Life and Work of Kurt Gödel – A Brief Sketch

Kurt Friedrich Gödel, perhaps the greatest logician of all time, was born on 28 April 1906 at Brünn to Rudolf Gödel and Marianne Handschuh in the Austro-Hungarian province of Moravia (later absorbed in Czechoslovakia).

Gödel's special intellectual talents emerged early. In the family, Kurt was called *Herr Warum* (Mr. Why) because of his constant inquisitiveness. At the age of six, he was enrolled in the Evangelische Volksschule, a Lutheran school in Brünn. From 1916 to 1924, Kurt carried on his school studies at the Deutsches Staats-Realgymnasium, where he showed himself to be an outstanding student, receiving the highest marks in all subjects; he excelled particularly in mathematics, languages and religion. Following his graduation from the Gymnasium in Brünn in 1924, Gödel went to Vienna to begin his studies at the university. There he hardly ever spoke, but was very quick to understand problems and to point the way through solutions. It became evident that he was exceptionally talented.

At the University of Vienna, Gödel came in touch with several eminent mathematicians including Hans Hahn, Karl Menger, Eduard Helly, Walter Mayer and Leopold Vietoris. Hahn was his principal teacher, who also introduced him to the group of philosophers around Moritz Schlick, holder of the chair in the Philosophy of Inductive Sciences. Schlick's group was later baptized the 'Vienna Circle' and became identified with the philosophical doctrine called logical positivism or logical empiricism. However, Gödel developed strong philosophical views of his own which were, in large part, almost diametrically opposed to the views of the logical positivists.

In 1927, he met his wife to be, Adele Nimbusky, who was six years older than him. Mainly due to objections from his parents, Gödel could marry her only in 1938.

In 1928, *Grudzüge der theoretischen Logik* by David Hilbert and Wilhelm Ackermann was published. This publication seems to have the most direct influence on Gödel in his choice of direction for creative work. Posed as an open problem in the book was the question whether a certain system of axioms for the first-order predicate calculus is complete. In other words, does it suffice for the derivation of every statement that is logically valid (in the sense of being true under every possible interpretations of its basic terms and predicates). Gödel gave a positive solution to the completeness problem. One of the impacts of this theorem was that it clarified what one means by a 'proof' which till then was understood only vaguely (see Srivastava's article, p.29). With this notable achievement he commenced his research career and came to be regarded as a rising star.

The ten years 1929-39 was a period of intense work which resulted in Gödel's major achievements in mathematical logic and set theory. In 1931, he published his famous *incompleteness theorem* (also see article on p.22 by Vijay Chandru in this issue): in any formal system S in which a certain amount of arithmetic can be developed and which satisfies some minimal consistency conditions, one can construct an arithmetical statement A such that neither A nor its negation is provable in S. In particular, no fixed system, no matter how complicated could represent the complexity of the whole numbers 0, 1, 2, ... This result of Gödel had a stunning effect upon logicians, mathematicians and philosophers interested in the foundations of mathematics. Modern readers may not be as nonplussed by this as readers of

1931 were, since in the interim our culture has absorbed Gödel's theorem, along with the conceptual revolutions of relativity and quantum mechanics, and their philosophically disorienting messages have reached the public. But back in 1931, this came as a bolt from the blue.

If the discovery of the incompleteness theorem shocked the world, its proof drew very special interest. The proof hinges upon the writing of a self-referential mathematical statement, in the same way as the so called *liar paradox*: "I am lying". Gödel had the insight that a statement of number theory could be about a statement of number theory or even about itself. This is done by making numbers stand for symbols and sequences of symbols. This way, each statement of number theory, being a sequence of symbols, is represented by a number itself. This coding trick enables statements of number theory to be understood in two ways: as statements of number theory and also as statements about statements of number theory. Now it requires only some subtle mathematical argument to write self-referential statements in number theory. This argument has been at the core of many very interesting results in logic, computer science and mathematics involving self-reference. The discovery and the proof of the incompleteness theorem led the world to recognize Gödel as a leading thinker of our time.

Gödel visited the Institute for Advanced Study, Princeton thrice during 1933-40 before taking up permanent residence there in 1940. During this period, he concentrated mainly on set theory. His intense efforts finally bore fruits in the summer of 1937 when he established that both the axiom of choice and the continuum hypothesis are consistent with the Zermelo–Fraenkel axioms without the axiom of choice (ZF). In particular, these statements cannot be disproved from them if the axioms of ZF are consistent. With the modest techniques then available, the details that Gödel needed to establish were formidable. This deep and complicated work caused him much effort, especially in its final parts.

At the Institute for Advanced Study, he continued to grapple with the difficult problems in set theory and at the same time began to think and write in-depth about the philosophy of mathematics. Later in the 1940s he arrived at his unusual but less well-known contributions to relativistic cosmology, in which he produced solutions of Einstein's equations permitting 'time travel' into the past. Gödel published comparatively little but always to maximum effect; his papers are models of precision and incisive presentation.

Gödel had recurrent bouts of mental depression and exhaustion. Towards the end, he became very paranoid. He had an abiding distrust of doctors' advice. He developed fears about being poisoned and would not eat. He passed away on 14 January 1978 of "malnutrition and inanition caused by personality disturbance", leaving behind him an intellectual achievement which will be difficult to surpass in its beauty or depth or impact.

S M Srivastava
Stat-Math Unit
Indian Statistical Institute
Kolkata 700 035, India.