apply Kepler's law to derive the masses (*Box* 2). This is true for a very small percentage of stars. The best examples are Alpha Centauri and Mizar. The Triple system Alpha Centauri has components A and B separated by about 24 AU (AU = the astronomical unit = the mean distance of the Earth from Sun = 150 million kms.), with masses 1.1 $M_\odot$ and 0.9 $M_\odot$. The third component, Proxima Centauri orbits around the center of mass (of A and B) at a distance of about 10,000 AU with a mass of 0.1 $M_\odot$.

There are other types of binary stars, which look like a single dot, even through the largest telescope.

More than two hundred years ago, a deaf and dumb boy called John Goodricke, first observed the 'eclipse' of a star.

The technique of monitoring a star continuously was used to identify the variability in light long ago. Thus, more than two hundred years ago, a deaf and dumb boy called John Goodricke, first observed the 'eclipse' of a star. This star Beta Persei was called Algol, meaning the 'devil's eye' because its brightness decreased suddenly quite often. This young man, keeping track of the light night after night, deduced that its light variation is a precise clock of about two and a half days. He postulated the presence of an unseen star treating the decrease of light as due to

Box 2. Mass Estimate in a Visual Binary

Among the binaries, the simplest is the visual binary (*Figure* A). The two components are resolved in a telescope and one can measure the angular separation at different time intervals. It is possible to measure the inclination of the orbital plane with reference to the line of sight. Now one can apply Kepler's law with the revised values of the angular separation.

$$(M_1 + M_2)\, T^2 = a^3 , \qquad (1)$$

where M_1 and M_2 are in solar masses, T in years and a in astronomical units.
The distance to the binary can be obtained with the measurement of parallax p as

$$a = a''/\,p''. \qquad (2)$$

The equation gets modified to

$$(M_1 + M_2)\, T^2 = (a''/p'')^3 \ . \qquad (3)$$

Further, we can estimate the individual masses from the relative positions of the stars with respect to the center of mass.

$$M_1 \,/\, M_2 = a_1 + a_2, \qquad (4)$$

where $a_1 + a_2 = a$.

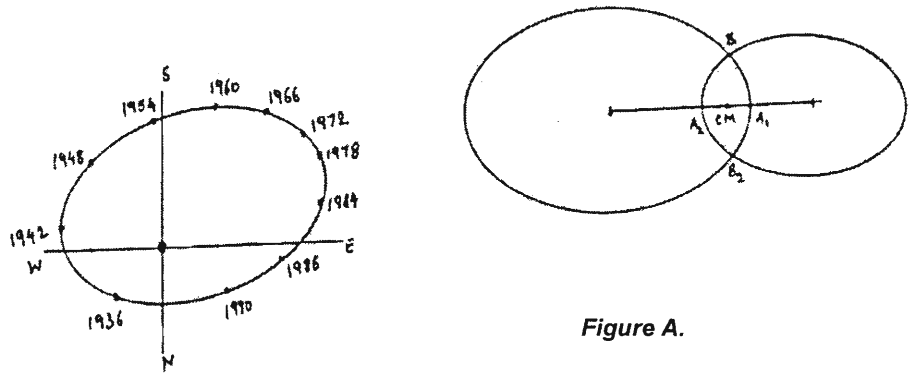


Figure A.

a periodical 'eclipse'. Little did he realize that he was venturing into a new avenue.

The light variability due to eclipse is an important tool today to derive the masses. Depending on the orientation of the orbital plane, the binary stars may render a total eclipse or a partial eclipse for us (*Figure* 3).

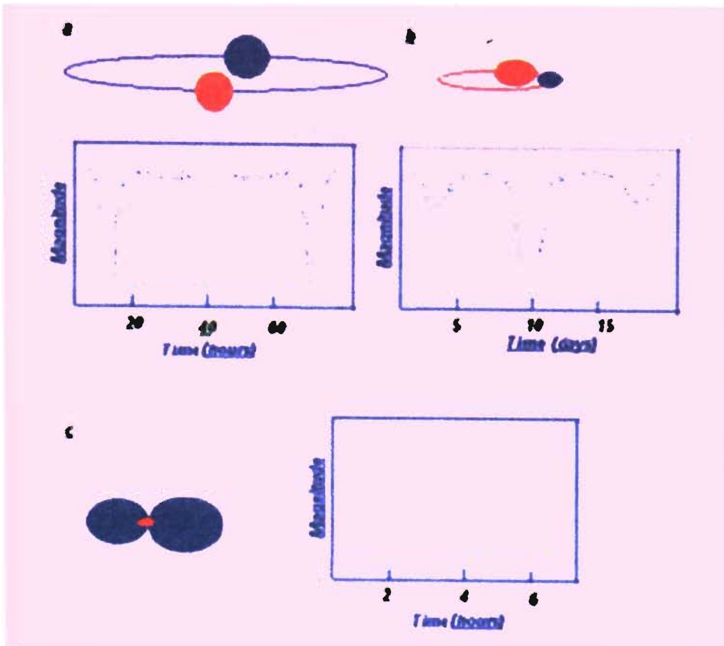


Figure 3. The eclipsing binaries show a periodic light variation, which can be interpreted in terms of the orbital parameters. Here the typical light curves of Algol, (Beta Persei), Beta Lyrae and W Ursa Majoris are shown. The nature of the orbits are reflected in the shape of the light curves.

The Doppler effect is another important tool for investigating binaries, especially those, which do not show eclipses. The periodic blue and red shift of the spectral lines give a clue on the nature of orbit (*Figure* 4). The mass can be derived as a function of the orbital inclination.

Thus, a reliable estimate of the mass is available only for the binary star systems. An interesting application of this tech-

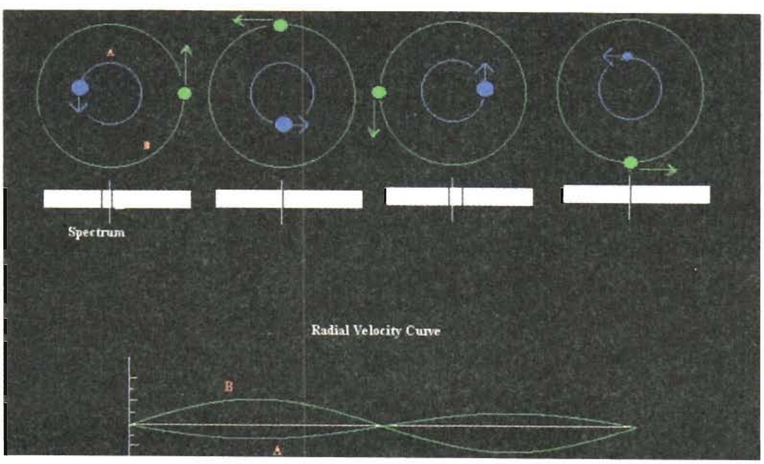


Figure 4. The spectroscopic binaries show periodic variation in the Doppler shifts of the spectral lines. In this diagram stars A and B revolve around the center of mass in a circular orbit. The spectrum shows the doubling of a spectral line, when they appear to move in opposite directions with reference to the observer. Here, the shifts are exaggerated. The spectral shifts corresponding to four configurations yield the velocities in the radial direction. The variation of the radial velocity with for the two components is also shown.

nique confirms the existence of compact objects, which otherwise remain hypothetical entities. For example, a star like SS Cyg shows light variations clearly indicating the phenomenon of eclipse. The spectral variations also have been recorded. The well established techniques help us to estimate the masses of the components as 1.1 $M_\odot$ and about 0.6 $M_\odot$. This, combined with other information, will give a clue on the nature of the smaller companion – it has to be a compact object. In this case it is a white dwarf. Similarly, several X-ray sources show eclipses and periodic spectral variations. The calculations reveal that one of the components of such binaries is a neutron star.

A further extension of this technique takes us to the elusive case of black holes. The absence of a detectable massive companion in an X-ray binary compels us to invoke the presence of a black hole. As it stands now, these binaries are the only confirmation of the existence of a black hole.

The absence of a detectable massive companion in an X-ray binary compels us to invoke the presence of a black hole. As it stands now, these binaries are the only confirmation of the existence of a black hole.

Thus the detection of all these compact objects (if I may be permitted to use the word object for a black hole) has been possible only in binary systems.

The Mass-Luminosity Relation

The measurement of masses of many eclipsing binaries soon gave way to an empirical relation. Since a star behaves like a blackbody, its luminosity can be written as

$$L = 4\pi R^2 \sigma T^4,$$

where R is its radius and T is the equivalent blackbody temperature of the star as deduced from its continuous spectrum.

Arthur Eddington observed that the stars on the main sequence whose distances and masses were both known, obeyed a relation

$$L \propto M^\alpha,$$

where α is an exponent.

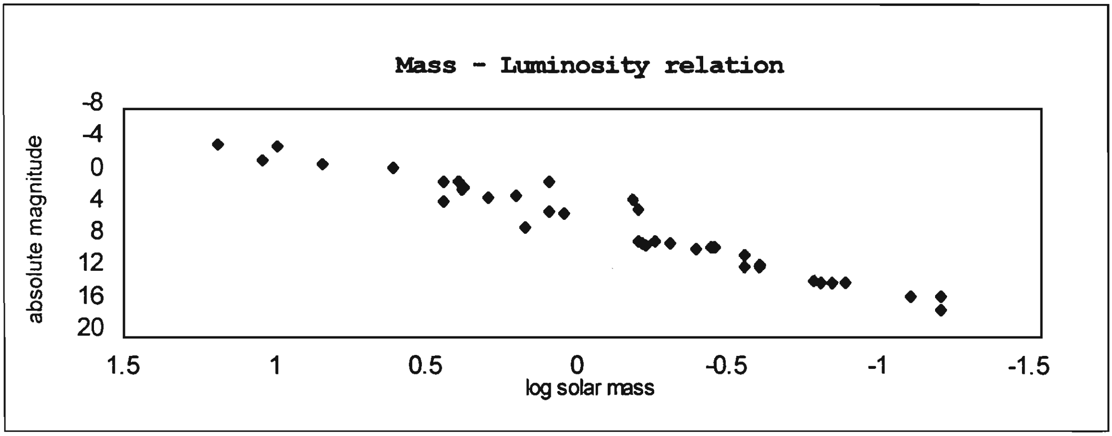
Based on the theoretical considerations, Eddington derived the rate of leakage of photons from the photosphere of a star and showed that luminosity is proportional to the fourth power of mass. The inference could now be generalized that the stellar spectral sequence of O B A F G K M also represented a sequence in mass. Thus O, B stars, the most luminous are also the most massive on the main sequence; while M stars, the least luminous are also the lightest.

It is nuclear fusion in the hot interiors that provides the energy to the stars.

The list of stars with precise estimates of mass and luminosity grew and the value of α could be fixed quite precisely. Interestingly, it was found that the value of α was different for different ranges of mass. For $1 < M/M_{\odot} < 10$, the value of α was $3 < \alpha < 4.5$ (*Figure* 5). This curve represents the mass luminosity relation. The data points are collected from the various sources that are cited in [1].

Eddington's work showed that stars in hydrostatic and radiative equilibria should obey relation of this type. Long after W Huggins had declared that the stars are all distant 'Suns', the mechanism of energy production remained a mystery. H Bethe and others nearly seventy years ago, came out with the right answer – it is nuclear fusion in the hot interiors that provides the energy to the stars. Today, we know that the proton-proton chain is the most important set of reactions prevalent at the cores of stars like the Sun at lower temperatures. The CNO cycle operates in the cores of massive stars at higher temperatures.

Figure 5. The mass-luminosity relation appears linear; a careful study shows the difference in the upper and lower regions. The data points have been collected from various references.



Normal stars are in both thermal and hydrostatic equilibrium. Hydrostatic equilibrium is inferred from the balance between the inward gravitational force and the outward pressure. If one of the two is greater, the structure of the star will not be stable.

Here carbon, nitrogen and oxygen act as catalysts to convert protons into helium nuclei.

Normal stars are in both thermal and hydrostatic equilibrium. Hydrostatic equilibrium is inferred from the balance between the inward gravitational force and the outward pressure. If one of the two is greater, the structure of the star will not be stable. Thermal equilibrium implies that the rate of production of energy is equal to the loss (or dissipation) of energy. The exact equations can be written down after defining the mechanism of energy transport (which may be radiation or convection) as well as the opacity. The latter is decided by the various atomic processes prevalent in the star. For example, it may be electron scattering, free-free absorption, bound-free absorption, etc. These processes act like hurdles. The photon, which is generated at the core of the star, finally makes its way to the photosphere, after several encounters with these hurdles. The photosphere is not the 'end' of the star. It is the region where the photon has a long mean free path and a high probability of escape.

The 'walk' of the photon to the photosphere is a combination of various complicated processes. (See *Box* 3 for deriving the relation between the photon diffusion time and the mean free path.) The mean free path can be calculated in the stellar interior by assuming a certain physical process. Keeping aside the mathematical details, it is possible to show that l, the mean free path, varies as a function of temperature (T) and density (ρ) in low mass stars; but is dependent only on density in high mass stars,

$$l \propto T^{3.5}/\rho^2 \quad \text{for low mass stars}$$
$$l \propto 1/\rho \quad \text{for high mass stars.}$$

In deriving the mechanism of energy transportation, details of the pressure, temperature and density gradients inside the star are necessary. Thus incorporating the differences in the structures it turns out that the mass-luminosity variation will have different types of indices for different mass ranges.

Box 3. Random Walk of Photons

Photons from the core of a star radiate out (*Figure* B). However, they have a good number of hurdles on the way like absorption, scattering, emission, etc. Thus, a photon gets degraded in energy by the time it reaches the photosphere. From simple principles of statistical mechanics, one can derive the time taken for this 'random walk'.

Let the position of the photon change from X_1 to X_{N+1} after N random steps. If we represent the change from X_1 to X_2 as $\langle X_1\rangle$ and from X_2 to X_3 as $\langle X_3\rangle$, keeping in mind that there is a change in the direction involved, it can be easily seen that

$$\langle X_{N+1}\rangle = \langle X_N\rangle = \langle X_{N-1}\rangle = \ \dots\ = \langle X_1\rangle = 0.$$

However, if we take the square of this quantity,

$$\langle X_{N+1}\rangle^2 = (X_{N+1} - X_N)^2, \text{ and so on,}$$

$$\langle X_{N+1}\rangle^2 = ½\,\langle(X_N - l)\rangle^2 + ½\,\langle(X_N + l)\rangle^2$$
$$= X_N^2 + l^2.$$

This is true because $\langle X_N\rangle = 0$.

Extending the same argument we can show that

$$\langle X_N\rangle^2 = N\,l^2.$$

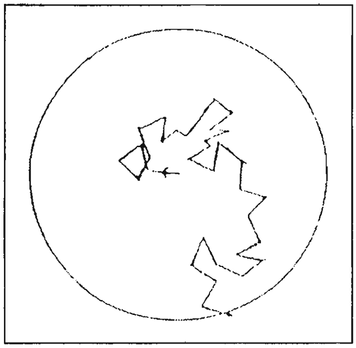


Figure B

Therefore,

$$l = (\langle X_N^2\rangle / N)^{1/2}$$
$$\text{or } N = \langle X_N^2\rangle / l^2.$$

This argument can be extended to three dimensions to show that

$$N = \langle X_N^2\rangle / l^2 + \langle X_N^2\rangle / l^2 + \langle X_N^2\rangle / l^2$$
$$= 3\,R_0^2 / l^2.$$

The time required for N steps is,

$$t = Nl/c = 3\,R_0^2 / lc.$$

It may be shown that in case of the Sun the time required for a photon to reach the photosphere is about 10^4 years. This path traced out by the photon is called random walk.

Thus the mass dependence of the luminosity, as well as the observed fact that it will be different in different mass ranges led to the understanding of the interiors of stars.

The 'life' of a star, i.e. the time during which hydrogen burning is in progress is decided by the rate of reaction itself.

The Meaning of Mass-Luminosity Relation

For qualitative discussion let us ignore the difference in the indices and assume that

$$L \propto M^4.$$

Now let us understand the implication. Let us consider a star, which is ten times as massive as the Sun ($10\,M_\odot$). Its luminosity therefore will be 10,000 times that of the Sun. ($L/L_\odot = (M/M_\odot)^4 = 10^4 = 10{,}000$).

This also implies that the star burns its fuel 10,000 times faster than the Sun. This means that the mechanism of energy production itself will have to be different. As we know today, two schemes of nuclear reactions are prevalent in stellar interiors – the CNO cycle and the *p-p* reactions. The rates of energy production have been derived in the two cases. The rate varies as T^{18} in the CNO cycle, while in the other it varies as T^4. The interior temperatures of the early spectral type O, B stars are higher than those of the later spectral type stars, e.g. K, M etc. Hence the rate of energy production also will be high in the early type stars. Therefore they are the most luminous, they are also much hotter on the surface. Quite naturally, their colour is blue.

By a simple logic we can arrive at the life span of a star. Although stars are known to be holding large amount of hydrogen, which serves as fuel for the thermonuclear reactions, they cannot survive to consume the entire fuel. By the time about 10% of the fuel is consumed, evolutionary processes take over. Therefore, the 'life' of a star, i.e. the time during which hydrogen burning is in progress is decided by the rate of reaction itself. This is the youth of the star and it is said to be on the main sequence. The later stages of evolution into a red giant differs considerably. It may lead to the formation of a white dwarf via a planetary nebula phase (for stars like the Sun) or the formation of a neutron star via a supernova explosion (for massive stars). The exact course is decided by the mass again.

The energy produced is proportional to the mass available at the core. $E \propto M$; mass in turn is proportional to the luminosity as $L^{1/4}$. Therefore, one can approximately write,

Life span = energy produced/rate of energy leakage
$= E/L \propto M/M^4 \propto M^{-3}$.

Out of the 10% of the mass that is used up, only 0.7% is converted in to energy. Therefore, the time for which the star lives (or it is on the main sequence) is given by

$$t = 0.007 \times 0.1 \times Mc^2 / L,$$

where L is the luminosity.

This can be expressed in terms of the solar quantities as

$$t = [(M/M_{\odot})/(L/L_{\odot})] \times 10^{10} \text{ years}.$$

Thus the Sun 'lives' for 10 billion years, while a star of mass $30M_{\odot}$ will 'live' for 2 million years only. This is the basis of the generalised statement – *larger star is more luminous and hence shortlived* – found in all astrophysics books. This sets a limit on the analogy of stellar evolution with that of biological systems. This is a situation exactly opposite to that in biology – larger animals live longer than smaller animals.

The most massive stars are of spectral type O, with predominant lines of ionized helium in their spectra.

The Upper and Lower limits

From the basic thermodynamic principles, we can deduce the average thermal gas energy per particle. In massive stars, radiation pressure is very large compared to gas pressure. We can arrive at an upper limit of the mass based only on this consideration. Not many stars are seen in the mass range > 50 $M_{\odot}$. Hydrodynamic considerations show that stars with masses higher than this pulsate and therefore, are unstable.

The most massive stars are of spectral type O, with predominant lines of ionized helium in their spectra. Among them a particular group of stars are designated Wolf–Rayet (WR) stars. These show emission lines in their spectra unlike the normal stars (which show absorption lines). They were recognized as a

The life span (the duration on the main sequence lifetime) of the massive stars is quite small, in accordance with evolutionary calculations. This is of the order of a few million years.

special group, more than 100 years ago, by the two astronomers, whose names are attached to them. The mass, temperature and luminosity are all the highest for this class. Therefore, they have been identified at farthest distances in our Galaxy, as well as in other galaxies.

Most of the WR stars are associated with nebulosities surrounding them. The mass loss rate also is quite high about $10^{-6}\,M_{\odot}$ per year. (The Sun also loses mass in the form of solar wind at a rate which is almost a million times lesser.) This gives us an important parameter to be included for the evolution of massive stars – the mass loss.

As seen earlier, the rate of energy production is proportional to T^{18} for very massive stars. This implies that the life span (the duration on the main sequence lifetime) of the massive stars is quite small, in accordance with evolutionary calculations. This is of the order of a few million years. Therefore, the shortlived WR stars cannot be seen in 'old' regions. Even in the Milky Way they are associated with the young open clusters, most of which are in the spiral arms. The implications are multi faceted – that spiral arms are the regions where star formation is in progress, that those galaxies which contain WR stars have been forming the stars in the recent past (a few million years) and so on.

At the other extreme, we have very low mass stars with very low luminosities. Detecting them is indeed very difficult. Owing to their low temperature and low mass, their energy production rate also is very low. Thus they live longer than 10 billion years. However, the low luminosity limits the detectability to very small distances.

It is possible to show theoretically that a star will not be able to maintain nuclear reaction if its mass is lower than $0.08M_{\odot}$. Such objects are called brown dwarfs. There may be many more stars of this mass range, but it is difficult to see them.

The mass luminosity curve looks scattered in the lower mass region, owing to observational difficulties. It is here that we can take a clue from Jupiter. It emits more energy than it receives from the Sun – only in the infrared. This is because the stored heat is slowly radiating out. Therefore, IR observations will provide a powerful tool in detecting small stars, which are large in number.

IR observations will provide a powerful tool in detecting small stars, which are large in number.

The technique of measuring Doppler shifts and measuring the infrared emission are being used now to detect the planets around other stars – the extra-solar planets. Gravitational lensing technique also is being explored for this purpose.

More than 30 cases of positive detection exist as on date. The programs, which started decades ago, have yielded results today. As the list of extra-solar planets is growing, precise estimates of masses of stars become available. That reduces the scatter in mass-luminosity curve and leads to a better understanding of the interiors of the stars.

Thus, a simple empirical relation between the mass and luminosity of the stars resulted in a clearer understanding of the physics of the stars. Further, it offers the best way of understanding the structure of our Galaxy as well as other galaxies, in terms of the prevalent star formation rate.

Acknowledgement

I am grateful to the referee, whose comments have greatly helped in improving the presentation. It is a pleasure to thank Mr Ananthakrishna, for creating the graphics.

Suggested Reading

[1] D M Popper, *Annual Reviews of Astronomy and Astrophysics*, Vol. 18, p. 115, 1980.
[2] K D Abhyankar, *Astrophysics – Stars and Galaxies*, Tata McGraw Hill, 1982.
[3] J M A Danby, R Kouzes and C Whitney, *Astrophysics simulations*, CUPS software, John Wiley and Sons 1995.
[4] F Shu, *The Physical Universe – An introduction to astronomy*, University Science Books, 1982.

Address for Correspondence
B S Shylaja
Bangalore Association for Science Education
Jawaharlal Nehru Planetarium
Bangalore 560 001, India.
Email: taralaya@vsnl.com